Multi-product firms and gains from trade through intra-firm adjustments

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Abstract
This paper investigates welfare gains associated with trade induced adjustments within multi-product firms. To disentangle the welfare gains, I focus on two distinct channels: investments in i) product variety and ii) the degree of product differentiation. Trade integration enables firms to exploit economies of scale in innovation and induces more investments in product scope. To reduce cannibalization among varieties, multi-product firms have incentives to invest in the degree of differentiation. I show how variety loving consumers benefit from a wider and more diversified product range.

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1 Introduction

In 1942, Joseph Schumpeter argued that innovation activity is carried out by large firms, for whom R&D is endogenous. R&D projects often go hand in hand with high development costs whereas a sufficiently large scale is required to cover these costs. Recent contributions in the trade literature emphasize the importance of intra-firm adjustments through innovation in explaining the welfare gains from trade.\(^1\) Since innovation is costly, changes in market size tend to encourage firms to incur these costs and exploit economies of scale in innovation.

I focus on innovation activities that affect the variety of products in an economy. Variety-loving consumers benefit from making a choice out of a broad and diversified product portfolio. Recent evidence suggests that product innovation by incumbent firms is the main channel through which new products enter the market (see Broda and Weinstein, 2010). Motivated by this, I analyze the effects of trade liberalization on product variety in a simple framework of multi-product firms (MPFs). The key element is an investment in the degree of product differentiation and the consequent welfare gains through more diversified varieties. MPFs have incentives to invest in product differentiation to mitigate the cannibalization effect among varieties. Investments in product specific attributes and promotion activities such as advertisement or marketing campaigns help to highlight the differences between products. All these measures come along with fixed costs, however, they are implemented to differentiate the products within the portfolio and to reduce cannibalization between varieties. To conclude my analysis, I disentangle the welfare implications of an increase in market size. I show that consumers benefit from investments in new products (love of variety) which are characterized by a higher degree of differentiation (love of diversity).

My paper is mostly related to recent models that study the innovation behavior of MPFs. Dhingra (2013) explains how firms react to trade liberalization in terms product and process innovation. In her model, firms reduce product innovation to mitigate internal competition but increase investments in production processes following an instance of trade liberalization. Eckel et al. (2015) analyze a different type of innovation and incorporate an endogenous investment in product quality into the framework by Eckel and Neary (2010). Flach and Irlacher (2018) show both theoretically and empirically, that firms in industries with a larger scope for product differentiation invest more in product innovation because of lower cannibalization effects. This finding motivates me to endogenize the degree of differentiation which is a main component of the industry structure that most studies treat as an exogenous variable.\(^2\) One exception is Lorz and Wrede (2009) who endogenize the degree of product differentiation in a notably different framework. However, the focus of my paper is different, as I split up the R&D portfolio of a MPF to disentangle the welfare gains from globalization.

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1. Lileeva and Trefler (2010), as well as Bustos (2011) find a complementarity between market size and the innovation behavior of firms.

2 The Model

2.1 Consumers

$L$ consumers maximize utility over the consumption of a homogeneous good $q_0$ and a differentiated good:

$$U = q_0 + aQ - \frac{1}{2} b \left[ (1 - e(s)) \int_{i \in \tilde{\Omega}} q(i)^2 di + e(s) Q^2 \right]. \quad (1)$$

Consumption per variety is denoted by $q(i)$ with $i \in \tilde{\Omega}$ and total consumption is given by $Q \equiv \int_{i \in \tilde{\Omega}} q(i) di$. Variables $a$ and $b$ represent preference parameters and $e(s) \in [0, 1]$ is an inverse measure of product differentiation. This parameter is of central interest, as it is chosen endogenously by a firm (the investment is denoted by $s$). Further assumptions on $e(s)$ will be discussed later.

Market demand for variety $i$ consists of the aggregated demand of all consumers $x(i) = L q(i)$. The inverse demand function is given by:

$$p(i) = a - b' \left[ (1 - e(s)) x(i) + e(s) X \right], \quad (2)$$

with $b' \equiv \frac{b}{L}$ being an inverse measure for the market size and $X \equiv \int_{i \in \Omega} x(i) di$ representing total demand in the differentiated industry.

2.2 Firm behavior

I introduce a simple two stage model of a single MPF that is solved by backwards induction. In the first stage, a MPF chooses its spending on product differentiation, anticipating the effects on optimal scale and scope in the second stage. A firm invests in product differentiation to reduce the cross elasticity between varieties which is given by:

$$\varepsilon_{i,j} \equiv \left| \frac{\partial x(i)}{\partial x(j)} \right| = e(s) x(j) / (1 - e(s)) x(i).$$

**Lemma 1** By investing in the degree of product differentiation, a MPF can lower the magnitude of the cannibalization effect through a lower cross elasticity of demand between varieties.

**Second stage** In the second stage, profits are given by:

$$\pi = \int_0^{\delta} [p(i) - c(i)] x(i) di - \delta r_\delta, \quad (3)$$

where $\delta$ denotes product scope and $r_\delta$ represents the fixed costs for a new production line. Variable production costs for variety $i$ are given by $c(i)$. For the sake of simplicity, I impose symmetry on the production costs: $c(i) = c(j) = c$.\footnote{Given the quasi-linearity there is no income effect, implying that $\lambda = 1$.} Maximizing profits with respect to

\footnote{See Flach and Irlacher (2018) for a variant of the Eckel and Neary (2010) framework with both flexible manufacturing and fixed costs of product innovation.}
scale leads to the optimal output of a single variety:

\[ x = \frac{a - c}{2b'(1 - e(s) + e(s)\delta)}. \quad (4) \]

Note that total output \( X = \delta x \) rises in the product range \( \delta \), however output of each variety \( x \) is decreasing in \( \delta \) due to the cannibalization effect. Maximizing profits with respect to scope gives:

\[ \delta = \frac{(a - c)\sqrt{\frac{1-e(s)}{b'r_\delta}} - 2(1 - e(s))}{2e(s)}. \quad (5) \]

Product scope is limited through the fixed costs \( r_\delta \), as well as the cannibalization effect associated with the launching of new products. Inspecting Eq. (5) reveals a multiplicative structure of the fixed costs \( r_\delta \) and the inverse measure for the market size \( b' \). Hence, an increase in the market size \( L \) has the same effect as decreasing fixed costs \( r_\delta \). I interpret \( b'r_\delta \) as the perceived costs of product innovation which are lower in a larger market. Furthermore, it can be shown that a larger scope for product differentiation induces the firm to enlarge its product range \( \delta \). I summarize the main results in the following proposition.

**Proposition 1** A larger market size \( L \), as well as a rising degree of product differentiation increase the optimal product range, i.e.

\[ \frac{\partial \ln \delta}{\partial \ln L} = \frac{(a - c)\sqrt{\frac{1-e(s)}{b'r_\delta}}}{4e(s)\delta} > 0 \text{ and } \frac{\partial \ln \delta}{\partial \ln e} = -\frac{(a - c)\sqrt{\frac{1}{4b'r_\delta(1-e(s))}} + 2(\delta - 1)}{2\delta} < 0. \quad (6) \]

**First stage** I assume that the firm correctly foresees how output levels and product range are determined in the second stage. To derive the firm’s profit function in this stage, I combine optimal scale and scope with the gross profits \( \tilde{\pi} \):

\[ \Pi = \tilde{\pi} - sr_s, \text{ where } \tilde{\pi} = \frac{(a - c)(a - c - 2\sqrt{b'r_\delta(1 - e(s))})}{4be(s)}. \quad (7) \]

Recall that \( s \) denotes the investment in product differentiation which is carried out at costs \( r_s \). The level of \( e(s) \) is determined by:

\[ \frac{\partial e}{\partial s} = e'(s) < 0 \text{ and } \frac{\partial^2 e}{\partial s^2} = e''(s) > 0, \quad (8) \]

where \( e(0) = 1 \) and \( e(\infty) = 0 \). The curvature of \( e(s) \) is of interest as it captures the innovation efficiency of firms. The elasticity of \( e(s) \) with respect to innovation input \( s \) is

\[ e(s) = \frac{e'(s)}{e(s)}. \]

\[ e''(s) = \frac{e''(s)}{e(s)} + \left( \frac{e'(s)}{e(s)} \right)^2. \]

\[ e''(s) < 0 \] for \( e(s) > 0 \) and \( e''(s) > 0 \) for \( e(s) < 0 \).

\[ \frac{\partial^2 e}{\partial s^2} = e''(s). \]

The first-order condition is given by: \( \frac{\partial \pi}{\partial s} = [p - c]x - b'e(s)\delta x^2 - r_\delta = 0. \) Inserting the optimal price \( p = \frac{a + c}{2} \) leads to \( x = \sqrt{\frac{r_\delta}{b'(1 - e(s))}} \). Inserting Eq. (4) gives optimal scope.

\[ \text{Note that the following proposition derives partial derivatives (i.e. for a constant } s). \text{ Later, I will also derive the total derivative.} \]
given by: \( \varepsilon_e(s) \equiv |d \ln e / d \ln s| \equiv \left| \frac{e'(s)}{e(s)} \right| \). The percentage change of \( e(s) \) following an one percentage point increase in \( s \) will be larger, the larger is \( |e'(s)| \).

Maximizing profits with respect to \( s \) leads to the first-order condition:  
\[
\frac{\partial \Pi}{\partial s} = \frac{\partial \tilde{\pi}}{\partial e} e'(s) - r_s = 0, \quad \text{where} \quad \frac{\partial \tilde{\pi}}{\partial e} = -\frac{(a - c) (\delta - \frac{1}{2})}{2e(s)} \sqrt{\frac{r_s}{b'(1 - e(s))}} < 0. \tag{9}
\]

Eq. (9) suggests that it is optimal to invest until the marginal benefits equal the marginal costs of the investment. The marginal benefit of the investment consists of two elements. The first element is the direct effect of a change in the degree of product differentiation on the operating profits: \( \frac{\partial \tilde{\pi}}{\partial e} \). Profits are rising in the degree of product differentiation as this reduces cannibalization. Most importantly, the magnitude of this effect depends on firm size meaning that larger firms will benefit more from the investment. The second element embodies the responsiveness of the differentiation parameter with respect to investments: \( e'(s) \). According to this, the marginal benefit of an investment also depends on the efficiency of transforming research input into output. The larger \( |e'(s)| \), the greater is the impact of the marginal unit of investment.

**Lemma 2** The marginal benefit of an investment depends on (i) the total firm size (determined by scale and scope) and (ii) the efficiency of research input utilization.

Eq. (9) implicitly determines the equilibrium level of product differentiation: \( s^* = s^*(r_s, r_s, b') \). In the next step, I present comparative statics with respect to an increase in market size (globalization). A larger market encourages a firm to introduce additional products (Proposition 1). This makes additional spending on product differentiation attractive to reduce cannibalization among varieties. Furthermore, trade liberalization increases firm size which raises investments through economies of scale as fixed costs can be spread over a larger scale of output.

**Proposition 2** In a larger market, the equilibrium level of investment is higher, i.e.

\[
\frac{d s^*}{dL} = (a - c) \frac{a - c - \frac{2 - e(s)}{2} \sqrt{\frac{b'r_s}{(1 - e(s))}}}{4be(s)^2 \frac{\partial^2 \Pi}{\partial s^2}} e'(s) > 0. \tag{10}
\]

\(7\)Note that the derivative is given by: \( \frac{\partial \tilde{\pi}}{\partial e} = -\frac{(a - c) (\delta - 1)}{4e(s)^2} \sqrt{\frac{e'(s)^2}{1 - e(s)}} \). To derive the result in Eq. (9), I substitute \( a - c = \frac{2(1 - e(s)) + e(s) \delta}{\sqrt{\frac{1 - e(s)}}} \) which follows from Eq. (5).

\(8\)Note that the producer is a MPF which produces more than one variety (i.e. \( \delta > 1 \)).

\(9\)To derive an explicit solution for \( s^* \), I would have to assume a specific functional form for \( e(s) \) that fulfills the properties in Eq. (8).

\(10\)I totally differentiate Eq. (9) and apply the following second-order condition: \( \frac{\partial^2 \Pi}{\partial s^2} = \frac{\partial^2 \tilde{\pi}}{\partial e^2} e'(s) + \frac{\partial \tilde{\pi}}{\partial e} e''(s) < 0 \). Moreover, from Eq. (5) follows that \( a - c = 2 (1 - e(s)) + e(s) \delta \sqrt{\frac{b'r_s}{(1 - e(s))}} \) which guarantees that \( \frac{d s^*}{dL} > 0 \).
2.3 Consumer welfare

To study the impact on consumer welfare, I follow Melitz and Ottaviano (2008) and derive the indirect utility function:

\[ V = I + \frac{\delta (a - c)^2}{8b [(1 - e(s) + e(s)\delta)]}. \]  

Eq. (11) displays "love of variety", i.e.

\[ \frac{dV}{d\delta} = \frac{(a - c)^2}{8b(1 - e(s) + e(s)\delta)^2} \left(1 - e(s)\right) - \delta (\delta - 1) \frac{de}{d\delta} > 0. \]  

Of particular importance is the following property which I call "love of diversity":

\[ \frac{dV}{de} = \frac{(a - c)^2}{8b(1 - e(s) + e(s)\delta)^2} \left(1 - e(s)\right) \frac{d\delta}{de} - \delta (\delta - 1) < 0. \]  

The utility is increasing in the degree of product differentiation whereas consumers value a given product range more when products are more differentiated.

**Lemma 3** Consumer welfare increases in the number of available products (love of variety) and the degree of product differentiation (love of diversity).

The following expression disentangles the gains from trade which are induced by an increase in the market size \( L \):\(^{13}\)

\[ \frac{dV}{dL} = \frac{(1 - e(s))b'x^2 d\delta}{2L \frac{dL}{d\delta}} - \frac{(\delta - 1)b'xX}{2L} e'(s) \frac{ds}{dL} > 0. \]

Eq. (14) highlights two distinct channels of gains from product variety and shows how trade liberalization affects welfare through within-firm adjustments. Consumers benefit from newly introduced varieties which are more diversified. The endogenous choice of investment in product differentiation is the key element of the theory. Trade liberalization enables firms to exploit economies of scale in innovation and increases incentives to invest. Given the opportunity to serve a larger market, a MPF will spend more resources on research for new blueprints or product specific attributes which increase consumer welfare through the "love of diversity" channel. Finally, welfare gains depend on the efficiency of research

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\(^{11}\)I substitute information from Eqs. (2) and (4) into indirect utility: \( V = I + \frac{\delta (a - p)^2}{2b(1 - e(s) + e(s)\delta)}. \)

\(^{12}\)Recall from Proposition 1 that \( \frac{d\delta}{de} < 0. \)

\(^{13}\)Note that: \( \frac{d\delta}{dL} = \frac{a - c}{4e(s)L} \sqrt{\frac{1 - e(s)}{b\sigma}} - \left( \frac{a - c}{4e(s)\sqrt{b\sigma(1 - e(s))}} + \frac{(\delta - 1)e(s)}{c(s)} \right) e'(s) \frac{ds}{dL} > 0. \)
input utilization determined by $e'(s)$. If trade induced investments do not generate more differentiated products because of inefficient innovation (low value of $|e'(s)|$), the welfare gains will be low.

3 Conclusion

This paper is motivated by recent evidence on the importance of intra-firm adjustments. In a simple framework of MPFs, I provide novel insights into the variety gains of globalization. To distinguish between the different welfare channels, I allow firms to choose both product scope and the degree of differentiation among varieties. The latter helps firms to reduce cannibalization among varieties within the portfolio. I highlight this investment as an additional channel through which globalization affects product variety. Consumers benefit from the additional gains as the marginal benefit of any new variety rises in the degree of differentiation.

An interesting avenue for future research would be to allow for import competition of foreign producers. On the one hand, competition reduces the incentives to invest in product differentiation as it is an opposing force to the market size effect and hence, reduces the economies of scale in innovation. On the other hand, external competition from other firms induces a MPF to cut back on internal competition by investing in the degree of product differentiation between varieties.14

4 Bibliography

References


14This result is related to Dhingra (2013) where firms respond to external competition by reducing the number of varieties which lowers internal competition. My framework would suggest investments in the degree of product differentiation as an alternative channel to reduce internal competition.


