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Return on Investment in Contest

Yizhaq Minchuk
Industrial Engineering and Management Department, Shamoon College of Engineering

Baruch Keren
Department of Industrial Engineering and Management, Shamoon College of Engineering

Yossi Hadad
Department of Industrial Engineering and Management, Shamoon College of Engineering

Abstract

This paper considers a contest with one organizer and two (or more) contestants that compete to win a prize that is set by the organizer (the winner takes all). In the first stage, the organizer determines the amount of money he needs to borrow in order to establish the prize of the contest. In the second stage, each contestant determines his efforts in the contest. The contest prize is determined according to the abilities of all the contestants and the cost of the loan. It is shown that if the organizer borrows the optimal amount of money for the prize, in some situations this setup can yield 100% return on investment (ROI).

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Contact: Yizhaq Minchuk - yizhami@sce.ac.il, Baruch Keren - baruchke@sce.ac.il, Yossi Hadad - yossi@sce.ac.il.
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1. Introduction

Many studies focus on strategies in contests. In most of them the prize that was awarded to the winner had no cost to the organizer (see, for example, Konrad 2009). But in real life situations this is rarely true, and the organizer must borrow the money in order to award the prize. Even if the money for the prize is obtained from internal resources, this money always has alternative uses, so awarding a prize is never a costless activity. Another related issue is to set the right amount of money as a prize and to adjust the prize according to the contestants' abilities.

This study combines two elements in order to calculate the optimal size of the prize, the cost of the prize for the organizer and contestants' abilities. Many contest studies deal with the issue of the optimal amount of effort that the contestants should invest in the contest. Several studies tried to find the optimal strategy from the point of view of the organizer; specifically, they tried to determine how to build a contest that would increase organizer revenue. For example, Cohen and Sela (2005) presented a simple contest mechanism based on the Tullock success function, in which the organizer pays back the effort costs (reimbursing) to the winner. They showed that despite the cost of returning to the winner his effort costs, this would nevertheless increase the revenue of organizers when compared to the basic Tullock contest model. Matros and Armanos (2009) studied a more general reimbursement model in contests. They also showed that by reimbursing the winner's costs the organizers' profit would increase. In general, designing a contest setup with incentives to the contestants, in order to achieve some of the organizers' goals, is not an easy job (see Wang 2010). In the above models, the prize awarded to the winner imposed zero cost on the organizer and thus, the issue of return on investment was not analyzed.

Return on investment (ROI) is a performance measure that is widely used to evaluate the efficiency of an investment. Applications that used the ROI measure can be found, for example, in Millar and Hall (2013).

In this paper, the prize awarded to the winner in the contests is borrowed by the organizer, and the organizer must pay the proper amount of interest for the money he borrows. The size of the prize (the amount of money that the organizer borrows) is determined according to the interest rate and the contestants' abilities and efforts. Chung (1996) made a connection between the prize and the contestants' efforts. Chung considered a model where the prize is increased with respect to the aggregated effort. He showed that this form of prize will result in excessive efforts by the contestants, thereby generating social waste. Our analysis follows that research by showing that if the prize is determined according to the abilities of the contestants and the cost of the loan, the reward for the organizer is a multiplication of the loan cost by a positive constant, which yields a positive return on investment.

The balance of the paper is organized as follows: Section 2 introduces the model; Section 3 analyzes the case of two heterogeneous contestants; Section 4 focuses on several homogeneous contestants; Section 5 concludes the paper.

2. The Contest Model

Consider a contest with risk-neutral players: a contest organizer and n contestants. The contestants compete to win prize V, which is to be awarded by the organizer. The organizer borrows the money he needs to award the prize. The cost of the prize for the organizer is $c(V)$.

The expected revenue of the organizer, R, is the total efforts of the contestants exerted in the contest, minus the cost of the prize. Given

$$R = \sum_{i=1}^{n} x_i - c(V), \tag{1}$$

where x_i is the effort in the contest by contestant $i = 1,..,n$. We assume that the prize cost function has the following form: $c(V) = \gamma V^{\beta}$ where $\gamma > 0, \beta > 1$. A special case where $\gamma > 0, \beta = 2$ is the known incentive cost was developed by Baker et al. (1994). The cost of the prize is considered as a loan that the organizer takes in order to award the prize.

The expected utility of the contestants is modeled as a variation of the Tullock contest, given by

$$U_i = V P_i - a_i x_i, \tag{2}$$

where $a_i > 0$ is the ability factor of contestant $i = 1,..,n$; a low a_i value indicates high ability.

The winning probability (based on a Tullock contest) for a contestant i is given by

$$P_i = \frac{x_i}{\sum_{i=1}^{n} x_i}.$$

The stages of the game are as follows: In the first stage, the organizer determines the value of the prize (V). In the second stage, each contestant sees the value of the prize and sets his efforts in the contest, taking into the value of the prize, his ability, and the ability of the other contestant.

3. The case of two heterogeneous contestants

In order to analyze a subgame perfect equilibrium of this contest, a backward induction is used beginning with the second stage. The second stage is an asymmetric Tullock contest with a well-known solution of the maximization problem in (2) for $n = 2$ and $a_1 \neq a_2$. (See, for example, Nti (1999), Konrad (2009)). Thus, the efforts in the contest are

$$x_1 = \frac{\left(\frac{V}{a_1}\right)^2 \frac{V}{a_2}}{\left(\frac{V}{a_1} + \frac{V}{a_2}\right)^2} = \frac{a_2}{(a_1 + a_2)^2} V \tag{3}$$

$$x_2 = \frac{\left(\frac{V}{a_2}\right)^2 \frac{V}{a_1}}{\left(\frac{V}{a_1} + \frac{V}{a_2}\right)^2} = \frac{a_1}{(a_1 + a_2)^2} V, \tag{4}$$

and the utilities are

$$U_1 = \frac{\left(\frac{V}{a_1}\right)^3}{\left(\frac{V}{a_1}+\frac{V}{a_2}\right)^2} = \frac{a_2^2}{\left(a_1+a_2\right)^2}V \tag{5}$$

$$U_2 = \frac{\left(\frac{V}{a_2}\right)^3}{\left(\frac{V}{a_1}+\frac{V}{a_2}\right)^2} = \frac{a_1^2}{\left(a_1+a_2\right)^2}V. \tag{6}$$

Notice that when $a_i < 1$ the prize valuation to the contestants is higher than the valuation to the organizer.

Substituting (3) and (4) into (1) yields

$$R = \frac{a_2}{\left(a_1+a_2\right)^2}V + \frac{a_1}{\left(a_1+a_2\right)^2}V - \gamma V^{\beta}.$$

Thus, the maximization problem of the organizer is

$$\max_V R = \max_V\left(\frac{V}{\left(a_1+a_2\right)} - \gamma V^{\beta}\right). \tag{7}$$

Differentiating (7) with respect to V yields

$$\frac{\partial R}{\partial V} = \frac{1}{\left(a_1+a_2\right)} - \beta\gamma V^{\beta-1} = 0,$$

by rearranging

$$V^* = \sqrt[\beta-1]{\frac{1}{\beta\gamma}\left(\frac{1}{\left(a_1+a_2\right)}\right)}. \tag{8}$$

Note that $\frac{\partial^2 R}{\partial V^2} = -\beta\gamma\left(\beta-1\right)V^{\beta-2} < 0$, i.e., the second order condition for a local maximum is satisfied. It easy to see that one of the prize components is the sum of contestants' abilities. This means that one contestant with lower ability and one with higher ability may each face the same price as two contestants, each with mid-level abilities.

Note that if $c(V^*) = \gamma V^{*\beta} < V^*$ (a negative interest rate), the loan will not be given by the lender and the contest will not be carried out. To guarantee that the contest will be accomplished, the following inequality must be satisfied: $c(V^*) = \gamma V^{*\beta} > V^*$.

Namely, $V^* > \sqrt[\beta-1]{\frac{1}{\gamma}}$ and $\left(a_1 + a_2\right) < \frac{1}{\beta}$. In other words, the contestants must have high abilities such that $\left(a_1 + a_2\right) < \frac{1}{\beta}$ in order to satisfy $c(V^*) > V^*$. From this point on it is assumed that the contestants' abilities satisfy this condition and that the contest organizer can obtain the needed loan.

Substituting (8) into (7) yields

$$R = V^*\left(\frac{1}{\left(a_1 + a_2\right)} - \frac{\beta - 1}{\beta}\right). \tag{9}$$

Then, substituting (8) into (3,4,5,6) yields the contestant efforts and the utilities. It easy to see from (9) that the revenue to the organizer increases with respect to the sum of contestants' abilities. It is also easy to see by (9) that as long as the prize in the contest is determined by the contestants' abilities, the organizers will always have a positive expected revenue. The reason for that is because the model adjusts the prize according to the cost of the loan, the contestants' abilities, and the contestants' efforts in the contest.

Proposition 1: *The optimal expected revenue of the organizer in a contest with two heterogeneous contestants is*

$$R = \left(\beta - 1\right)c(V^*).$$

Proof: By (8) and (9) one sees that

$$R = V^*\left(\frac{1}{\left(a_1 + a_2\right)} - \frac{\beta - 1}{\beta}\right) = V^*\left(\frac{1}{\left(a_1 + a_2\right)} - \frac{\beta - 1}{\gamma\beta}\gamma\right) = V^*\left(\left(V^*\right)^{\beta-1}\left(\beta - 1\right)\gamma\right) = \left(\beta - 1\right)\gamma\left(V^*\right)^{\beta}.$$

Note that $\gamma\left(V^*\right)^{\beta} = c(V^*)$, which proves the proposition. ■

Corollary: *The return on investment in the contest is* $(\beta - 1)$

Proof: The return on investment defined as the expected revenue divided by the loan cost. So, by applying Proposition 1 the result, $\frac{R}{c(V^*)} = \beta - 1$, is obtained. ■

Thus, the net revenue of the organizer is $(\beta - 1) > 0$, multiplied by the amount of the loan he took in order to grant the prize. Additionally, it is easy to see that total effort equals $x_1 + x_2 = \beta c(V)$. If, for example, $\beta = 2$, as proposed by Baker et al. in 1994, and if the contestants have high abilities, the organizer's return on investment (the loan) is 100%.

4. The case of several homogeneous contestants

This section considers a contest with n contestants that have identical abilities $\left(a_1 = a_2 = ... = a_n = a\right)$.

By applying the backward induction approach that was used in the previous section, the solution of (2) is the well-known symmetric equilibrium of a Tullock contest (see, for example, Konrad, 2009). The efforts and the utilities are

$$x_i = \frac{n-1}{n^2}\frac{V}{a} \quad i = 1,...,n \tag{10}$$

$$\mathrm{U}_i = \frac{V}{n^2} \quad i = 1,...,n \ . \tag{11}$$

Substituting (10) into (1) yields:

$$R = \frac{n-1}{n}\frac{V}{a} - \gamma V^{\beta}. \tag{12}$$

Differentiating (12) with respect to V yields: $\frac{\partial R}{\partial V} = \frac{n-1}{an} - \beta\gamma V^{\beta-1} = 0$.

Rearranging the derivative gives:

$$V^* = \sqrt[\beta-1]{\frac{1}{\beta\gamma}\left(\frac{n-1}{an}\right)}, \tag{13}$$

As mentioned, the contest will be accomplished only if $c(V^*) = \gamma V^{*\beta} > V^*$. To ensure that (13) fulfills this condition, it is assumed that the contestants' abilities satisfy $a < \frac{n-1}{n\beta}$.

The second derivative is $\frac{\partial^2 R}{\partial V^2} = -\beta(\beta-1)\gamma V^{\beta-2} < 0$. Thus, the second order condition for a local maximum is satisfied. Substituting (13) into (12) yields:

$$R = V^*\left(\frac{n-1}{an}\cdot\frac{\beta-1}{\beta}\right). \tag{14}$$

Proposition 2: *The optimal expected revenue of the organizer in a contest with n homogeneous contestants is:*

$$R = \left(\beta-1\right)c(V^*).$$

Proof: By (13) and (14) one sees that

$R = V^*\left(\frac{n-1}{an}\cdot\frac{\beta-1}{\beta}\right) = V^*\left(\frac{n-1}{an}\cdot\frac{\beta-1}{\gamma\beta}\right)\gamma = V^*\left(V^*\right)^{\beta-1}(\beta-1)\gamma.$ Note that $\gamma\left(V^*\right)^{\beta} = c(V^*)$, which proves the proposition. ■

As in the section above, it easy to see that total effort equals $\sum_{i=1}^{n} x_i = \beta c(V)$. Namely, the organizer's net revenue is $(\beta-1) > 0$ times the loan he took in order to grant the prize.

5. Conclusions

This paper presents the contest mechanism as an investment opportunity. The contest organizer can borrow money to award a prize, pay the loan and its extra costs, and end the contest with a profit and a positive return on his investment, which is exactly $(\beta - 1) > 0$ times the cost of the loan he takes. As long the organizer optimally determines the prize according to the proposed model and the abilities of the contestants are high enough to enable the loan being given, he can expect that his contest organizing activity would yield him a profit. This profit depends on the cost of the loan (low values of γ, β mean a cheaper loan) and on the abilities of the contestants (low values of a_i mean higher abilities). The proposed model is simple to operate and can therefore be useful in many managerial applications where incentives are needed and offered in order to urge workers and contestants toward a desired result.

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