

## Volume 38, Issue 4

### Market frictions, misallocation of talent and development

Fernando A Barros Jr

Bruno R Delalibera

*Department of Economics - FEARP/USP and FGV EPGE FGV EPGE Brazilian School of Economics and Finance  
Brazilian School of Economics and Finance*

#### Abstract

We identify a negative relationship between teachers' wage and regional income in the Brazilian data. To explain this fact, we propose a general equilibrium model where workers' decision are distorted due to market frictions and teachers' quality is calculated endogenously as an input for the formation of human capital. Our model is calibrated to Brazilian scenario, matching data and closely reproducing share of workers and the average wage for each state and occupation. Our benchmark economy suggests that there is a misallocation of workers in the Brazilian economy and a reallocation of high skilled workers to teachers' occupation could increase GDP due to a multiplicative effect of teachers' human capital. We also find that the outside options to teachers' career in less developed states are worst than in richer states. This contributes to higher talented workers to choose the teacher's career in poorer states.

---

We thank the editor, John P. Conley, an anonymous referee, Jefferson Bertolai, Bernardo Blum, Ricardo Cavalcanti, Flávio Cunha, Pedro Ferreira, Fábio Gomes, Márcio Laurini, Diego Restuccia and seminar participants at EPGE/FGV-RJ, FEARP/USP-RP and 36th Meeting of the Brazilian Econometric Society for helpful comments. Also, we thank the work of Daniel Galvêas as our research assistant. This work was previously circulated as "The importance of teachers' human capital in an economy with market frictions". Delalibera acknowledges the financial support from CAPES and FAPERJ and Barros Jr. from CNPq/INCT and FAPERJ. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

**Citation:** Fernando A Barros Jr and Bruno R Delalibera, (2018) "Market frictions, misallocation of talent and development", *Economics Bulletin*, Volume 38, Issue 4, pages 2410-2430

**Contact:** Fernando A Barros Jr - fabarrosjr@gmail.com, Bruno R Delalibera - brunodelalibera@gmail.com

**Submitted:** July 18, 2017. **Published:** December 13, 2018.

# 1 Introduction

Teacher quality is extremely relevant to the human capital formation and, thus, it is an important aspect of economic development. Indeed, as teacher quality affects the human capital of all individuals, it might generate an externality for the whole workforce. Thus, the educational sector might be thought as the main intermediate sector of an economy.

Teacher quality as an intermediate good is not a new idea in development literature. Jones (2011) documents that once intermediate goods are used in the production of final goods, the quality of the former can explain part of economic development and determine the failure of a nation. In this paper, we present a general equilibrium model where teacher's human capital is a key input of human capital of other professionals. We show how frictions in the labor and educational markets can influence the allocation of workers among occupations and how a reallocation in the microeconomic level could impact the macro scenario. Next, we calibrate our model to match the Brazilian economy and find evidence that there is a misallocation in the labor force: a higher GDP could be achieved if more talented workers were allocated to teachers' career.

If we consider the idea that the teacher's human capital has a direct impact on students' human capital, we should expect that more developed areas should present a body of teachers with higher quality than less developed regions. Indeed, most of our knowledge was handed down to us by previous generations and a good part of it is due to teachers. Then, qualified teachers may be essential in an economy where better-educated workers tend to be more productive than less educated ones.

In the context of learning on the job, Jovanovic and Nyarko (1995) argue that knowledge transferred from the old to the young is, therefore, a cornerstone of productivity growth. If we can extend this idea to a more general framework, teacher's human capital could be an important determinant of other professionals' human capital. That is, in a society where there are good teachers, we expect to find more productive workers in other occupations. In addition, wages are strongly related to productivity and, therefore, related to human capital when we consider competitive markets. If teacher's human capital is important for development, we should expect that more developed areas should present higher relative teachers' wages than less developed areas. However, according to National Household Sample Survey (PNAD) 2013, state-level Brazilian data display a negative relationship between the GDP per worker and teachers' average wage as proportion of others occupations' average wage, as we can see in Figure 1.

We argue that this inverse relationship between relative teachers' wages and workers' income may be explained by different allocations of talented workers to teachers' career in these areas. There are two basic ideas we would like to link in this paper. First, the occupational choice of multi-ability workers is driven by labor market incentives (net wage) and the costs of investment in specific human capital (acquisition of education). Second, teacher's human capital is an important input for the formation of the human capital of the entire workforce in an economy. Thus, we could infer that there is an *externality* in the occupational choice problem of workers. When many people with lower idiosyncratic ability choose to be a teacher, the average of teachers' human capital is reduced and, therefore, all workers human capital formation is compromised.

In addition, market frictions can determine the occupational choice of individuals. We

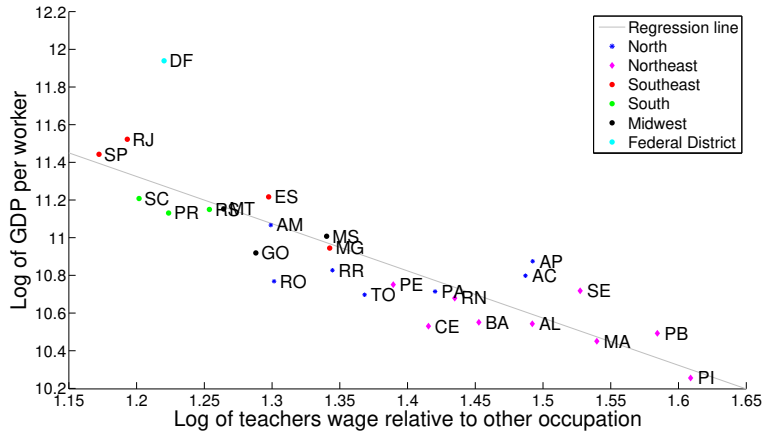


Figure 1: GDP per worker and the ratio of the average wage of teachers to the average wage of others occupation - 2013

interpret the net wage of an occupation as the gross monetary payment minus a distortion, which we call a labor market friction. This friction can be interpreted as a non-observable part of the return of an occupation such as discrimination, prestige of a carrier, the difficulty of finding a job in an occupation, labor market regulation, idiosyncratic power of trade unions, or other forms of expropriation (Jones, 2011). Also, the education cost varies among different occupations and involves both monetary payment for the educational goods and non-observational costs: some courses are not easily available to all individuals or a specific occupation requires an extra amount of effort from the student, etc. Again, we call those non-observable costs of the formation of the human capital as a friction in the educational market.

We combine those ideas in a general equilibrium model where people are heterogeneous concerning abilities for different occupations and there are market frictions distorting their occupational choice. The calibration exercise indicates that relationship in figure 1 can be generated by the model. This happens because, among other calibrated parameters, the market frictions to teachers are higher than other occupations' friction in less developed areas, when compared to more developed states. Therefore, in less developed regions, teachers are more qualified relative to other professions in these states.<sup>1</sup> In a counterfactual exercise, we show that a higher output may be reached by changing those frictions: if we set market frictions of all the Brazilian states equal to the frictions in the state where the average of teachers' human capital is the highest, Brazilian GDP would increase around 60%.

## 2 Model

We consider a Roy model based on Hsieh et al. (2013). There is a continuum of people in the economy working in  $N$  different occupations. People are heterogeneous in two dimensions:

<sup>1</sup>This result can be part of the explanation of the convergence between Brazilian states found by several works. See Ellery and Ferreira (1996), Ferreira (2000) and Azzoni (2001) for some examples.

each person has idiosyncratic abilities and is born in a region  $r \in \{1, \dots, R\}$ . We assume that people cannot choose to supply labor or study in a location different from where they are born.<sup>2</sup> About abilities, people have a different talent for the different occupations: some people can present a high talent for many occupations, some people have a high talent for only one occupation and some people lack skills for any occupation in the economy. An individual values consumption and leisure - which we model as the time not spent in school. Each person is endowed with one unit of time to study or to leisure. The utility of a person is given by

$$U(c, s) = c^\beta(1 - s) \quad (1)$$

where  $c$  represents consumption,  $s$  is the time spent at school and  $\beta$  is a parameter to balance the trade-off between consumption and accumulation of human capital.

There is a distortion in the labor market. People working in occupation  $i$  and region  $r$  is paid a net wage of  $(1 - \tau_{ir}^w)w_{ir}$  where  $w_{ir}$  is the wage per efficiency unit of labor paid by a firm and  $\tau_{ir}^w$  is a distortion specific for occupation  $i$  and location  $r$ . Human capital choices are also distorted due to a ‘tax’ on educational goods. For each good invested in education, a person pays  $\tau_{ir}^h$  as a ‘tax’.

The formation of human capital of a worker in a region  $r$  is given by:

$$h_r(e, s) = H_{tr}^\varphi s^{\phi_i} e^\eta, \quad (2)$$

where  $e$  is the consumption of educational goods,  $s$  is the time spent in school,  $H_{tr}$  the aggregate human capital of teachers and  $\phi_i$  is the elasticity of human capital with respect to time in school. Notice that this parameter varies among occupations and generates differences in schooling as shown below.

Following McFadden (1974) and Eaton and Kortum (2002), abilities dispersion is modeled as a multivariate Fréchet distribution. Let  $\epsilon_i$  be the ability of an individual in occupation  $i$ , then the distribution of abilities is given by

$$F(\epsilon_1, \dots, \epsilon_N) = \exp \left[ - \left( \sum_{i=1}^N \epsilon_i^{-\frac{\tilde{\theta}}{1-\rho}} \right)^{1-\rho} \right], \quad (3)$$

where  $\tilde{\theta}$  governs the skill dispersion and  $\rho \in [0, 1]$  gives the correlation of individual’s skills among occupations. A higher  $\tilde{\theta}$  implies a smaller dispersion in abilities. Also,  $\rho = 1$  indicates that skills are perfect correlated, while a  $\rho = 0$  means that individual’s skills are uncorrelated among occupations. For convenience, let  $\theta = \tilde{\theta}/(1 - \rho)$ .

Finally, the firms hire workers in all regions and occupations to produce a single good. We assume that there is large number of homogeneous firms, i.e., there is no firm with market power. The representative firm has the following production function

$$Y = \sum_{r=1}^R \sum_{i=1}^N A_r H_{ir}, \quad (4)$$

---

<sup>2</sup>In accord to the Brazilian Census of 2010, almost 80% of the population have never moved from home state. And if we consider people who are in the same state for more than 20 years, this number increases to almost 88%. Therefore, we believe that the assumption that people can not move from the region where they are born is not a extreme one.

where  $Y$  is the output,  $A_r$  is the Total Factor Productivity (TFP) of region  $r$  and  $H_{ir}$  is the aggregate human capital of people working in occupation  $i$  at region  $r$ . Then, the firms problem is choosing labor in terms of efficient units (aggregate human capital) to maximize profit taking as given the wages of each occupation:

$$\text{Max}_{H_{ir}} \sum_{r=1}^R \sum_{i=1}^N A_r H_{ir} - \sum_{r=1}^R \sum_{i=1}^N w_{ir} H_{ir}. \quad (5)$$

The solution of the problem described in (5) is trivial. The demand for human capital is

$$H_{ir}^d = \begin{cases} 0 & \text{if } A_r < w_{ir} \\ x \in \mathbb{R}_+ & \text{if } A_r = w_{ir} \\ \infty & \text{if } A_r > w_{ir} \end{cases}. \quad (6)$$

The worker's problem can be solved in two steps. First, given the occupational choice  $i$ , for which the individual has an idiosyncratic ability  $\epsilon$ , and taking wage  $w_{ir}$  as given, each worker chooses consumption  $c$ ,  $e$  and  $s$  to solve the following problem:

$$\text{Max}_{c,s,e} c^\beta (1-s) \quad (7)$$

$$\text{s.t. } c = (1 - \tau_{ir}^w) h_r(e, s) \epsilon w_{ir} - (1 + \tau_{ir}^h) e.$$

Solving this problem above, we find the amount of time and goods spent on human capital accumulation:

$$s_i^* = \left( 1 + \frac{1-\eta}{\beta \phi_i} \right)^{-1}, \quad (8)$$

$$e_{ir}^*(\epsilon) = \left( \frac{1 - \tau_{ir}^w}{1 + \tau_{ir}^h} \epsilon w_{ir} \eta s_i^{*\phi_i} H_{ir}^\varphi \right)^{\frac{1}{1-\eta}}. \quad (9)$$

Given an occupation  $i$ , the higher is the elasticity of human capital concerning time, the higher is the time spent accumulating human capital. Individuals in high  $\phi_i$  occupations acquire more schooling and have higher wages to compensate their time spent on schooling. Note that the wage and distortions do not affect schooling because they have the same effect on the return and on the cost of time.<sup>3</sup> However, they can change the returns of investment in goods in human capital, relative to the cost, with an elasticity that is increasing in  $\eta$ .

We can substitute the expressions in equations (8) and (9) and the budget constraint into the utility function and get the following expression for indirect utility function for occupation  $i$ :

$$U_{ir} \equiv U(\tau_{ir}^w, \tau_{ir}^h, H_{tr}, w_{ir}, \epsilon_i) = \left[ \left( \frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} \right) \eta^\eta (1-\eta)^{1-\eta} H_{tr}^\varphi w_{ir} \epsilon_i s_i^{\phi_i} (1-s_i)^{\frac{1-\eta}{\beta}} \right]^{\frac{\beta}{1-\eta}}. \quad (10)$$

---

<sup>3</sup>In the appendix B we present some data showing that the average schooling is similar in the Brazilian data giving an occupation.

Therefore, the occupational choice problem reduces to picking the occupation that delivers the highest value of  $U_{ir}$ . Since talent is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution (McFadden, 1974). The overall occupational share can then be obtained by aggregating the optimal choice, as we show in the next proposition.

**Proposition 1.** *Aggregating among people, the solution of individual's occupational choice problem allows us to write*

$$p_{ir} = \frac{\tilde{w}_{ir}^\theta}{\sum_{j=1}^N \tilde{w}_{jr}^\theta}, \quad (11)$$

where  $p_{ir}$  is the fraction of people that work in occupation  $i$  in region  $r$  and

$$\tilde{w}_{ir} = \left( \frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} \right) H_{tr}^\varphi w_{ir} s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}.$$

*Proof.* See appendix A □

We can interpret  $\tilde{w}_{ir}$  as a liquid reward for a person with mean ability from region  $r$  working in occupation  $i$ .  $\tilde{w}_{ir}$  is composed by wage per efficiency unit in the occupation  $w_{ir}$ , schooling, teacher's human capital and frictions. The occupational sorting depends on  $\tilde{w}_{ir}$ . Moreover, it depends on the relative returns and not absolute returns, as we can see from equation (11).

Also, we can write an expression for the average quality of worker in occupation  $i$  for each region.

**Proposition 2.** *For a given region, the average quality of workers in occupation  $i$ , including both human capital and idiosyncratic abilities, is*

$$\mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \gamma \left[ H_{tr}^\varphi \left( \frac{(1 - \tau_{ir}^w)w_{ir}}{1 + \tau_{ir}^h} \right)^\eta \eta^\eta s_i^{\phi_i} p_{ir}^{-\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}, \quad (12)$$

where  $\gamma = \Gamma(1 - (\theta(1 - \rho))^{-1}(1 - \eta)^{-1})$  is related to the mean of the Fréchet distribution for abilities.

*Proof.* See appendix A □

This result shows that there is a selection effect in the economy. The average quality is inversely related to the share of the group in the occupation  $p_{ir}$ . Giving a region, if the distortion is high for a occupation  $i$ , than only the most qualified workers are select to that occupation. For example, in a region where it is easy to become a teacher, the average quality of a teacher will be low.

Next, given a region and an occupation, we solve the model for the average wage.

**Proposition 3.** *Let  $W_{ir}$  be the gross average earnings in occupation  $i$  in region  $r$ . Then,*

$$W_{ir} = w_{ir} \mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \frac{(1 - s_i)^{-1/\beta}}{(1 - \tau_{ir}^w)} \gamma \eta \left( \sum_{s=1}^N \tilde{w}_{sr}^\theta \right)^{\frac{1}{\theta(1-\eta)}}. \quad (13)$$

The proof of this proposition is straightforward given the results of Proposition 2.

Equation (13) states that gross average earnings for a given region differs among occupations due to schooling and labor market frictions. Occupations in which schooling is especially productive or occupation where labor market friction are high have a higher gross average earnings. In addition, equation (13) has important consequences for explaining differences between average earnings across regions giving an occupation. From equation (6), we can conclude that in equilibrium that  $A_r = w_{ir}$ . Then,  $\tilde{w}_{ir}$  is a function of  $A_r$  and consequently  $W_{ir}$  is a function of regional TFP too. Therefore, frictions, schooling and TFP are important sources of variation in the average wage across states.

The competitive equilibrium in our model is the straightforward: it consists optimal choices for individuals and firms such that, given prices, wages and market frictions, there is market clearing in each market.

### 3 An empirical investigation

Our calibration strategy involves choosing values for the parameters in our model such that the equilibrium implications of the model are consistent with brazilian data at state level for the year of 2013.<sup>4</sup> We split the parameters into two groups. The first group is calibrated in accordance to Table 1. The remaining parameters,  $\phi$ 's,  $\tau$ 's and  $A$ 's, are calibrated using a minimization of the distance between statistics of our simulated model and brazilian economy. Also, we set the number of occupations to 7. See the appendix B for more details of our calibration strategy.

Table 1: Constant parameters between occupation and region

Parametes	Value	Description	Source
$\beta$	0.69	Consumption preference	Hsieh et al. (2013)
$\eta$	0.25	Elasticity of education goods in the human capital function	Assumption
$\varphi$	0.25	Elasticity of Professor human capital in the human capital function	Assumption that $\eta = \varphi$
$\theta$	3.44	Dispersion of skills	Hsieh et al. (2013)
$\rho$	0.19	Correlation of an individual's skill	PNAD 2013

The calibrated model has an excellent fit to GDP per worker data, which we do not use as a target, as Figure 2 shows.<sup>5</sup> In addition, our calibrated model suggests a positive relation between TFP and GPD per worker at the state level as it is shown in Figure 3. This feature is in accordance to the development literature which claims that more developed areas have stronger institutions and better infrastructure, which supports a higher Total Factor Productivity.<sup>6</sup> We interpret these two features of our empirical exercises as the first check that our model can be useful to study problems of development.

<sup>4</sup>We choose this specific year because it is the last year before a recession hit the brazilian economy.

<sup>5</sup>The GDP per worker for each state in our model is generated by making a weighted mean of each occupational wages multiplied by TFP of that region, where the weights are the share of workers in that occupation.

<sup>6</sup>For some references on this subject see Klenow and Rodriguez-Clare (1997), Hall and Jones (1999) and Erosa et al. (2010).

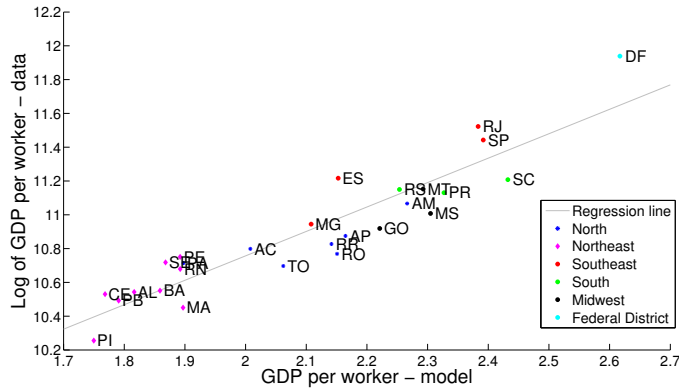


Figure 2: GDP per worker - Data and Model

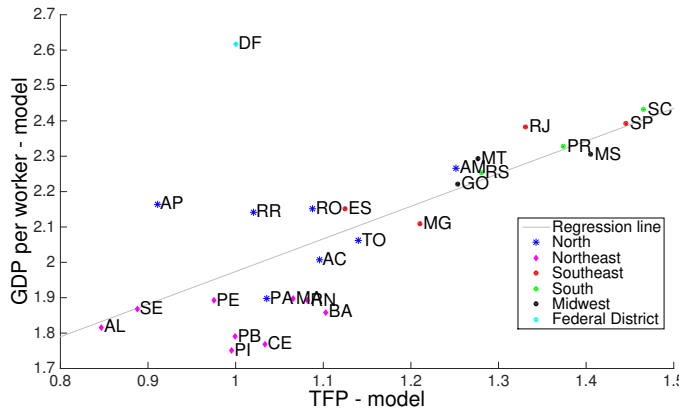


Figure 3: GDP per worker and TFP

For a better understanding of why we see the inverse relationship between GDP per worker and relative teachers' wage, note that there is a strong positive relationship between the share of people working as a teacher and the relative wage of teachers to other occupations, as we can see in figure 4. This is because the teachers' career becomes relatively more attractive when the average of teachers' wage is higher than the average wage in other occupations, therefore more people choose this occupation.

In richer states the relative teacher's wage is lower than in poorer states, as we observe in the data (see Figure 1). There are two reasons behind this. First, in Brazil a significant share of teachers works in the public sector, which has a minimum wage regulated by law and is equal for all Brazilian states.<sup>7</sup> Then, in less developed states this public policy has a direct impact on the relative wage because given their stage of development the average wage is slightly above the minimum wage for teachers. Second, the teacher's occupation is labor intensive and it is not so affected by technological transformation, infrastructure and trade as other occupation such as engineering for example. Then, the teacher's occupation is less sensitivity to global economic change. Thus, in developed states, where institutions

<sup>7</sup>See Brazilian law 11.738 of 2008.



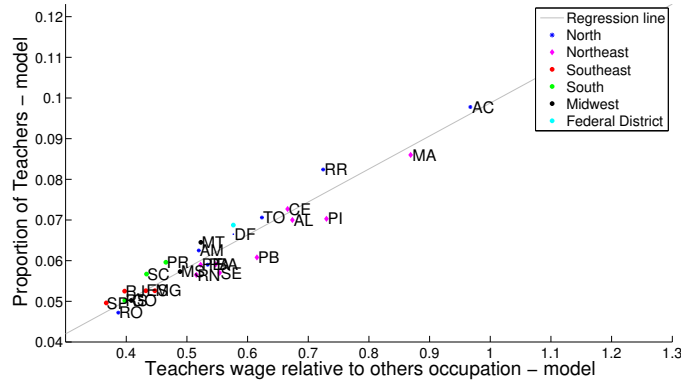


Figure 4: Share of workers in the teacher occupation and teachers' relative wage

and technology are more advanced, teacher's wages relative to other professional are lower than in less developed areas.

There is an interesting result about the allocation of talent among occupations. In poor states, relatively to the richer states, a bigger share of talented workers choose to be a teacher due to the less attractive outside options. As a result, there is a negative relationship between GDP per worker at the state level and average teachers' human capital. Figure 5 presents this result.

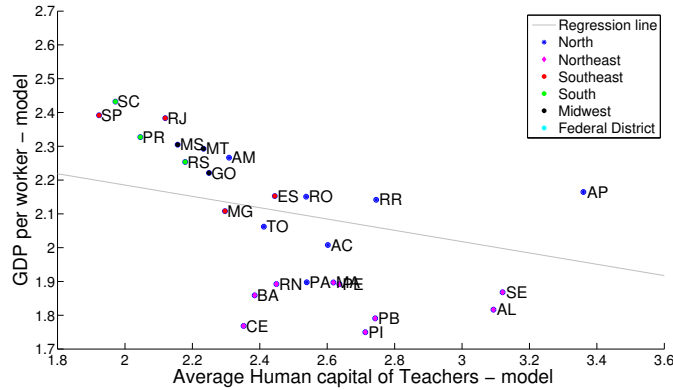
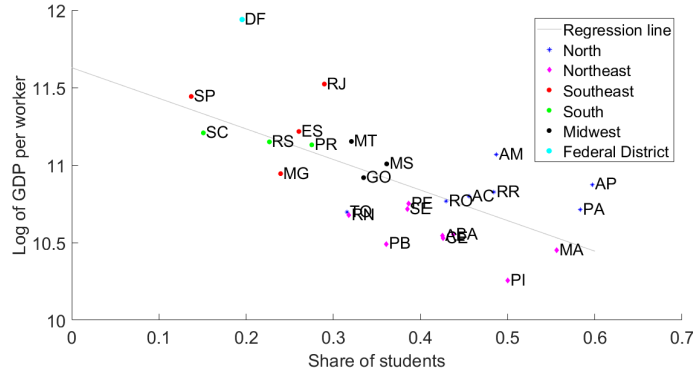


Figure 5: GDP per worker and the average of teachers' human capital

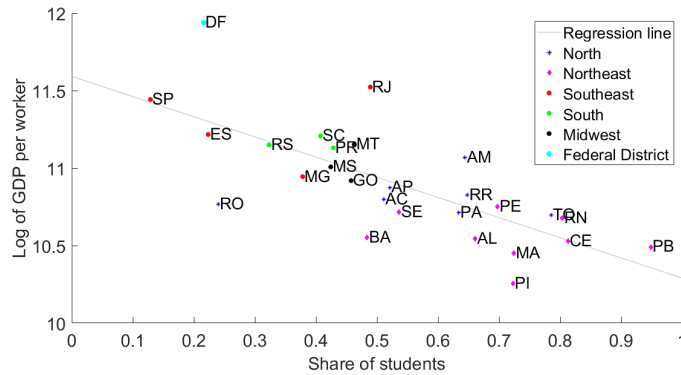
The relation presented in Figure 5 is captured in the model due to the market frictions:  $\tau^w$  and  $\tau^h$ . Friction  $\tau^w$  indicates labor market requirement to get and keep a job and it is the main factor to select workers with high human capital to occupations. Then,  $\tau^w$  has a direct impact on the average human capital of workers implying that occupations with a high professional requirement present a higher average human capital.

On the other hand,  $\tau^h$  summarizes several facts related to educational requirement such as difficulty to be accepted by an university and difficulty to accomplish such course. Furthermore, public policies can play a role in creating or increasing distortions in the educational sector. For example, if the government offers relatively more vacancies in courses related to teaching in poorer states, then we should expect people choosing their career according to

this incentive.<sup>8</sup> Indeed, Figure 6 display the negative relation between GDP per worker and the share of students enrolled in courses related to teaching in brazilian public universities at state level. Figure 6a presents the ratio of students in teaching courses in public universities to the total of students in public universities and Figure 6b presents the ratio of students in teaching courses in public universities to the total of students in teaching courses in both public and private universities.



(a) Relative to the total of students enrolled in the public universities



(b) Relative to the total of students enrolled in teaching courses (public and private universities)

Figure 6: GDP per worker and teaching related courses enrollment in public universities in 2013

The negative relation presented in Figure 6a helps us to understand why in poorer states teachers'  $\tau^h$  is lower than in more developed states while Figure 6b gives us an intuition of why teachers'  $\tau^w$  is higher in poorer states. As public education is subsidized by government, the higher share of students enrolled in teaching related courses in poorer states (see Figure 6a) represents the relative higher subsidies to become a teacher. Also, given that the public budget is limited, this mean that teaching related courses consumes a higher share of the public budget in poorer states.<sup>9</sup> For instance, in Figure 6a "São Paulo" (SP), one of the

<sup>8</sup>Public education is completely subsidized by government, i.e., public education is free in Brazil.

<sup>9</sup>As consequence, other courses are responsible for o lower share of public budget in poor states relative to richer states.

richest Brazilian states, has approximately 14% of the students in teaching related courses, while "Maranhão" (MA), one of the poorest Brazilian states, has approximately 56%. On the other hand, public universities students have higher scores on average in standardized tests relative to private universities students in all Brazilian states (Cavalcanti et al., 2010), i.e, an evidence that public universities students have higher idiosyncratic human capital when compared to private universities students. Thus, the higher share of students in public education vis-à-vis private education (see Figure 6b) implies a higher average human capital of students in poorer states. For example, in Figure 6b "São Paulo" (SP) has approximately 13% of the students in public universities, while "Maranhão" (MA) has approximately 72%.<sup>10</sup>

Next, we address the following question: how would Brazilian GDP be in 2013 if some market frictions were equal across all regions? This question also helps us to study how differences across states had affected the Brazilian economy. In this sense, we analyze a workforce reallocation impact among occupations observing teachers' case closely.

Table 2 presents an experiment where we use the calibrated market frictions,  $\tau$ 's, of two states, Distrito Federal and São Paulo, which are, respectively, the states with the highest and lowest Average of Teachers' Human Capital (ATHC). In this exercise, we equalize the frictions of all the Brazilian states and solve the model for regional and aggregate GDP.

Table 2: Variation of GDP - Changing frictions of the highest and lowest ATHC

	ATHC	ATHC for each state	
		Minimum Value	Maximum value
Brazil	-	- 10.9%	69.44%
Richest states			
Distrito Federal (DF)	3.45	- 47.41%	0
Santa Catarina (SC)	1.97	0.4%	90.93%
São Paulo (SP)	1.92	0	90.16%
Rio de Janeiro (RJ)	2.12	- 11.38%	68.52%
Paraná (PR)	2.05	- 4.78%	81.07%
Poorest states			
Bahia (BA)	2.38	- 14.26%	63.05%
Alagoas (AL)	3.09	- 40.94%	12.31%
Pernambuco (PB)	2.74	-23.25%	45.94%
Ceará (CE)	2.35	- 18.27%	55.41%
Piauí (PI)	2.71	- 21.92%	48.48%

The first line in table 2 presents the variation in the Brazilian GDP while the other lines present the same variation for the five richest and the five poorest states. If the market frictions (labor and educational markets) of all states were equal to the Distrito Federal (DF), GDP would be 69.44% higher, while if those distortions were equal to São Paulo's distortions, GDP would be 10.9% lower. We can interpret it as a misallocation of talent in the economy. When we change market frictions, there is a reallocation of talent among

<sup>10</sup>We believe it is important to find similar explanations for frictions for other occupations. Unfortunately, we do not address that in this paper. Our main goal here is proposing a model for a better understanding of the negative relationship between teachers' wage and regional income in the Brazilian data and we believe that in the heart of this problem are the frictions in labor and education markets.

occupations. Since there is an externality in choosing the teacher occupation, a change that gives incentive to more talented people to become a teacher, increases the overall human capital of the economy and, therefore, a higher GDP can be attained. Note that the effect is heterogeneous among states. In the richest Brazilian states, with exception of DF which have the highest ATHC, increasing the incentives for high talent people to become a teacher has a significant impact on the GDP. For example, São Paulo would have a increase in GDP greater than 90%. The opposite occurs when there are incentives to less talented people choosing the teacher career. The only exception is the state of Santa Catarina, all other states would present a negative impact on GDP.<sup>11</sup>

## 4 Final remarks

This paper studies the negative relationship between relative teacher's wages and GDP per worker across Brazilian states for 2013. Its novelty is to incorporate teacher's endogenous quality as a human capital formation input into an otherwise standard Roy model. In this sense, teachers' human capital may benefit workforce as a whole, hence, educational sector may be thought as the main intermediate sector of an economy. We use the model to measure teachers' average human capital which generates the negative relationship between relative teacher's wage and GDP per worker across Brazilian states.

We also find that there is a better outside option to teachers' career in more developed regions when compared to less developed ones which contributes for higher talented workers choose teaching careers in less developed areas. Furthermore, there is a multiplicative impact of a small change in the teacher's human capital, so that if we give the same incentives of the state with highest teachers' human capital to every worker in all other Brazilian states, Brazilian GDP would increase around 60%.

This work may be improved in several dimensions. For example: (i) one could extend the model in order to allow workers moving from one region to another and then study the reallocation effect due to migration; (ii) This model could be used to understand how different steps of human capital might generate different outcomes for children. Therefore, a different reallocation of teachers among educational stages may produce different results and some misallocation could be identified. Thus, our exercise is a provocative one for those who are interested in the interaction between human capital and development. Empirically, the main challenge is to find evidences that justify the (relative) values of the calibration of the friction on the labor and educational markets. In this paper, we present only a piece of information that can be interpreted as part of the friction in the educational market. Future research could address this topic.

---

<sup>11</sup>In our calibrated model, GDP and TFP are strongly related. Then, the result of the experiment with talent reallocation could be a direct effect of a better use of productivity. We run a simple exercise where we increase the TFP of all states to the level of the highest TFP at the state level. As a result, GDP would increase about 28%. Therefore, the effect of a workforce reallocation is almost three times stronger than an increase in the TFP.

## References

- Azzoni, C. R. (2001, Feb). Economic growth and regional income inequality in brazil. *The Annals of Regional Science* 35(1), 133–152.
- Bils, M. and P. J. Klenow (2000). Does schooling cause growth? *American economic review*, 1160–1183.
- Cavalcanti, T., J. Guimaraes, and B. Sampaio (2010). Barriers to skill acquisition in Brazil: Public and private school students performance in a public university entrance exam. *The Quarterly Review of Economics and Finance* 50(4), 395–407.
- Eaton, J. and S. Kortum (2002). Technology, Geography, and Trade. *Econometrica* 70(5), 1741–1779.
- Ellery, J. R. d. G. and P. C. Ferreira (1996). Convergência entre a renda per-capita dos estados brasileiros. *Brazilian Review of Econometrics* 16(1), 83–103.
- Erosa, A., T. Koreshkova, and D. Restuccia (2010). How important is human capital? a quantitative theory assessment of world income inequality. *Review of Economic Studies* 77(4), 1421–1449.
- Ferreira, A. (2000). Convergence in brazil: recent trends and long-run prospects. *Applied Economics* 32(4), 479–489.
- Hall, R. E. and C. I. Jones (1999). Why do some countries produce so much more output per worker than others? Technical report, National bureau of economic research.
- Hsieh, C.-T., E. Hurst, C. I. Jones, and P. J. Klenow (2013). The Allocation of Talent and U.S. Economic Growth. NBER Working Papers 18693, National Bureau of Economic Research, Inc.
- Jones, C. I. (2011). Intermediate goods and weak links in the theory of economic development. *American Economic Journal: Macroeconomics* 3(2), 1–28.
- Jovanovic, B. and Y. Nyarko (1995). The transfer of human capital. *Journal of Economic Dynamics and Control* 19(5), 1033 – 1064.
- Klenow, P. and A. Rodriguez-Clare (1997). The neoclassical revival in growth economics: Has it gone too far? In *NBER Macroeconomics Annual 1997, Volume 12*, pp. 73–114. MIT Press.
- McFadden, D. (1974). Conditional Logit Analysis of Qualitative Choice Behavior. In P. Zarembka (Ed.), *Frontiers of Econometrics*, pp. 105 – 142. New York, NY: Academic Press.

## A Proofs

**Proposition 1.** *Aggregating among people, the solution of individual's occupational choice problem allows us to write*

$$p_{ir} = \frac{\tilde{w}_{ir}^\theta}{\sum_{j=1}^N \tilde{w}_{jr}^\theta}, \quad (11)$$

where  $p_{ir}$  is the fraction of people that work in occupation  $i$  in region  $r$  and

$$\tilde{w}_{ir} = \left( \frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} \right) H_{tr}^\varphi w_{ir} s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}.$$

*Proof.* Let

$$\tilde{w}_{ir} = \left( \frac{1 - \tau_{ir}^w}{(1 + \tau_{ir}^h)^\eta} \right) H_{tr}^\varphi w_{ir} s_i^{\phi_i} (1 - s_i)^{\frac{1-\eta}{\beta}}, \quad (14)$$

where  $s_i$  is given by equation (8). Then, we can rewrite equation (10) as

$$U_{ir} = [\tilde{w}_{ir} \epsilon_i]^{\frac{\beta}{1-\eta}}.$$

Therefore, the solution of individual's problem in region  $r$  involves picking the occupation with the highest value of  $\tilde{w}_{ir} \epsilon_i$ .

Without loss of generality, consider the probability of an individual choose occupation 1

$$\begin{aligned} p_{1r} &= \Pr(\tilde{w}_{1r} \epsilon_1 > \tilde{w}_{ir} \epsilon_i) \quad \forall i \neq 1 \\ &= \Pr\left(\epsilon_i < \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}} \epsilon_1\right) \quad \forall i \neq 1 \\ &= \int F_1(\alpha_1 \epsilon, \alpha_2 \epsilon, \dots, \alpha_N \epsilon) d\epsilon \end{aligned} \quad (15)$$

where  $F_1$  represents the derivative of 3 with respect to its first argument and  $\alpha_i = \tilde{w}_{1r}/\tilde{w}_{ir}$  for  $i \in \{1, \dots, N\}$ . Taking the derivative of 3 with respect to  $\epsilon_1$  and evaluating at the appropriate arguments gives

$$F_1(\alpha_1 \epsilon, \alpha_2 \epsilon, \dots, \alpha_N \epsilon) = \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp\left[-(\hat{S} \epsilon^{-\theta})^{1-\rho}\right]$$

where  $\hat{S} = \sum_{i=1}^n \alpha_i^{-\theta}$ . Then, (15) can be written as

$$\begin{aligned}
p_{1r} &= \int \frac{\hat{S}^{1-\rho}}{\hat{S}} \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp \left[ - \left( \hat{S} \epsilon^{-\theta} \right)^{1-\rho} \right] d\epsilon \\
&= \frac{1}{\hat{S}} \int \hat{S}^{1-\rho} \hat{S}^{-\rho} \theta \epsilon^{-\theta(1-\rho)-1} \exp \left[ - \left( \hat{S} \epsilon^{-\theta} \right)^{1-\rho} \right] d\epsilon \\
&= \frac{1}{\hat{S}} \int dF(\epsilon) \\
&= \frac{1}{\hat{S}} \\
&= \frac{1}{\sum_i \left( \frac{\tilde{w}_{1r}}{\tilde{w}_{ir}} \right)^{-\theta}} \\
&= \frac{\tilde{w}_{1r}^\theta}{\sum_{i=1}^N \tilde{w}_{ir}^\theta}.
\end{aligned} \tag{16}$$

This argument can be easily extended to occupation  $i$ . □

**Proposition 2.** *For a given region, the average quality of workers in occupation  $i$ , including both human capital and idiosyncratic abilities, is*

$$\mathbb{E}[h(e_{ir}, s_i)\epsilon_i] = \gamma \left[ H_{tr}^\varphi \left( \frac{(1 - \tau_{ir}^w)w_{ir}}{1 + \tau_{ir}^h} \right)^\eta \eta^\eta s_i^{\phi_i} p_{ir}^{-\frac{1}{\theta}} \right]^{\frac{1}{1-\eta}}, \tag{12}$$

where  $\gamma = \Gamma(1 - (\theta(1 - \rho))^{-1}(1 - \eta)^{-1})$  is related to the mean of the Fréchet distribution for abilities.

*Proof.* First, notice that

$$H_{ir} = p_{ir} \mathbb{E}(h(e_{ir}, s_i)\epsilon_i | \text{person chooses } i), \text{ and} \tag{17}$$

$$h(e_{ir}, s_i)\epsilon_i = H_{tr}^\varphi \left[ \frac{(1 - \tau_{ir}^w)}{(1 + \tau_{ir}^h)} w_i \epsilon_i \eta H_{tr}^\varphi s_i^{\phi_i} \right]^{\frac{\eta}{1-\eta}} s_i^{\phi_i} \epsilon_i, \tag{18}$$

where  $H_{ir}$  is the total efficiency units of labor supplied to occupation  $i$  in region  $r$ . Then,

$$H_{ir} = p_{ir} \tilde{h}_{ir} \left[ \frac{(1 - \tau_{ir}^w)}{(1 + \tau_{ir}^h)} w_i \right]^{\frac{\eta}{1-\eta}} \mathbb{E} \left( \epsilon_i^{\frac{1}{1-\eta}} | \text{person chooses } i \right) \tag{19}$$

where  $\tilde{h}_{ir} = (H_{tr}^\varphi s_i^{\phi_i} \eta^\eta)^{(1-\eta)^{-1}}$ .

To calculate this last conditional expectation, we use the Fréchet distribution. For now, we suppress the region index  $r$ , because this calculation is similar for all regions. Let  $y_i = \tilde{w}_i \epsilon_i$ . Since  $y_i$  is the thing we are maximizing, it inherits the extreme value distribution:

$$\Pr\left(\max_i y_i < z\right) = \Pr(\epsilon_i < z/\tilde{w}_i) \quad \forall i \quad (20)$$

$$= F(z/\tilde{w}_1, \dots, z/\tilde{w}_N) \quad (21)$$

$$= \exp\left[-\left(\sum_{i=1}^N (z/\tilde{w}_i)^{-\theta}\right)^{1-\rho}\right] \quad (22)$$

$$= \exp\left[-\left(\hat{S}z^{-\theta}\right)^{1-\rho}\right] \quad (23)$$

$$(24)$$

That is, the extreme value also has a Fréchet distribution.

Straightforward algebra then reveals that the distribution of  $\epsilon^*$ , the ability of people in their chosen occupation, is also Fréchet:

$$G(x) = \Pr(\epsilon^* < x) = \exp\left[-\left(\hat{S}^*x^{-\theta}\right)^{1-\rho}\right], \quad (25)$$

where  $\hat{S}^* = \sum_{i=1}^N (\tilde{w}_i/\tilde{w}^*)^\theta$ .

Finally, one can then calculate the expectation we needed above, back in equation (19). Let  $i$  denote the occupation that the individual chooses, and let  $\lambda$  be some positive exponent. Then,

$$E(\epsilon_i^\lambda) = \int_0^\infty \epsilon_i^\lambda dG(\epsilon) \quad (26)$$

$$= \int_0^\infty \theta(1-\rho)\hat{S}^{*(1-\rho)}\epsilon^{-\theta(1-\rho)-1+\lambda} \exp\left[-\left(\hat{S}^*\epsilon^{-\theta}\right)^{1-\rho}\right] d\epsilon \quad (27)$$

$$= \hat{S}^{*\lambda/\theta} \int_0^\infty x^{-\frac{\lambda}{\theta(1-\rho)}} \exp(-x) dx, \quad (28)$$

where  $x = (\hat{S}^*\epsilon^{-\theta})^{1-\rho}$ . The last part of (28) is a gamma function which amounts to  $\Gamma(1 - \lambda(\theta(1-\rho))^{-1})$ .<sup>12</sup> Therefore, we have

$$\mathbb{E}\left(\epsilon_i^{\frac{1}{1-\eta}} \mid \text{person chooses } i\right) = \left(\frac{1}{p_i}\right)^{\frac{1}{\theta(1-\eta)}} \Gamma\left(1 - \frac{1}{\theta(1-\rho)} \frac{1}{1-\eta}\right). \quad (29)$$

Using this result in the equation (19) completes the proof.  $\square$

## B Calibration

We use microdata from 2013 PNAD - Pesquisa Nacional por Amostras de Domicílios de 2013. From this dataset we extract four variables: years of schooling, worked hours, gross earnings

---

<sup>12</sup>Remember that a gamma function is

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$



and occupation. We drop out our sample individuals with no earnings or without occupation. Furthermore, we just select individuals that are between 25 and 55 years old. Also, we exclude individuals with earnings inferior to R\$1.00/h.<sup>13</sup> With respect to occupations, we drop individual with ‘bad defined’ occupations and individuals working in the army.<sup>14</sup> After all these exclusions we have a sample with 101740 individuals distributed among 8 big groups of occupation: 1) managers (except public sector); 2) professionals of sciences and arts; 3) middle level technicians; 4) administrative service workers; 5) service-sector workers; 6) sellers and service providers; 7) agricultural workers; 8) workers of goods and industrial production, services and repairs-maintenance. We aggregate groups 4, 5 and 6 into the service-sector workers. Finally, we create the teacher occupation, where we put all workers in this career. Thus, we get the following categories of occupation:

1. managers (except public sector);
2. professionals of sciences and arts (except teachers);
3. middle level technicians(except teachers);
4. service-sector workers;
5. agricultural workers;
6. workers of goods and industrial production, services and repairs-maintenance;
7. teachers.

We choose the number of regions in model such that it accounts for the 26 states and the federal district ( $R = 27$ ) in Brazil. Thus, in our calibrated model presents  $N = 7$  and  $R = 27$ .

The parameter  $\beta$  is the consumption preference relative to leisure. This parameter is equal to 0.69, which is the value found by literature (Hsieh et al., 2013). The elasticity of education goods in the human capital function,  $\eta$ , is set equal to 0.25, which follows the chosen value by Hsieh et al. (2013). Bils and Klenow (2000) assume some different values to the elasticity of teachers’ human capital in the human capital production function,  $\varphi$ . They use values varying between 0 and 0.67. Thus, we chose 0.25 which is a conservative value (and it is equal to  $\eta$ ). The parameter  $\theta$  gives the dispersion of comparative advantage of an individual in a specific occupation and  $\rho$  is the correlation of an individual’s skill among occupation. To estimate these parameter we follow Hsieh et al. (2013) and use the coefficient of variation of wages within an occupation which is, considering our model, given by

$$CV = \frac{\Gamma(1 - \frac{2}{\theta(1-\rho)(1-\eta)}) - (\Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)}))^2}{(\Gamma(1 - \frac{1}{\theta(1-\rho)(1-\eta)}))^2} \quad (30)$$

---

<sup>13</sup>If we consider a week with 44 working hours and that the minimum wage in Brazil was R\$678.00 per month in 2013, this is equivalent to a wage of R\$3.59/h. Therefore, our manipulation only drops individuals that receives much less than the minimum wage.

<sup>14</sup>There is a code in the CBO - Código Brasileiro de Ocupações - that classify some occupations as ‘bad defined’.

where  $\Gamma(\cdot)$  represents Gamma function distribution with shape governed by  $\theta(1 - \rho)(1 - \eta)$ . Using PNAD data, we run an OLS regression from log wage by hour to dummies that represents the all occupation of our model and region (7x27 dummies). With this regression we are interested in the part of wage dispersion that is not explained by occupation our region, i.e, we are interested in wage dispersion within occupation that is free of difference in schooling, frictions and TFP. We use the mean and variance of exponential of this wage residuals to estimate  $\theta(1 - \rho)(1 - \eta)$  by equation (30). The coefficient of variation of the residuals estimated is 2.05. Then, we set  $\theta$  equal to 3.44 and, with the last estimation, we obtain the  $\rho$  value equal to 0.19.

We use equation (8) and years of schooling available in PNAD to directly estimate  $\phi$  for each occupation. First, we compute the average years of schooling of each occupation and then calculate the effectively time spent in education.<sup>15</sup> For the latter, we suppose that an individual spend 8 hours a day studying during weekdays, which gives 2857 hours by year in a total of 365\*24 hours available. Thus, a individual spends 24% of his available time studying in a given year. As our educational period is composed by the first 25 years of life cycle, we divide the average years of schooling by 25 and, then, multiply by the time really invested in education (0.24).

In accord to our model, years of schooling does not depend on the region where the worker lives, but this is not necessarily empirically true. We made some descriptive statistics using each occupation average years of schooling in each state. As we can see in Table 3, the variance compared to the mean is small in almost all occupations. The only exception is occupation 5, that includes agriculture workers. Also, the amplitude (max - min) is at most 2.3 years if we do not consider occupation 5. The numbers confirms that the average of years of schooling is similar across states.

Table 3: Average year of schooling descriptive statistics across regions

Occupation	Mean	variance	min	max	median
1	11.27	0.38	10.21	12.51	11.42
2	14.34	0.04	13.78	14.76	14.34
3	11.31	0.18	10.34	12.32	11.28
4	8.71	0.30	7.74	9.74	8.87
5	4.44	1.07	2.96	6.29	4.35
6	7.11	0.43	5.99	8.26	7.22
7	13.83	0.13	12.89	14.31	13.92

Table 4 presents the estimated parameters of schooling elasticity.

The remaining parameters are educational and labor distortions,  $\{\tau_{ir}^h, \tau_{ir}^w\}_{i=1, r=1}^{N, R}$  and TFP parameters,  $\{A_r\}_{r=1}^R$ , which amounts to  $2NR + R$  parameters. Our strategy to calibrate them is minimizing the distance between statics generated by the baseline model and statics observed in the data. We define two kind of statistics for each occupation and region: the share of workers and the average gross wage. In our model those statistics are described in

<sup>15</sup>Remember that an agent can consume leisure in the education step.

Table 4: Parameters of schooling elasticity in the human capital function

$\phi_1$	0.138
$\phi_2$	0.174
$\phi_3$	0.136
$\phi_4$	0.100
$\phi_5$	0.051
$\phi_6$	0.084
$\phi_7$	0.168

propositions 1 and 3. Remember that:

$$W_{ir} = w_{ir} E(h_{ir} \epsilon_i | \text{agent chooses occupation } i),$$

this equation uses proposition 3 and the FOC's of firm's problem, where  $w_{i,r} = A_r \quad \forall i, r$ .

Since each occupation's share of workers sum a unit in each region,  $\sum_{i=1}^N p_{ir} = 1$ , we only have  $(N - 1)R$  independent statistics in each region. Thus, we assume that  $\tau_{1r}^h = 0 \quad \forall r$ . Beside that, we assume that  $\tau_{1r}^w = \tau_1^w \quad \forall r$ , i.e we assume that frictions in occupation 1's labor market are equal across regions. Also, we set  $A_R = 1$ , i.e., the TFP of last region is normalized to 1. Thus, we have the same number of statistics and parameters to be fitted by the model  $(2(N - 1)R + R)$ . Thus, we define the following objective function for our calibration:

$$D = \sum_{i=1, r=1}^{N, R} \left( \frac{W_{ir}^M - W_{ir}^T}{W_{ir}^T} \right)^2 + \sum_{i=1, r=1}^{N, R} \left( \frac{p_{ir}^M - p_{ir}^T}{p_{ir}^T} \right)^2 \quad (31)$$

where superscript  $M$  and  $T$  indicate model and target(s) statistics, respectively.

The calibrated model matches our targets very well. We find a  $D = 0.12$ , which we consider it a small number, because we have 378 different targets. Figure 7 presents the quality of adjustment to log wage and Figure 8 presents the quality of adjustment to the share of workers in each occupation. When a point is lined in the 45 degree line in both figures, it indicates that our model matches data perfectly for those targets.

Next, in tables 5 and 6 we present the calibrated values for parameters  $\tau^w$  and  $\tau^h$

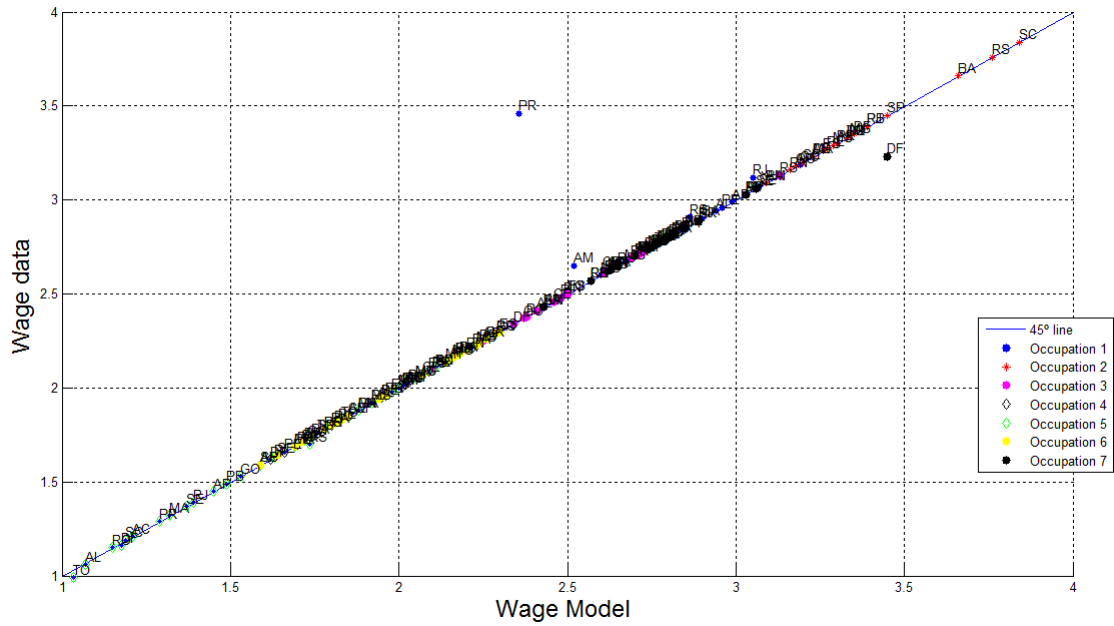


Figure 7: Model adjustment to data - wages

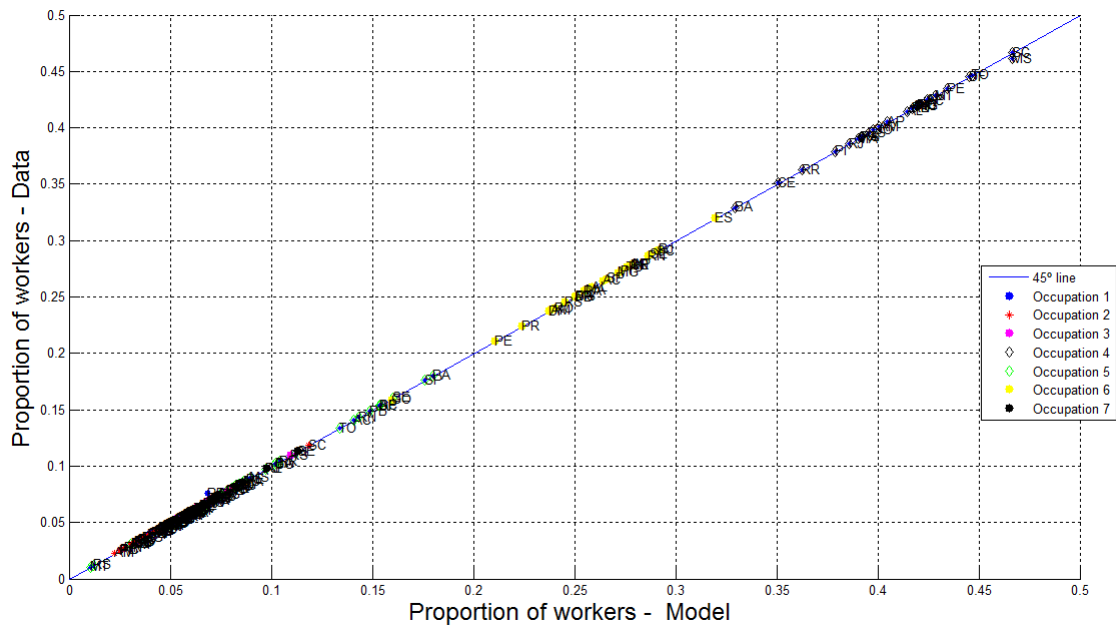


Figure 8: Model adjustment to data - share of workers

Table 5: Calibrated frictions on educational market

State	Occupation						
	1	2	3	4	5	6	7
Low-income States							
mean	0	-0.3779	0.1612	-0.5574	9.1555	-0.2037	-0.4489
RO	0	-0.0930	0.3462	-0.6176	0.5182	-0.5676	-0.1259
AC	0	0.3463	0.4621	-0.5358	4.2539	0.2770	-0.6397
AM	0	-0.4420	-0.0413	-0.4651	17.7555	-0.2046	-0.0518
RR	0	-0.7004	-0.3894	-0.7323	4.6021	-0.3582	-0.6402
PA	0	0.0479	0.4310	-0.5530	3.2446	-0.3143	-0.3271
AP	0	-0.5186	-0.4416	-0.7662	3.0124	-0.5370	-0.8637
TO	0	-0.3404	0.3434	-0.4752	2.7433	-0.0080	-0.3181
MA	0	-0.3486	0.1489	-0.3716	10.9313	0.0299	-0.5814
PI	0	-0.5407	0.4119	-0.4020	15.2703	-0.1276	-0.5819
CE	0	-0.3301	0.7234	-0.5172	24.3925	-0.1057	-0.3617
RN	0	-0.3693	0.1131	-0.4958	15.2786	-0.0412	-0.2231
PB	0	-0.5339	0.0375	-0.4734	18.8011	0.1363	-0.5052
PE	0	-0.5658	0.1572	-0.6754	8.2998	-0.4256	-0.4297
AL	0	-0.6851	0.1925	-0.7487	1.5356	-0.5303	-0.7196
SE	0	-0.7349	-0.2998	-0.7236	4.9728	-0.4384	-0.6364
BA	0	-0.2378	0.3833	-0.3652	10.8754	-0.0445	-0.1773
High-income States							
mean	0	-0.3079	0.5119	-0.3973	16.9484	-0.2118	0.2516
mean without DF	0	-0.2453	0.6442	-0.3529	16.3016	-0.1874	0.3565
MG	0	-0.3035	0.5525	-0.3327	5.7925	-0.2646	0.0890
ES	0	-0.4708	0.4780	-0.5889	3.7278	-0.5043	-0.1157
RJ	0	-0.6079	0.3477	-0.5055	99.0000	-0.1083	0.4925
SP	0	-0.3165	0.5636	-0.2693	24.9904	-0.0104	1.2307
PR	0	-0.2283	0.5312	-0.3231	8.3516	-0.1341	0.4034
SC	0	0.0046	0.6582	-0.1958	7.3690	-0.2116	0.6440
RS	0	-0.3616	0.3236	-0.4096	4.5470	-0.2247	0.3707
MS	0	0.2153	0.9606	-0.1166	3.8790	0.2026	0.2711
MT	0	-0.3687	0.8548	-0.3359	1.1250	-0.2891	-0.0883
GO	0	-0.0158	1.1718	-0.4518	4.2333	-0.3290	0.2673
DF	0	-0.9340	-0.8112	-0.8410	23.4167	-0.4561	-0.7968

Note: DF is an outlier because it presents a higher share of public sector in all activities when compared to other regions.

Table 6: Calibrated frictions on labor market

State	Occupation						
	1	2	3	4	5	6	7
Low-Income States							
mean	0.5429	0.6054	0.4857	0.3448	0.1663	0.3674	0.5300
RO	0.5429	0.5998	0.4916	0.4106	0.4076	0.4601	0.5353
AC	0.5429	0.5577	0.4793	0.3466	0.2249	0.3387	0.5334
AM	0.5429	0.5999	0.4892	0.3579	0.0923	0.3940	0.4973
RR	0.5429	0.6575	0.5302	0.4099	0.2747	0.4069	0.5437
PA	0.5429	0.5817	0.4605	0.3419	0.2319	0.3657	0.5124
AP	0.5429	0.5998	0.5193	0.3956	0.3410	0.4017	0.5809
TO	0.5429	0.5859	0.4712	0.3397	0.2504	0.3662	0.5009
MA	0.5429	0.5847	0.5034	0.2761	0.0000	0.3220	0.5140
PI	0.5429	0.6393	0.4710	0.3077	0.0000	0.3618	0.5587
CE	0.5429	0.6069	0.4529	0.3142	0.0000	0.3199	0.4928
RN	0.5429	0.6094	0.4886	0.3368	0.1494	0.3312	0.5232
PB	0.5429	0.5907	0.4999	0.2984	0.0708	0.2996	0.5509
PE	0.5429	0.6033	0.4793	0.3617	0.1998	0.3884	0.5268
AL	0.5429	0.6497	0.4595	0.3672	0.2530	0.3870	0.5502
SE	0.5429	0.6328	0.5104	0.3590	0.0986	0.3719	0.5541
BA	0.5429	0.5875	0.4643	0.2929	0.0660	0.3633	0.5052
High-Income States							
mean	0.5429	0.5859	0.4990	0.3856	0.3773	0.4375	0.5231
mean without DF	0.5429	0.5737	0.4832	0.3704	0.3622	0.4253	0.5078
MG	0.5429	0.5740	0.4821	0.3396	0.2839	0.4123	0.5165
ES	0.5429	0.5945	0.4742	0.3768	0.3357	0.4487	0.5216
RJ	0.5429	0.5780	0.4828	0.3676	0.2772	0.4137	0.4878
SP	0.5429	0.5458	0.4673	0.3391	0.3445	0.3954	0.4570
PR	0.5429	0.5717	0.4970	0.3805	0.3358	0.4179	0.4964
SC	0.5429	0.5722	0.5021	0.4171	0.4066	0.4488	0.5153
RS	0.5429	0.5823	0.4952	0.3777	0.3814	0.4216	0.5098
MS	0.5429	0.5660	0.4922	0.3443	0.4070	0.4047	0.5268
MT	0.5429	0.5910	0.4908	0.4006	0.4605	0.4703	0.5400
GO	0.5429	0.5618	0.4485	0.3606	0.3898	0.4197	0.5066
DF	0.5429	0.7072	0.6567	0.5379	0.5274	0.5591	0.6766

Note: DF is an outlier because it presents a higher share of public sector in all activities when compared to other regions.