Economics Bulletin

Volume 38, Issue 4

Market foreclosure, output and welfare under second-degree price discrimination

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Abstract

We compare second-degree price discrimination with uniform pricing using two linear demands, with and without the Spence-Mirrlees condition. We find that second-degree price discrimination can result in a welfare-enhancing market foreclosure (one market is excluded under second-degree price discrimination when both markets would be served under uniform pricing) because the resulting foreclosure can increase both the total output and the total surplus. Moreover, the total surplus under second-degree price discrimination could also be lower without the foreclosure. Our results are in stark contrast with the well-known results related to output and welfare effects under third-degree price discrimination.

We would like to thank the Editor, the Associate Editor and two anonymous referees for valuable suggestions that have significantly improved this paper

Citation: Yong Chao and Babu Nahata, (2018) "Market foreclosure, output and welfare under second-degree price discrimination", *Economics Bulletin*, Volume 38, Issue 4, pages 2116-2127
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Submitted: September 19, 2018. **Published:** December 02, 2018.

1 Introduction

It is well known that when a monopolist charges a uniform price in two different markets, one market may not be served because the optimal uniform price is too high. However, the excluded market under uniform price will always be served under third-degree price discrimination. The opening up this previously not served market, because of third-degree price discrimination, not only is sufficient for the increase in total welfare, but it could also result in a Pareto improvement.[1] Without the market opening, the impacts of third-degree price discrimination on welfare and output are ambiguous under general demands.[2] But for linear demands in each market, the output remains unchanged and welfare is always lower compared to when the monopolist charges a uniform price.[3]

Firms also practice second-degree price discrimination in real life. One form is the quantity-payment packages on take-it-or-leave-it basis. For example, in many pizza chain stores, such as Little Caesars, Papa John's, and Pizza Hut, pizzas are not sold at a fixed price per slice. Instead, they are only available as packages with fixed sizes, e.g. small, medium and large. Another form is quantity discounts. For example, one for \$3, two for \$5, and three for \$6, or "buy-one-get-one-free." For more examples of second-degree price discrimination, readers are referred to Wilson (1993, Chapter 2). One can find examples when a seller uses uniform pricing only, or second-degree price discrimination only, or some segments of the market pay a uniform price while others pay a tariff not strictly proportional to the quantity purchased. Nahata and Ringbom (2007) analyze the welfare effects when both linear and nonlinear pricing are used *concurrently*, and provide many real life exampes too.

What happens to the welfare and the total output when uniform price is compared to second-degree price discrimination has not been explored. Our reason for comparing second-degreee price discrimination with uniform pricing is two-fold: First, unlike in third-degree price discrimination where the monopolist employs linear pricing, the pricing used under second-degree is nonlinear. The question that so far has remained unexplored is: how does the difference in pricing strategies—linear when price is uniform and nonlinear pricing under discrimination—affect the results related to output and total welfare? Second, when pricing is linear, as it is the case under third-degree, only a new market can be opened but a market that is being served under uniform pricing can never be foreclosed. However, because of nonlinear pricing a market served under uniform pricing may be foreclosed under second-degree because it may be more profitable to serve only one market. It is important to note that under second-degree price discrimination, it is well known that the monopolist may find it *more* profitable to serve *only* one market and exclude the other. However, this conclusion is independent of whether both markets are served or not under uniform pricing. Thus, conclusions regarding change in welfare and output because of foreclosure cannot be made without comparing the two regimes. The question that has also remained unexplored is: when *a foreclosure defined as when both markets were served under uniform pricing but only one of the two markets is served under second-degree price discrimination* how do the total output and the social welfare (measured as the sum of consumer and producer surplus) change vis-a-vis uniform

[1] Opening of the market, however, in general, is not necessary for Pareto improvement under third-degree price discrimination, see Nahata, Ostaszewski and Sahoo (1990).

[2] See Varian (1985) and Aguirre, Cowan, and Vickers (2010).

[3] When demands in all markets are of constant elasticity type, Formby, Layson, and Smith (1983) and Aquirre (2006) show that, total output always increases under third-degree price discrimination than under uniform pricing; Aquirre and Cowan (2015) demonstrate that social welfare could increase too.

pricing?

The main objective of our paper is to make a first attempt to investigate the welfare changes from uniform price to second-degree price discrimination with and without Spence-Mirrlees (SMC) condition. However, relaxation of the SMC creates technical hardships because when the SMC is violated the local incentive-compatibility constraints are not sufficient to sort out different types of consumers and hence it is difficult to obtain all solutions.[4] Perhaps, this is the main reason why no comparison between uniform price and second-degree price discrimination has been made in the literature.[5] To overcome technical complexities and contrast the standard results of third-degree price discrimination, we restrict our attention to two linear demands but present a comprehensive comparison between the output and welfare impacts under second- and third-degree price discrimination.

We show some counter-intuitive results that have not been contemplated in the literature. First, we find that second-degree price discrimination *can* foreclose a market that was previously served under uniform pricing. We state both the sufficient and necessary (S&N) conditions for such market foreclosure. Since the current literature has not contemplated the possibility that price discrimination can foreclose a market previously served *without* price discrimination, our comparison provides this new insight.

Second, compared to uniform pricing, the total output *always* goes up under second-degree price discrimination whether the SMC holds or not. Surprisingly, the total output also increases, in spite of market foreclosure. Our results provide these two new insights related to the direction of output change with and without the foreclosure.

Third, although in general the direction of change in the total surplus remains ambiguous, for large parameter range we show it will be higher than lower. But, what is more surprising is that, even when one market is foreclosed, still, the total surplus could be higher. Also, *without any foreclosure*, the total surplus could be lower. We derive both the sufficient and necessary conditions and show the likelihood for the total surplus to be lower under second-degree price discrimination.

The paper is organized as follows. Section 2 presents two motivating examples followed by the formal model in Section 3. In Section 4, we characterize the optimal solutions under uniform pricing and second-degree price discrimination for two linear demands. Section 5 presents the comparison between the two regimes and our main proposition, followed by a brief discussion of our results. Section 6 concludes. Appendix contains the details of the proof for Proposition 1.

2 Two motivating examples

The following two examples motivate our analysis. Consider two types of consumers with gross valuation functions as: $v_1(q) = q \cdot (1 - 0.5q)$ and $v_2(q) = 0.5q \cdot (1 - \theta q)$ ($\theta = 0.6$ or 1). It follows that the inverse demands are: $p_1(q) = 1 - q$ and $p_2(q) = 0.5 - \theta q$. Clearly, the SMC holds, type 1 is the high-demand consumer and type 2 is the low-demand consumer. Further, assume there are equal numbers of each type of consumers and zero marginal cost. It is easy to calculate the optimal

[4]See Kokovin, Nahata and Zhelobodko (2011) and Kokovin and Nahata (2016) for both the theoretical and computational difficulties in characterizing the solution structures in a general setting.

[5]For two recent comprehensive surveys of price discrimination see, Armstrong (2006) and Stole (2007). For an earlier survey, see Varian(1989).

uniform price (UP) and the two tariffs, (t_i, q_i) $(i = 1, 2)$ under second-degree price discrimination (SPD) as shown in Table 1.

Table 1: Two Examples of Total Output and Total Surplus Changes from UP to SPD when One Market is Foreclosed under SPD

		Price/Tariff	Total Output	Total Surplus
$\theta = 0.6$	**UP**	$p^{UP} = 0.34375$	0.916667	0.550781
TS ↓	**SPD**	$\begin{cases} (t_1, q_1) = (0.5, 1) \\ (t_2, q_2) = (0, 0) \end{cases}$	$\mathbf{1}$	$\mathbf{0.5}$
$\theta = 1$	**UP**	$p^{UP} = 0.375$	0.75	0.484375
TS ↑	**SPD**	$\begin{cases} (t_1, q_1) = (0.5, 1) \\ (t_2, q_2) = (0, 0) \end{cases}$	$\mathbf{1}$	$\mathbf{0.5}$

These examples clearly show that under SPD, type-2 consumers previously served under UP are excluded—a market foreclosure e.g., $(t_2, q_2) = (0, 0)$ and $p^{UP} < 0.5$. Such foreclosure could result in a higher total profit, higher total output, and an *increase* ($\theta = 1$) as well as a decrease ($\theta = 0.6$) in the total welfare.

3 Model

A monopolist sells a homogeneous product to two types of consumers, say 1 and 2, on a take-it-or-leave-it basis. Their gross valuation functions are quadratic and given by $v_1(q) = q \cdot (1 - bq/2)$ and $v_2(q) = q \cdot (\alpha - b\beta q/2)$, where $1 \geq \alpha > 0, \beta > 0, b > 0$. Equivalently, the two general linear demands are $p_1(q) = 1 - bq$ and $p_2(q) = \alpha - b\beta q$ (see Figure 5). The number of type i consumers is n_i $(i = 1, 2)$, and their ratio is $\gamma \equiv n_2/n_1$. For expositional simplicity, we restrict our attention to the case when $\gamma = 1$.[6] Without any loss of generality, we normalize the monopolist's marginal cost to be zero. Note that when $\alpha \leq \beta$, the SMC holds and the two demands do *not* cross; when $\alpha > \beta$, the SMC is violated and the two demands cross.

When the monopolist chooses UP, she sets its per-unit price p to

$$\max_p p \cdot Q(p), \tag{UP}$$

where

$$Q(p) \equiv \begin{cases} \frac{1-p}{b} & \text{if } \alpha \leq p \leq 1 \\ \frac{\alpha+\beta-(1+\beta)p}{b\beta} & \text{if } 0 \leq p \leq \alpha \\ 0 & \text{otherwise} \end{cases}$$

is the aggregate demand.

[6] The results for the cases of $\gamma = \frac{1}{2}$ and $\gamma = 1$ are provided in Section 5.1. We found that our results continue to hold for a wider range of γ.

Under SPD, the monopolist offers a menu of quantity-tariff packages (t_i, q_i) $(i = 1, 2)$ to

$$\max_{(t_i,q_i),i=1,2} t_1 + t_2 \tag{SPD}$$

$$s.t.\ v_i(q_i) - t_i \geq v_i(q_j) - t_j \tag{IC_i}$$

$$v_i(q_i) - t_i \geq 0 \tag{IR_i}$$

4 Characterization of Optimal UP and SPD

Below we characterize the optimal UP and SPD for different parameter values of the two linear demands.

4.1 Optimal UP

Under UP, it is possible that one market is foreclosed. $\alpha = \sqrt{\beta(\beta+1)} - \beta$ is the boundary curve below which market 2 is not served. The optimal UP outcomes for the monopolist are given in Table 2, and the partitions in $(\alpha, \beta)-$space are shown in Figure 1.

Table 2: Optimal UP Outcomes for the Monopolist

	Total Output (Q^{UP})	**Total Surplus** (TS^{UP})
(U1)	$\frac{\alpha+\beta}{2b\beta}$	$\frac{3(\alpha+\beta)^2+4\beta\cdot(1-\alpha)^2}{8b\beta\cdot(\beta+1)}$
(U2)	$\frac{1}{2b}$	$\frac{3}{8b}$

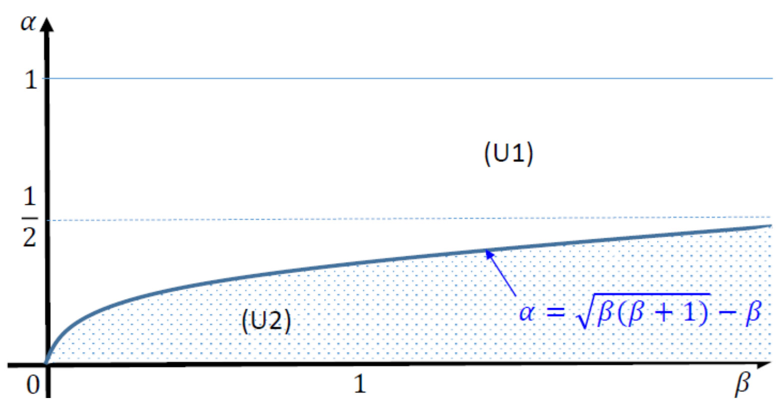

Figure 1: Partitions under Optimal UP

$(U1) : \sqrt{\beta(\beta+1)} - \beta < \alpha$ Both markets are served

$(U2) : \alpha \leq \sqrt{\beta(\beta+1)} - \beta$ Market 2 is not served

4.2 Optimal SPD

When $\alpha \leq \beta$, the SMC holds and the two demands do not cross. It is easy to compute the optimal SPD outcomes for such standard case.[7]

[7]For a complete characterization of all solution structure for two linear demands see Nahata, Kokovin and Zhelobodko (2002).

When $\alpha > \beta$, the two valuation functions cross (at $\widehat{q} = \frac{2}{b} \cdot \frac{1-\alpha}{1-\beta}$); and so do the demand curves. For this non-standard case, we calculate the optimal SPD outcomes based on all possible allocations identified in Chao and Nahata (2015).

The optimal SPD outcomes are given in Table 3, and the partitions are shown in Figure 2.[8]

Table 3: Optimal SPD Outcomes for the Monopolist

		Total Output (Q^{SPD})	**Total Surplus** (TS^{SPD})
Non-crossing Demands $\alpha \leq \beta$	(S1)	$\frac{2(\alpha+\beta-1)}{b\cdot(2\beta-1)}$	$\frac{3(\alpha+\beta)^2+4\beta\cdot(1-\alpha)^2}{8b\beta\cdot(\beta+1)}$
	(S2)	$\frac{1}{b}$	$\frac{1}{2b}$
Crossing Demands $\alpha > \beta$	(S3)	$\begin{cases} \frac{1}{b}\cdot\left(\frac{\alpha}{\beta}+\frac{2-\alpha}{2-\beta}\right) & \text{if } \alpha \geq \frac{2}{3-\beta} \\ \frac{1}{b}\cdot\left[\frac{\alpha}{\beta}+\frac{2(1-\alpha)}{1-\beta}\right] & \text{if } \alpha < \frac{2}{3-\beta} \end{cases}$	$\begin{cases} \frac{\frac{\alpha^2}{\beta}+\frac{(2-\alpha)(2+\alpha-2\beta)}{(2-\beta)^2}}{2b} & \text{if } \alpha \geq \frac{2}{3-\beta} \\ \frac{\frac{\alpha^2}{\beta}+\frac{4(1-\alpha)(\alpha-\beta)}{(1-\beta)^2}}{2b} & \text{if } \alpha < \frac{2}{3-\beta} \end{cases}$
	(S4)	$\frac{1}{b}\cdot\left(1+\frac{\alpha}{\beta}\right)$	$\frac{1+\frac{\alpha^2}{\beta}}{2b}$
	(S5)	$\begin{cases} \frac{1}{b}\cdot\left[1+\frac{2(1-\alpha)}{1-\beta}\right] & \text{if } \alpha \geq \frac{3-\frac{1}{\beta}}{2} \\ \frac{1}{b}\cdot\left(1+\frac{2\alpha-1}{2\beta-1}\right) & \text{if } \alpha < \frac{3-\frac{1}{\beta}}{2} \end{cases}$	$\begin{cases} \frac{1+\frac{4(1-\alpha)(\alpha-\beta)}{(1-\beta)^2}}{2b} & \text{if } \alpha \geq \frac{3-\frac{1}{\beta}}{2} \\ \frac{1+\frac{(2\alpha-1)[\beta+2\alpha\cdot(\beta-1)]}{(2\beta-1)^2}}{2b} & \text{if } \alpha < \frac{3-\frac{1}{\beta}}{2} \end{cases}$

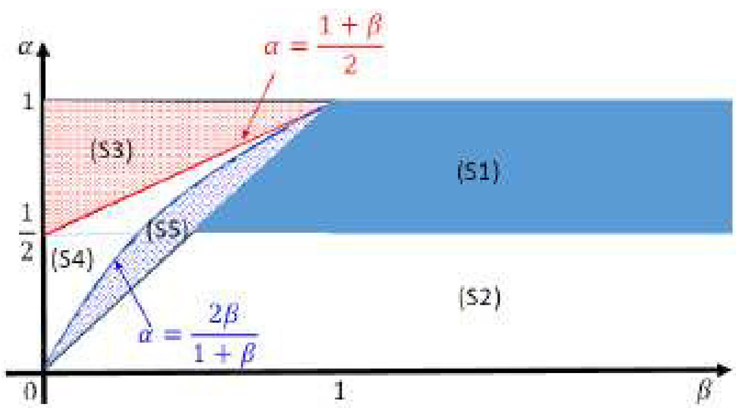

Figure 2: Partitions under Optimal SPD

$(S1) : \frac{1}{2} < \alpha$	Undersizing type 2
$(S2) : \alpha \leq \frac{1}{2}$	Market 2 is not served
$(S3) : \frac{1+\beta}{2} < \alpha$	Undersizing type 1
$(S4) : \frac{2\beta}{1+\beta} \leq \alpha \leq \frac{1+\beta}{2}$	Overall efficiency
$(S5) : \beta < \alpha < \frac{2\beta}{1+\beta}$	Oversizing type 2

5 Comparison

In this section, we first examine when will the markets be foreclosed under SPD, and then compare the welfare under SPD with that under UP.

[8]The detailed calculations of the results listed in Table 3 are available upon request from the authors.

Proposition 1 *Compared with UP, under SPD,*

(i) Market 2 is foreclosed if and only if

$$\sqrt{\beta(\beta+1)}-\beta<\alpha\leq\min\{\beta,\frac{1}{2}\}.$$

(ii) When $\alpha\leq\beta$, the total welfare is lower, $TS^{SPD}<TS^{UP}$ if and only if

$$\begin{cases}\sqrt{\beta(\beta+1)}-\beta<\alpha<\beta & \text{if } \beta<\frac{1}{2}\\ \max\left\{\sqrt{\beta(\beta+1)}-\beta, f_2(\beta)\right\}<\alpha<f_1(\beta) & \text{if } \frac{1}{2}\leq\beta\end{cases},$$

where $f_1(\beta)\equiv\frac{\beta}{3+8\beta-4\beta^2}\cdot\left[5+4\beta^2-4(2\beta-1)\cdot\sqrt{\beta+1}\right]$, *and* $f_2(\beta)\equiv\frac{\beta}{4\beta+3}\cdot\left(1+2\sqrt{\beta+1}\right)$.

When $\alpha>\beta$, the total welfare is lower, $TS^{SPD}<TS^{UP}$ if and only if

$$\max\left\{\beta,\sqrt{\beta(\beta+1)}-\beta,g_2(\beta)\right\}<\alpha<g_1(\beta),$$

where $g_1(\beta)\equiv\frac{\beta\cdot\left[9+14\beta+9\beta^2-2(3-2\beta-\beta^2)\cdot\sqrt{1+\beta}\right]}{3+\beta\cdot(14+11\beta+4\beta^2)}$, *and* $g_2(\beta)\equiv\frac{\beta\cdot\left[5+4\beta^2+4(2\beta-1)\cdot\sqrt{1+\beta}\right]}{3+8\beta-4\beta^2}$.

(iii) The total output never decreases, i.e., $Q^{SPD}\geq Q^{UP}$ with "=" if and only if $\alpha=\beta\in(\frac{1}{3},\frac{1}{2}]$.[9]

When the two demands do not cross ($\alpha\leq\beta$), it is important to note that even though market foreclosure under SPD could occur when $\sqrt{\beta(\beta+1)}-\beta<\alpha\leq\min\left\{f_2(\beta),\frac{1}{2}\right\}$, the total surplus will always be higher as demonstrated earlier in Table 1 ($\alpha=1/2,\beta=b=1$ for $\theta=1$ case there).

When the two demands cross ($\alpha>\beta$), no market will be foreclosed. The sufficient and necessary conditions clearly show that even though no market is foreclosed, the total surplus could still be lower.

5.1 Why Total Welfare could be Lower under SPD

Applying parts (ii) of Proposition 1, we depict the region where the total welfare is lower under SPD compared to UP for $\gamma=1$ in Figure 3. For a clear illustration of the lower total welfare area, we concentrate on the unit square parameter space because the total welfare can be lower *only* in this space, even though β can be any positive number exceeding one. Therefore, the region where $\beta>1$ is excluded. In Figures 4 and 5, we plot the lower welfare regions for $\gamma=\frac{1}{2}$ and $\gamma=2$. Three observations from these figures are worth noting. First, for different values of γ, the basic patterns for the reduction in the welfare are similar: they all are heart-shaped along the 45-degree line. Second, taking into account that β can exceed one, overall these regions are not that significant and thus a likely decrease in welfare under SPD within a narrow range of parameter

[9]Chao and Nahata (2015) have identified the optimal allocation patterns under SPD. Their Proposition 1 shows that, when the SMC does not hold, both types will always be served and thus foreclosure can never occur. Their Corollary 1 states the conditions for efficient and distorted——both downward and upward——allocations.

space need not be a major concern. Third, the impact of γ on the magnitude of the lower welfare areas does not appear to be monotonic and hence it is difficult to ascertain.

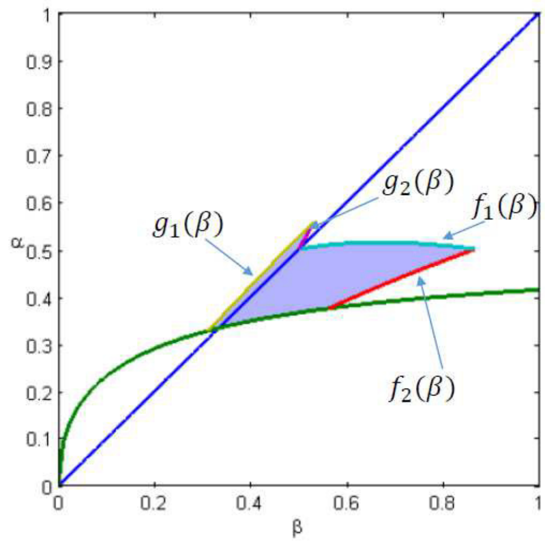

Figure 3: Parameter Area in which $TS^{SPD} < TS^{UP}$ for $\gamma = 1$

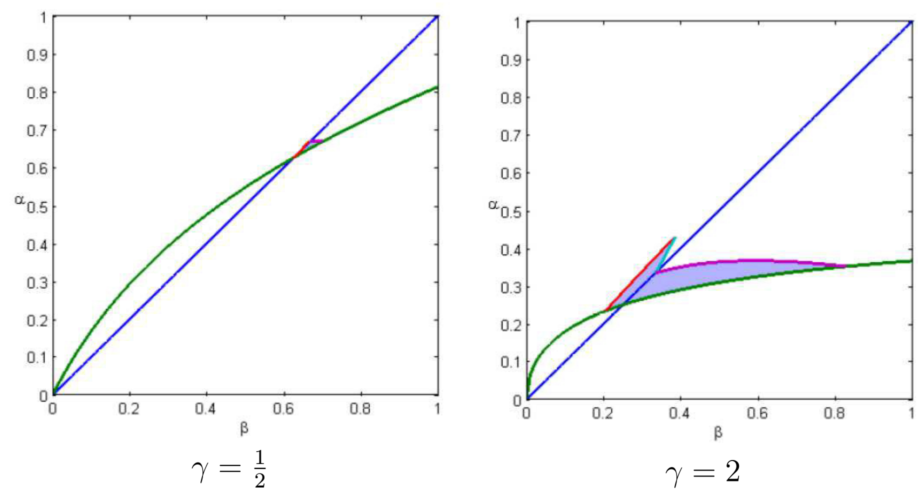

Figure 4: Parameter Areas in which $TS^{SPD} < TS^{UP}$ for different γs

Some economic intuition for the sufficient and necessary conditions can be illustrated using inverse demands in Figure 5. Essentially, there are two requirements for the total welfare to decrease under SPD: (1) the maximum willingness-to-pay for type-2 consumers must be in a narrow range that is neither too large nor too small (as shown by the red interval on the vertical axis in the figure); (2) the efficient quantities for both types of consumers should be more or less the same (as shown by the red interval on the horizontal axis). The second requirement implies that two types of consumers' marginal willingness-to-pay are nearly proportional to each other for any given quantity. Under SPD, one market is always served with efficient quantity and hence the welfare goes up in that market. Thus, the only way for the total welfare to decrease is that the distortion in the other market is so large that it cannot be offset from the efficiency gain from the former market. This will be the case when both (1) and (2) hold.

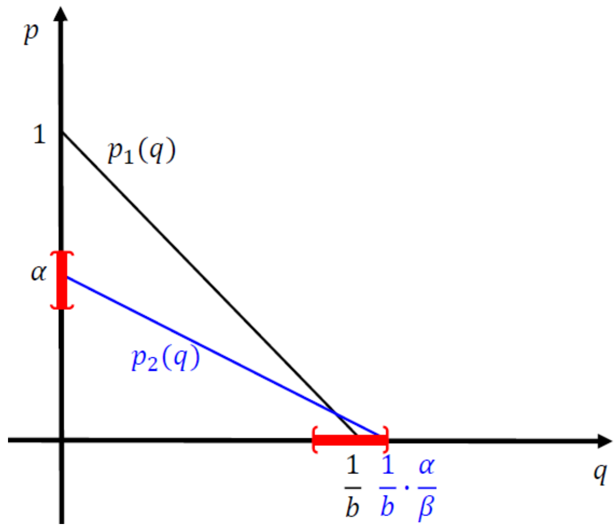

Figure 5: Illustrations of Two Demands Resulting $TS^{SPD} < TS^{UP}$

6 Conclusions

We have shown that when the preferences satisfy the SMC and both markets are served under uniform pricing, second-degree price discrimination not only could result in market foreclosure, but the resulting foreclosure also can increase both the total output and social welfare. The possibility of a welfare-enhancing foreclosure so far has not been contemplated in the literature. When the preferences violate the SMC, the total surplus can be *lower* under second-degree price discrimination, even when both markets are served under both uniform pricing and price discrimination. Further, we have shown that the range of parameters when the welfare could be lower is relatively narrow.

Our results also show an interesting contrast in the changes in total output and total welfare between uniform pricing and second-degree price discrimination, and between uniform pricing and third-degree price discrimination. For two linear demands, when both markets are served under uniform pricing, the total output remains unchanged and the welfare always goes down under third-degree price discrimination. But, under second-degree price discrimination this is not the case. The output always goes up but the welfare can either increase or decrease. Under third-degree price discrimination, no market would be excluded—market foreclosure can never occur, but second-degree price discrimination can foreclose one market. More importantly, even with the foreclosure the welfare can increase. Also, without the foreclosure the welfare can decrease, under second-degree price discrimination compared to uniform pricing.

Table 4 below highlights the important differences between second- and third-degree price discrimination. Further, none of the above results are isolated cases as they hold for a range of demand parameters.

A comparison in a general framework that does not restrict the number of consumer types with a general demand specification certainly would be more desirable. However, such analysis seems

Table 4: Comparison between 2nd-degree and 3rd-degree Price Discrimination for Linear Demands

	2nd-degree Price Discrimination	**3rd-degree Price Discrimination**
Market Foreclosure	possible	impossible
Total Output	higher	no change
Total Surplus	could be higher *with* foreclosure could be lower *without* foreclosure	always lower
S&N Condition for Lower Total Surplus	identified in this paper	impossible

not feasible due to theoretical hardships because as the number of consumer types increase the number of possible solutions increases exponentially. In spite of our analysis being specific, it does provide some new insights.

Acknowledgments: *We would like to thank the Editor, the Associate Editor and two anonymous referees for valuable suggestions that have significantly improved this paper.*

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Appendix

Proof of Proposition 1. (i) Under UP, the type-2 consumers are excluded if and only if $\alpha \leq \sqrt{\beta(\beta+1)} - \beta$. Under SPD, the type-2 consumers are excluded if and only if $\alpha \leq \min\{\beta, \frac{1}{2}\}$. Note that for any $\beta > \frac{1}{3}$, we have $\sqrt{\beta(\beta+1)} - \beta < \min\{\beta, \frac{1}{2}\}$, so $\sqrt{\beta(\beta+1)} - \beta < \alpha \leq \min\{\beta, \frac{1}{2}\}$ is non-empty. This part follows.

(ii) *Non-crossing Demand:* $\alpha \leq \beta$

For (S1), when $\frac{1}{2} < \alpha$, $TS^{SPD} - TS^{UP} \propto (4\beta^2 - 8\beta - 3) \cdot \alpha^2 + 2\beta \cdot (4\beta + 5) \cdot \alpha + (4\beta^2 - 8\beta - 3) \cdot \beta^2 \lesseqqgtr 0$ if and only if $\alpha \lesseqqgtr f_1(\beta)$, where $f_1(\beta)$ is the smaller root of equation $(4\beta^2 - 8\beta - 3) \cdot \alpha^2 + 2\beta \cdot (4\beta + 5) \cdot \alpha + (4\beta^2 - 8\beta - 3) \cdot \beta^2 = 0$, given in the proposition. Here we use $\propto$ to denote the "proportional to" relation.

For (S2), when $\alpha \leq \frac{1}{2}$ and $\alpha > \sqrt{\beta(\beta+1)} - \beta$, $TS^{SPD} - TS^{UP} \propto -(4\beta+3) \cdot \alpha^2 + 2\beta \cdot \alpha + \beta^2 \lesseqqgtr 0$ if and only if $\alpha \gtreqqless f_2(\beta)$. When $\alpha \leq \frac{1}{2}$ and $\alpha \leq \sqrt{\beta(\beta+1)} - \beta$, $TS^{SPD} = \frac{1}{2b} > TS^{UP} = \frac{3}{8b}$.

Crossing Demand: $\alpha > \beta$

For (S4) (Overall Efficiency), it is obvious to see $TS^{SPD} > TS^{UP}$.

For (S3) (Undersizing Type 1), when $\alpha \geq \frac{2}{3-\beta}$, both markets will be served under UP. $TS^{SPD} - TS^{UP} \propto h(\alpha)$, where $h(\alpha) \equiv (4 - 8\beta - 3\beta^2) \cdot \alpha^2 + 2\beta \cdot (4 + 5\beta^2) \cdot \alpha + \beta^2 \cdot (4 - 8\beta + 3\beta^2)$. $h'(\alpha) = 2(4 - 8\beta - 3\beta^2) \cdot \alpha + 2\beta \cdot (4 + 5\beta^2)$. If $4 - 8\beta - 3\beta^2 \geq 0$, then $h'(\alpha) \geq 0$. If $4 - 8\beta - 3\beta^2 < 0$, $g''(\alpha) < 0$, and thus $h'(\alpha) \geq g'(1) = 2(1+\beta)(4 - 8\beta + 5\beta^2) > 0$ for any $\beta \in [0, 1]$, where the first inequality follows from $\alpha \leq 1$. Thus, $h'(\alpha) \geq 0$ in either case. Since $\alpha \geq \frac{2}{3-\beta}$, we must have $h(\alpha) \geq h(\frac{2}{3-\beta}) = \frac{(2-\beta)^2 \cdot \left[4 + \beta(1+\beta) \cdot (8 + \beta - 3\beta^2)\right]}{(3-\beta)^2} > 0$. Hence, $TS^{SPD} > TS^{UP}$ when $\alpha \geq \frac{2}{3-\beta}$. Similarly, we can show $TS^{SPD} > TS^{UP}$ when $\alpha < \frac{2}{3-\beta}$.

For (S5) (Oversizing Type 2), when $\alpha \leq \sqrt{\beta(\beta+1)} - \beta$, only type 1 consumers are served under UP, while under SPD, both types are served. The total surplus is the sum of the surplus from market 1 being served with the efficient output and some non-negative amount from the market 2. So clearly, $TS^{SPD} \geq TS^{UP}$ when $\alpha \leq \sqrt{\beta(\beta+1)} - \beta$. The only case remains to be considered is when $\max\left\{\beta, \sqrt{\beta(\beta+1)} - \beta\right\} < \alpha < \frac{2\beta}{1+\beta}$. In this case, both the markets are served under UP.

When $\alpha \geq \frac{3-\frac{1}{\beta}}{2}$, $TS^{SPD} - TS^{UP} \propto l(\alpha)$, where $l(\alpha) \equiv -(3 + 14\beta + 11\beta^2 + 4\beta^3) \cdot \alpha^2 + 2\beta \cdot (9 + 14\beta + 9\beta^2) \cdot \alpha - \beta^2 \cdot (15 + 18\beta - \beta^2)$. For this subcase, we have $\max\left\{\beta, \sqrt{\beta(\beta+1)} - \beta, \frac{3-\frac{1}{\beta}}{2}\right\} < \alpha < \frac{2\beta}{1+\beta}$. Note that $l(\frac{2\beta}{1+\beta}) = \frac{(1-\beta)^2 \cdot \beta^2 \cdot (3+\beta)^2}{(1+\beta)^2} > 0$ and $l'(\frac{2\beta}{1+\beta}) = \frac{2(1-\beta)^2 \cdot \beta \cdot (3+\beta)}{1+\beta} > 0$. It follows that $l'(\alpha) > 0$ for any $\alpha < \frac{2\beta}{1+\beta}$. Therefore, $TS^{SPD} \gtreqqless TS^{UP}$ if and only if $\alpha \gtreqqless g_1(\beta)$, where $g_1(\beta)$ is the smaller root of equation $l(\alpha) = 0$, given in the proposition.

When $\alpha < \frac{3-\frac{1}{\beta}}{2}$, $TS^{SPD} - TS^{UP} \propto m(\alpha)$, where $m(\alpha) \equiv -(3 + 8\beta - 4\beta^2) \cdot \alpha^2 + 2\beta \cdot (5 + 4\beta^2) \cdot \alpha - \beta^2 \cdot (3 + 8\beta - 4\beta^2)$. Since this subcase occurs only for $\beta \in [\frac{1}{2}, 1]$, we have $\beta > \sqrt{\beta(\beta+1)} - \beta$ for this subcase, and thereby we consider $\beta < \alpha < \frac{3-\frac{1}{\beta}}{2}$. Note that $m(\beta) = 4(1-2\beta)^2 \cdot \beta^2 \geq 0$, and $m(\frac{3-\frac{1}{\beta}}{2}) \propto -3 - 2\beta + 9\beta^2 + 8\beta^3 + 4\beta^4 \gtreqqless 0$ if and only if $\beta \gtreqqless \widehat{\beta}$, where $\widehat{\beta} \simeq 0.5315$ is the second smallest root of equation $-3 - 2\beta + 9\beta^2 + 8\beta^3 + 4\beta^4 = 0$. Thus, for $\beta \geq \widehat{\beta}$, we have $TS^{SPD} \geq TS^{UP}$ for $\beta < \alpha < \frac{3-\frac{1}{\beta}}{2}$; for $\beta < \widehat{\beta}$, we have $TS^{SPD} \gtreqqless TS^{UP}$ if and only if

$\alpha \lesseqgtr g_2(\beta)$, where $g_2(\beta)$ is the larger root of equation $l(\alpha) = 0$, given in the proposition.

(iii) *Non-crossing Demand:* $\alpha \leq \beta$

For (S1), when $\frac{1}{2} < \alpha$, $Q^{SPD} - Q^{UP} = \frac{1}{b} \cdot \frac{4\beta(\alpha+\beta-1)-(\alpha+\beta)(2\beta-1)}{2\beta\cdot(2\beta-1)} \propto 2\beta^2 + (2\alpha - 3) \cdot \beta + \alpha$, where the "proportional to" relation ($\propto$) follows from $\frac{1}{2} < \alpha \leq \beta$. Note that $2\beta^2 + (2\alpha - 3) \cdot \beta + \alpha$ increases in β for $\frac{1}{2} < \alpha \leq \beta$, and $2\beta^2 + (2\alpha - 3) \cdot \beta + \alpha\big|_{\beta=\frac{1}{2}} = 2\alpha - 1 > 0$. It follows that $Q^{SPD} - Q^{UP} > 0$.

For (S2), when $\alpha \leq \frac{1}{2}$, because $\frac{1}{b} > \frac{1}{2b}$, we have $Q^{SPD} > Q^{UP}$ if $\alpha \leq \sqrt{\beta(\beta+1)} - \beta$; the fact that $\alpha \leq \beta$ leads to $\frac{1}{b} \geq \frac{\alpha+\beta}{2b\cdot\beta}$, so $Q^{SPD} \geq Q^{UP}$ if $\alpha > \sqrt{\beta(\beta+1)} - \beta$, with "$=$" if and only if $\alpha = \beta$. Taking into account $\alpha \leq \frac{1}{2}$ and $\alpha > \sqrt{\beta(\beta+1)} - \beta$, $Q^{SPD} = Q^{UP}$ can only occur when $\alpha = \beta \in (\frac{1}{3}, \frac{1}{2}]$.

For (S4) (Overall Efficiency), and (S5) (Oversizing Type 2) cases, $Q^{SPD} \geq Q^{efficient} = \frac{1}{b} \cdot \left(1 + \frac{\alpha}{\beta}\right) > Q^{UP}$. So the remaining case to be shown is (S3) (Undersizing Type 1).

Note that in (S3), both markets are served under UP, and thus $Q^{UP} = \frac{\alpha+\beta}{2b\cdot\beta}$. When $\alpha \geq \frac{2}{3-\beta}$, $Q^{SPD} - Q^{UP} \propto 2\alpha \cdot (1-\beta) + \beta^2 + 2\beta - \alpha \cdot \beta \geq 2\alpha \cdot (1-\beta) + \beta^2 + \beta > 0$, where the first inequality follows from $\alpha \leq 1$. So $Q^{SPD} > Q^{UP}$. When $\alpha < \frac{2}{3-\beta}$, $Q^{SPD} - Q^{UP} \propto \alpha \cdot (1-\beta) + \beta^2 + 3\beta - 4\alpha\beta \geq \alpha \cdot (1-\beta) + \frac{\beta\cdot(1-\beta^2)}{3-\beta} > 0$, where the first inequality results from $\alpha < \frac{2}{3-\beta}$. Thus, $Q^{SPD} > Q^{UP}$. ■