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Abstract

In a private values, open auction, we show that bidder surplus can be expressed as a simple difference between unconditional moments of order statistics. The strength of the result is its simplicity and generality, as we dispense with the typical assumptions of independence or symmetry. We show how to use the expression to derive closed-form expressions for the effects of a merger among bidders for any joint value distribution.

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In a private values, open auction, we show that bidder surplus can be expressed as a simple difference between unconditional moments of order statistics. The strength of the result is its simplicity and generality, as we dispense with the typical assumptions of independence or symmetry. We show how to use the expression to derive closed-form expressions for the effects of a merger among bidders for any joint value distribution.

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1 Introduction

Expressions for bidder surplus in private value auctions are used by economists in a variety of contexts, e.g., to predict the price effects of mergers (Werden and Froeb (2008),Brannman and Froeb (2000)), or as primitives in more complex models, like Nash-in-Nash Bargaining (Sheu and Taragin, 2017) or "Score Auctions," (Miller, 2014). In a private-values, open auction, we show that bidder surplus can be expressed as a simple difference between unconditional moments. The strength of the result is its simplicity and generality, as we dispense with typical assumptions like independence. We show how to use the expression to derive closed-form expressions for the effects of a merger among bidders for any joint value distribution, and illustrate its use by simulating the effects of the 2016 proposed Anthem-Cigna merger using a nested logit specification.

2 Expected Bidder Surplus as Difference Between Unconditional Moments

Consider an open auction where two bidders draw private values X and Y from a joint distribution. We assume that X and Y have continuously differentiable marginal distribution functions. The expected surplus for a bidder drawing from Y is the probability that Y is the highest value times the expected difference between the highest and second-highest values, conditional on the Y being the highest value. The following theorem shows that this expression simplifies to a difference of unconditional moments.

Theorem 1. *In a two-bidder, second-price auction, the expected surplus of the bidder Y is a simple difference between unconditional moments,*

$$S_Y \equiv Prob[Y > X] * E[\max(X, Y) - X|Y > X]) = E[\max(X, Y)] - E[X].$$

Proof. Let $p_x = \Pr(X > Y)$.[1] Then

$$(p_x)E[X|X > Y] + (1 - p_x)E[X|X < Y] = E[X]. \tag{1}$$

However, observe that

$$(p_x)E[X|X > Y] + (1 - p_x)E[Y|X < Y] = E[\max(X, Y)]. \tag{2}$$

[1] For random variables with continuous densities, we can ignore the case $\Pr(X = Y)$.

Subtracting the two equations yields

$$(1 - p_x)E[Y - X|X < Y] = E[\max(X, Y)] - E[X]. \tag{3}$$

The left side of the equation is bidder $Y's$ expected surplus, the probability that Y draws the high value times the expected difference between $Y's$ value and the second-highest value, conditional on Y winning.[2] The right side is the expected surplus to bidder Y, per auction. By a similar argument we have that bidder X's expected surplus is simply $E[\max(X, Y)] - E[Y]$. ∎

The result allows us to work with the simple unconditional moments on the right hand side of the expression instead of the conditional ones on the left. So, instead of assuming symmetry or putting restrictions on the distribution of values, as in Tschantz et al. (2000), we can work with more general distributions that admit, e.g., correlation among bidder values.

Generalizing the result is straightforward. For n bidders, replace Y by $\max\limits_{k\neq i}(X_k)$,

$$E[S_i] = E[\max_k(X_k)] - E[\max_{k\neq i}(X_k)]. \tag{4}$$

Because open auctions without reserve prices allocate efficiently, the expected total surplus is $E[\max\limits_k(X_k)]$. It follows that the auctioneer surplus (expected price), is just the expected total surplus minus expected bidder surplus which can be expressed as

$$\begin{aligned} \textit{Auctioneer Surplus} &= \textit{Total Surplus} - \textit{Bidder Surplus} \\ &= E[\max_k(X_k)] - \sum_i \left(E[\max_k(X_k)] - E[\max_{k\neq i}(X_k)] \right) \\ &= \sum_i E[\max_{k\neq i}(X_k)] - (n-1)E[\max_k(X_k)]. \end{aligned}$$

This formula can be used to easily compute suprlus an any auction. For example, in an auction with three bidders, each taking independent draws from a Uniform(0,1) distribution, $E[\max\limits_{k\neq i}(X_k)] = 2/3$, $E[\max\limits_k(X_k)] = 3/4$, so Total Surplus is $3/4$, Auctioneer surplus is $1/2$ and bidder surplus is $1/4$. To check this, note that bidder surplus is $S_i = 3/4 - 2/3 = 1/12$ per auction and

[2] Of course, the same result can be obtained for independent distributions by integrating the probability of winning for individual bidders Myerson (1981). Moreover, the result reflects the fact that in an efficient trading mechanism (open auctions) bidders recover their marginal contribution to the transaction.

since each wins 1/3 of the time, this corresponds to 1/4 per win.

3 A Merger Application

If bidders i and j merge, and bid the maximum of their values, e.g., Werden and Froeb (2008), then the expected surplus of the merging bidders earn $i+j$ is

$$E[S_{i+j}] = E[\max_k(X_k)] - E[\max_{k\neq i,j}(X_k)],$$

and the transfer of surplus from the auctioneer to the merging bidders is

$$\begin{aligned} E[\Delta S_{i+j}] &= E[S_{i+j}] - (E[S_i] + E[S_j]) \\ &= \left(E[\max_{k\neq i}(X_k)] + E[\max_{k\neq j}(X_k)]\right) - \left(E[\max_k(X_k)] + E[\max_{k\neq i,j}(X_k)]\right) \end{aligned}$$

This expression for merger effects generalizes the formulas in Waehrer and Perry (2003) and Tschantz et al. (2000), where bidders draw from independent "power-related" distributions,[3] to any joint value distribution.

3.1 A Nested Logit Auction Merger Model

To illustrate the utility of the result, we derive a nested logit merger model and apply it to the 2016 proposed Anthem-Cigna merger. Without loss of generality, we imagine first two bidders in a nest and a third non-merging bidder, representing the the maximimum of the non-merging bidder values, outside it. The joint distribution of values is

$$F(x_1, x_2, x_3) = \exp\left(-\left(\left(e^{(x_1-\eta_1)\beta/\theta} + e^{(x_2-\eta_2)\beta/\theta}\right)^\theta + e^{(x_3-\eta_3)\beta}\right)\right),$$

e.g., Train (2002), so that bidders draw values from marginal extreme value distributions with the same spread parameter β, but different location parameters η_i,

$$F(x_i) = \exp(-e^{-(x_i-\eta_i)\beta}), E[X] = \eta_i + \frac{\gamma}{\beta}, Var[X_i] = \frac{\pi^2}{6\beta^2},$$

where γ is Euler's Gamma, and π is the ratio of a circle's circumference to its

[3]Power-related distributions are modeled as bidders taking different numbers of draws from the same base distribution.

diameter. The advantage of this distribution for our application is that the expected maxima have closed-form expressions:

$$
\begin{aligned}
E[\max_k(X_k)] &= \frac{1}{\beta}\ln\Big((\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^\theta+\exp(\beta\eta_3)\Big)+\gamma/\beta\\
E[\max_{k\neq 1}(X_k)] &= \frac{1}{\beta}\ln(\exp(\beta\eta_2)+\exp(\beta\eta_3))+\gamma/\beta\\
E[\max_{k\neq 2}(X_k)] &= \frac{1}{\beta}\ln(\exp(\beta\eta_1)+\exp(\beta\eta_3))+\gamma/\beta\\
E[\max_{k\neq 3}(X_k)] &= \frac{1}{\beta}\ln(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^\theta+\gamma/\beta,
\end{aligned}
$$

from which "margins" and merger effects can be easily computed.

The probabilities of choosing each alternative also have closed form expressions, e.g., Train (2002):

$$
\begin{aligned}
p_1 = \mathrm{Prob}[X_1 = \max_k(X_k)] &= \frac{\exp(\beta\eta_1/\theta)(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^{\theta-1}}{(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^\theta+\exp(\beta\eta_3)}\\
p_2 = \mathrm{Prob}[X_2 = \max_k(X_k)] &= \frac{\exp(\beta\eta_2/\theta)(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^{\theta-1}}{(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^\theta+\exp(\beta\eta_3)}\\
p_3 = \mathrm{Prob}[X_3 = \max_k(X_k)] &= \frac{\exp(\beta\eta_3)}{(\exp(\beta\eta_1/\theta)+\exp(\beta\eta_2/\theta))^\theta+\exp(\beta\eta_3)}
\end{aligned}
$$

3.2 Calibrating the Merger Model

The model can be calibrated to observed margins (Equation 4), observed shares $\{s_i\}$:

$$
\begin{aligned}
\log\left(\frac{s_1}{s_2}\right) &= \log\left(\frac{p_1}{p_2}\right)\\
\log\left(\frac{s_1}{s_3}\right) &= \log\left(\frac{p_1}{p_3}\right)
\end{aligned}
$$

or to the frequency of order of finish. If X_1 and X_2 are are in the same nest (positively correlated), then when one wins, the second should finish more frequently than would be implied by independence, e.g.,

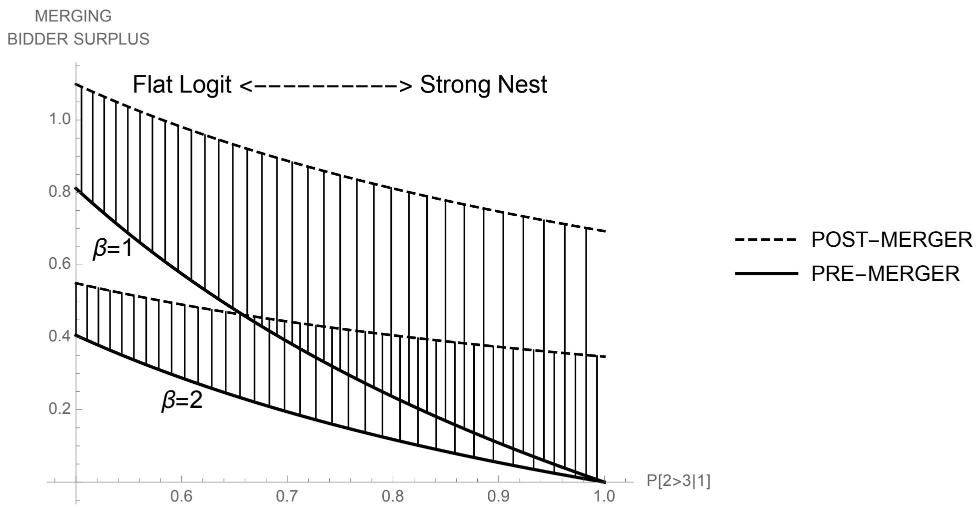

Figure 1: Merger Effects Vary with Correlation Between Merging Bidder Values

$$\begin{aligned} \text{Prob}[X_2 > X_3 | X_1 = \max_k(X_k)] &= \frac{\text{Prob}[X_1 > X_2 > X_3]}{p_1} \\ &= \frac{\text{Prob}[X_2 > X_3] - p_2}{p_1} \\ &= \frac{\frac{\exp(\beta\eta_2)}{\exp(\beta\eta_2)+\exp(\beta\eta_3)} - p_2}{p_1}. \end{aligned}$$

In Figure 1, we plot the pre- and post-merger merging bidder surplus as a function of $\text{Prob}[X_2 > X_3|X_1 = \max_k(X_k)]$, for $\beta = 1$ (larger variance) and for $\beta = 2$ (smaller variance), setting all of the location parameters η_i to zero. These parameter values imply that $\text{Prob}[X_2 > X_3|X_1 = \max_k(X_k)]$ varies from .5 (flat logit) to 1 (X_1 and X_2 are perfectly correlated).

To the right of the graph, where correlation between the merging bidders' values is perfect, there is no pre-merger surplus, because the merging bidders (1 and 2) compete their entire surplus away in the pre-merger equilibrium. In this case, a merger has a bigger effect (the gap between the pre- and post-merger bidder surplus) because more competition is eliminated by merger.

Moving to the left, as the merging bidders' values become less similar, the size of the merger effect is reduced, as there is less pre-merger competition to eliminate. Note that the level of surplus increases because the merging bidders are essentially taking an extra draw. At the very left, surplus is at its highest as the distribution becomes a flat logit, with three independent draws.

Note also that a bigger variance ($\beta = 1$) increases surplus because the difference between the order statistics is bigger, which both increases surplus as well as the change in surplus due to merger.

3.3 Example: Anthem-Cigna

In 2016, The Antitrust Division of the US Dept. of Justice sued to block the health insurer Anthem from acquiring one of its rivals, Cigna. In its complaint, the Division alleged that the "merger would substantially reduce competition for millions of consumers who receive commercial health insurance coverage from national employers throughout the United State."[4]

We use the auction model described here to determine the effect of the Anthem-Cigna merger under two assumptions: that diversion ratios are in proportion to shares (flat logit), or that diversion ratios are determined by win-loss data reported at trial, which we model by putting Anthem and Cigna in the same nest. The flat logit specification is supported by win-loss data showing that when Anthem lost, Cigna won 18% of the time. Assuming that when Anthem loses it is ranked second, then a merger between Anthem and Cigna under this specification raises the merged firm's profit by 13%.

Alternatively, the nested logit specification is supported by win-loss data showing that when Cigna lost, Anthem won about 61% of the time, above the 44% prediction of the flat logit. These data are consistent with a nest parameter of about 0.7.[5] When we simulate the merger using this nested logit specification, the merged firm's profit – and therefore consumer harm – increases by 46%, much bigger than the 13% under the flat logit.

Of course, we could do better by using a mixed logit with bidders drawn

[4]Dranove, David (2016). U.S., et al v. Anthem, Inc. and Cigna Corp: Testimony of David Dranove, Ph.D [redacted PowerPoint slides], pp. 46-47. Retrieved from https://www.justice.gov/atr/page/file/914606/download

[5]At page 16, Dr. Dranove reports various values of critical loss for specific price increases. These suggest a price cost margin of about 80% but do not state precisely what margin the expert found on any particular product. Also, in his testimony, Dr. Dranove indicates that ASO fees are about 6% of the fully insured premium (pg 1,057 of the trial transcript). Taking the 80% as indicative of what Anthem's margin might be and applying it and the 6% figure to publicly available 2016 premium information from Kaiser of $508/month (https://www.kff.org/other/state-indicator/single-coverage), we get a dollar margin of $24.50 that is sufficient for our purposes. For $\{p_1, p_2, p_3, margin_1,$ $\text{Prob}[X_2 > X_3 | X_1 = \max_k(X_k)]\ \} = \{0.389984, 0.109928, 0.500087, 24.50, 0.61\}$, we solve for $\{\eta 1, \eta_2, \eta_3, \beta, \theta\} = \{0, -43.5381, 8.55551, 0.0201346, 0.692285\}$.

from a mixture of distributions. In this case, the dominant-strategy equilibrium of the second-price auction implies that merger effects can be computed as a mixture of auctions. See Froeb and Tschantz (2002) for further details.

4 Discussion

Efficient mechanisms, like open auctions, cannot capture some significant features of competition. For example, in standard oligopoly models, the merged firm faces a market power/efficiency trade-off, e.g., as the merged firm raises price, output falls. In open auctions, however, the merged firm wins all the auctions that the pre-merger firms would have won, so there is no efficiency loss. As a consequence, mergers in open auctions can have bigger effects, sometimes much bigger, than in settings where the exercise of market power results in an efficiency loss, as in optimal auctions, e.g., Froeb et al. (2017).

References

Lance Brannman and Luke Froeb. Mergers, cartels, set-asides and bidding preferences in asymmetric second-price auctions. *Review of Economics and Statistics*, 82(2):283–290, 2000.

Luke Froeb and Steven Tschantz. Mergers among bidders with correlated values. In Daniel J. Slottje, editor, *Measuring Market Power*, volume 255 of *Contributions to Economic Analysis*, pages 31–46. Elsevier, 2002.

Luke Froeb, Vladimir Mares, and Steven Tschantz. Horizontal mergers in optimal auctions. January 2017. URL `https://ssrn.com/abstract=2742187`.

Nathan H. Miller. Modeling the effects of mergers in procurement auctions. *International Journal of Industrial Organization*, 37(November):201–208., 2014.

Roger B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6:58–73, February 1981.

Gloria Sheu and Charles Taragin. Simulating mergers in a vertical supply chain with bargaining. *U.S. Dept. of Justice, EAG 17-3*, October 2017.

Kenneth Train. *Discrete Choice Methods with Simulation*. Cambridge University Press, 2002.

Steven Tschantz, Philip Crooke, and Luke Froeb. Mergers in sealed vs. oral auctions. *International Journal of the Economics of Business*, 7(2):201–213, July 2000.

Keith Waehrer and Marktin K. Perry. The effects of mergers in open-auction markets. *RAND Journal of Economics*, 34(2):287–304, Summer 2003.

Gregory Werden and Luke Froeb. *Issues in Competition Law and Policy*, volume 2, chapter Unilateral Competitive Effects of Horizontal Mergers, page 1343. ABA Section on Antitrust Law, 2008.