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### On the distribution dynamics of human development: Evidence from the metropolitan regions of Bolivia

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#### Abstract

Bolivia has experienced large socioeconomic transformations in the last decades. Among them, almost half of the population currently lives in the main metropolitan regions of the country. Motivated by the potential for growth and development convergence in these regions, this article documents the evolution of human development disparities and convergence patterns over the 1992-2013 period. Using a distribution dynamics framework, this article evaluates both the transitional dynamics and the long-run equilibrium of the cross-regional distribution of human development. Results from the transitional dynamics analysis suggest that the formation of multiple clusters of convergence is a salient feature of inequality reduction in human development. On the other hand, results from the long-run equilibrium analysis suggest that the process of regional convergence is characterized by the transformation of a trimodal distribution into a left-skewed unimodal distribution. The article concludes emphasizing that the cross-regional distribution of human development in Bolivia is quite sticky at its left tail, and as a result, the least developed regions are still relatively far from achieving complete convergence in the long run.

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## 1 Introduction

Since the mid-1980s, Bolivia has experienced large political, social, and economic transformations. Among its social and demographic transformations, there has been a continuous movement of population toward the most urban and metropolitan areas of the country. By the year 2013, forty six percent of the total population are concentrated in the main metropolitan regions<sup>1</sup> of Bolivia (UNDP, 2016).

Given the notion that metropolitan regions within a country are more likely to share common technological and institutional environments,<sup>2</sup> the neoclassical growth model would predict that these regions are expected to converge in terms of their living standards. Motivated by this prediction and the observed socioeconomic progress of the metropolitan regions of Bolivia,<sup>3</sup> this article documents the evolution of human development disparities and convergence patterns over the 1992-2013 period. In particular, using the United Nations' human development index of 20 metropolitan municipalities, this article evaluates the process of regional convergence of through the lens of a distribution dynamics framework (Quah 1997; Johnson 2005).

Results from the transitional dynamics analysis suggest that the formation (and merge) of multiple clusters of convergence is a salient feature of inequality reduction in human development. The 1992-2001 period appears to be characterized by three separate convergence clusters. The 2001-2013 period, on the other hand, highlights the merge between the central cluster and the high-development cluster identified in the previous decade. Given these dynamics, results from the estimated long-run distribution suggest that the process of regional convergence is characterized by the transformation of a trimodal distribution into a left-skewed unimodal distribution. This unimodal transformation, however, largely depends on the continuation of the human development dynamics observed in the 2001-2013 period.

Although one could be skeptical about the strength of any empirical result that is obtained using only 20 observations in a nonparametric setting, this paper only suggests a set of clear and provocative empirical patterns that could be (re)tested and confirmed when larger datasets become available.<sup>4</sup> Furthermore, the Monte Carlo simulations performed by Gerolimetto and Magrini (2017) suggest that—even for very small samples<sup>5</sup>—the distribution dynamics approach appears to correctly identify trends of overall convergence. Nonetheless, the results of the present article should only constitute a first step towards a more comprehensive analysis of municipal convergence in Bolivia.

The rest of paper is organized as follows. Section 2 briefly describes the distribution dynamics framework and data of the study. Section 3 presents the results of the transitional dynamics and the long-run equilibrium analyses. Finally, Section 4 offers some concluding remarks.

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<sup>1</sup>Bolivia is administratively divided into nine departments. Of those nine, the urban and metropolitan regions of the largest three departments (La Paz, Santa Cruz, and Cochabamba) concentrate forty six percent of the total population. Although the other six departments also show a tendency toward urbanization, this study focuses only on the metropolitan regions of the three largest departments of Bolivia.

<sup>2</sup>For instance, compared to urban and rural differences within a country or high-income and low-income differences across countries, metropolitan regions within a country are expected to have a higher degree of homogeneity.

<sup>3</sup>See UNDP (2016) for a complete report on the human development progress of these regions.

<sup>4</sup>Although one could try polling the data over time to increase the number of cross-sectional observations, the limited number of years currently available in the dataset constraint this strategy.

<sup>5</sup>Gerolimetto and Magrini (2017) evaluate the robustness of the distribution dynamics results using three samples of size 200, 100, and 50 respectively. As reported by the authors, even when drastically reducing the sample size, the overall convergence trends were still identifiable.

## 2 Methodology and Data

### 2.1 Distribution Dynamics Framework

Building on the seminal work of Silverman (1986), Quah (1993, 1997) introduces the distribution dynamics framework as a modeling technique that describes the evolution of the entire income distribution across countries. At its core, this framework characterizes the dynamics of a system in terms of the transitional dynamics and long-run equilibrium of a non-parametric distribution function. Transitional dynamics are modeled via an estimated stochastic kernel, which is a continuous state-space representation of a Markovian transition matrix. The long-run equilibrium is modeled via an estimated ergodic distribution, which is a continuous representation of a Markov chain equilibrium.

In what follows, I sketch<sup>6</sup> more formally the main components of the distribution dynamics framework in the context of the variables of this article. First, denote  $f_t(x)$  as the initial cross-sectional distribution of human development<sup>7</sup> at time  $t$ . Likewise,  $f_{t+s}(y)$  is the human development distribution at some future time  $t + s$ . To model the evolution from time  $t$  to time  $t + s$ , the literature typically assumes a first-order autoregressive process of a time-homogeneous Markov chain. That is,

$$\underbrace{f_{t+s}(y)}_{\text{Future Distribution}} = \int \underbrace{f_{t+s|Y_t=x}(y)}_{\text{Transitional Operator}} \underbrace{f_t(x)}_{\text{Initial Distribution}} dx. \quad (1)$$

Where the transition between the initial distribution,  $f_t(x)$ , and the future distribution,  $f_{t+s}(y)$ , is mapped by a transitional probability operator,  $f_{t+s|Y_t=x}(y)$ , that is commonly referred in the literature as the stochastic kernel.

#### Transitional Dynamics via the Stochastic Kernel

To estimate the stochastic kernel, most recent studies exploit recent advances in non-parametric statistical methods. The first step in the estimation process is the definition of the stochastic kernel as a conditional distribution

$$f_{t+s|Y_t=x}(y) = \frac{f_{t,t+s}(y,x)}{f_t(x)}, \quad (2)$$

where  $f_{t,t+s}(y,x)$  is an unconditional joint distribution. The next step is to specify this joint distribution in terms of two kernel functions and a pair of smoothing parameters (bandwidths). A common first candidate<sup>8</sup> for this endeavor is

$$f_{t,t+s}(y,x) = \frac{1}{nh_y h_x} \sum_{i=1}^n K_y \left( \frac{y - y_i}{h_y} \right) K_x \left( \frac{x - x_i}{h_x} \right), \quad (3)$$

<sup>6</sup>See Epstein, Howlett, and Schulze (2003) or Magrini (2004, 2009) for a more complete presentation.

<sup>7</sup>For the rest of this analysis, the human development level of each region is expressed in relative terms. That is, the officially reported HDI level for each region is normalized by the cross-sectional average of the sample.

<sup>8</sup>An alternative estimator for the stochastic kernel has been proposed by Hyndman et al. (1996). Although this estimator has better asymptotic mean bias properties than the original estimator proposed by Quah (1997), the overall convergence patterns identified in this paper remain unchanged when the estimator of Hyndman et al. (1996) is applied. See the Appendix for further details. Thus, considering the similarity of the overall results, the graphical display of Quah (1997) is presented in the body of the paper. Moreover, the visualization of clusters appears slightly more evident and appealing through the lens of the Quah (1997) estimator.

where  $y$  and  $x$  denote (relative) human development in each region at time  $t$  and  $t + s$  respectively,  $K_y$  and  $K_x$  denote Gaussian kernel functions, and  $h_y$  and  $h_x$  denote the smoothing parameters (bandwidths) for  $y$  and  $x$  respectively. Following Magrini (1999, 2009) and Kar, Jha, and Kateja (2011), the optimal selection of the bandwidths is based on the minimization of the asymptotic mean integrated square error (AMISE). In addition, variable bandwidths have been used to deal with the sparseness of the data.<sup>9</sup> The final step is the specification of the marginal distributions. Similar to the estimation of the joint distribution, the marginal distributions ( $f_{t+s}(y)$  and  $f_t(x)$ ) are estimated using a single Gaussian kernel function and a smoothing parameter.<sup>10</sup>

## Long-run Equilibrium via the Ergodic Distribution

To estimate the ergodic distribution, the approach of Johnson (2000, 2005) is implemented. Considering the dynamics described in Equation 1, the long-run equilibrium of the system is given by the solution to the following problem:

$$f_{\infty}(y) = \int f_{t+s|Y_t=x}(y) f_{\infty}(x) dx = f_{\infty}(x). \quad (4)$$

If a solution exists, then the shape of the ergodic distribution,  $f_{\infty}(y)$ , provides valuable information regarding the long-run convergence patterns of the economic system under study. To compute this solution, this article uses the MATLAB functions developed by Magrini (2009).

## 2.2 Data

The dataset is from the 2016 Human Development Report for Bolivia.<sup>11</sup> The United Nations Development Program (UNDP, 2016) constructed a municipal-level Human Development Index (HDI) that covers 20 municipalities from the metropolitan regions of La Paz, Cochabamba, and Santa Cruz. The temporal dimension of this dataset comprises four years: 1992, 2001, 2005 and 2013. The construction of this dataset required census data, household surveys, and administrative records of public services.

To control for aggregate shocks that might affect all metropolitan regions, a relative (ratio) measure of the HDI is used as the main unit of analysis. More specifically, the HDI of each municipality was rescaled by the cross-sectional mean of each year. To facilitate the interpretation of the results, relative HDI of each municipality is presented in natural *log* terms. This transformation simply re-scales the HDI in a way that the sample average now takes a value of zero at each point in time.

Figure 1 presents a graphical summary of the dataset at three points in time.<sup>12</sup> First, the location of observations along the main scatterplot axes show a noticeable reduction in human development differences over time. Relative to the sample average of the year 1992, human development differences ranged between 17 percent below average (the case of Palca) and 20 percent above average (the case of Santa Cruz). By the year 2013, this range has noticeably

<sup>9</sup>For a more detailed exposition about the selection of bandwidths used in this paper, see the technical appendix of Magrini (2007).

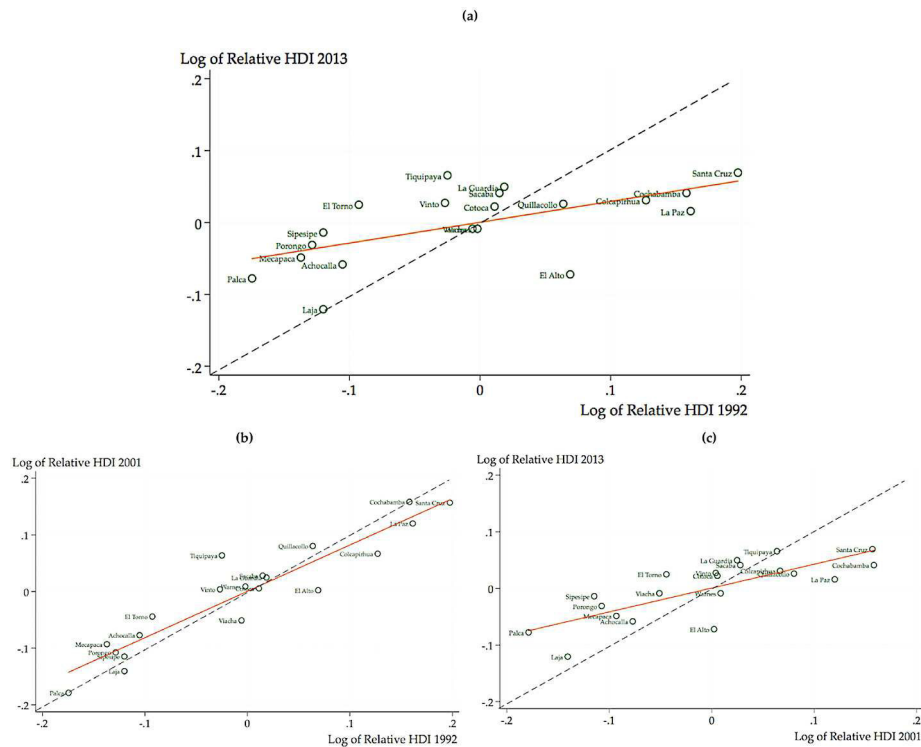
<sup>10</sup>The smoothing parameter for each marginal distribution is also derived through the minimization of the asymptotic mean integrated square error (AMISE).

<sup>11</sup>The report can be downloaded from the following website: [http://www.bo.undp.org/content/dam/bolivia/docs/undp\\_bo\\_IDH2016.pdf](http://www.bo.undp.org/content/dam/bolivia/docs/undp_bo_IDH2016.pdf). Table 1 of the appendix has been used to construct the dataset of this study.

<sup>12</sup>Although, given the data availability, it is possible to work with four reference points and three sub-periods, this article focuses only on two longer sub-periods that roughly cover two decades each.

decreased. Human development differences ranged between 12 percent below average (the case of Laja) and 7 percent above average (the case of Santa Cruz). The subperiod scatterplots point that the 2001-2013 period shows the largest reduction in human development differences across regions.

Figure 1: Regional Mobility and Convergence across Regions



The slope of the fitted regression lines in Figure 1 suggests that regions with relatively lower levels of development appear to be moving forward, whereas the regions with relatively higher levels of development appear to be moving backward.<sup>13</sup> Naturally, the outcome of these dynamics is a process of convergence, which was most notorious in the 2001-2013 period. Regions located above the dashed 45-degree line are those that improved their relative position and regions located below are those that deteriorated their position, relative to its initial level of human development. For instance, over the 1992-2013 period, the region of Tiquipaya improved its relative position from 3 percent below average<sup>14</sup> to 7 percent above average.<sup>15</sup> On the other hand, the region of El Alto deteriorated its position from 7 percent above average to 7 percent below average. Indeed, these patterns of forward and backward mobility appear to be a general characteristic of the development path of the regions in the sample.

Although the fitted regression lines of Figure 1 summarize—to some extent—the overall convergence pattern across metropolitan regions, there are some key aspects of the convergence process that require further investigation. The distribution dynamics framework provides valuable new insights regarding nonlinear dynamics and the formation of convergence clusters. In addition, a more complete dynamic analysis should include both notions of transition and long-run equilibrium. These two key features are presented in the next section.

<sup>13</sup>Note that a backward movement in relative terms does not imply a backward movement in absolute terms.

<sup>14</sup>This is the sample average of the year 1992, which in the scatterplot is normalized to zero.

<sup>15</sup>This is the sample average of the year 2013, which in the scatterplot is normalized to zero.

### 3 Results

#### 3.1 Transitional Dynamics

Figures 2 and 3 show the transitional dynamics of convergence through the lens of the estimated stochastic kernel. One of the main features of the estimation is the graphical identification of stagnation, transition, and clustering patterns. Building on top of the mobility patterns described in Figure 1, the stochastic kernel shows that the dynamics of convergence clusters (or clubs) is a salient feature of inequality reduction in human development across metropolitan regions in Bolivia. Moreover, these cluster dynamics are different across the two decades of the analysis.

Figure 2: Stochastic Kernel: Surface Plots

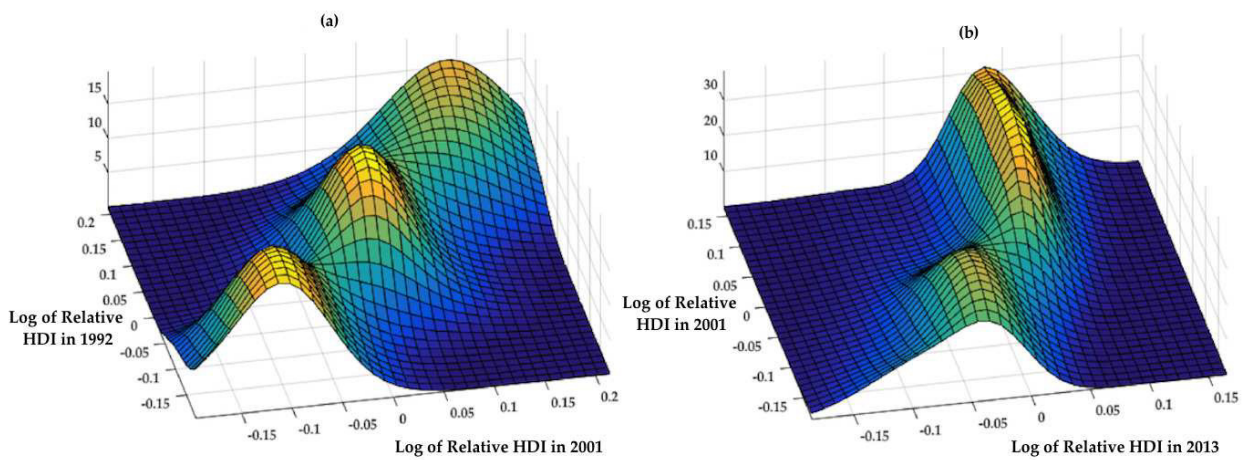
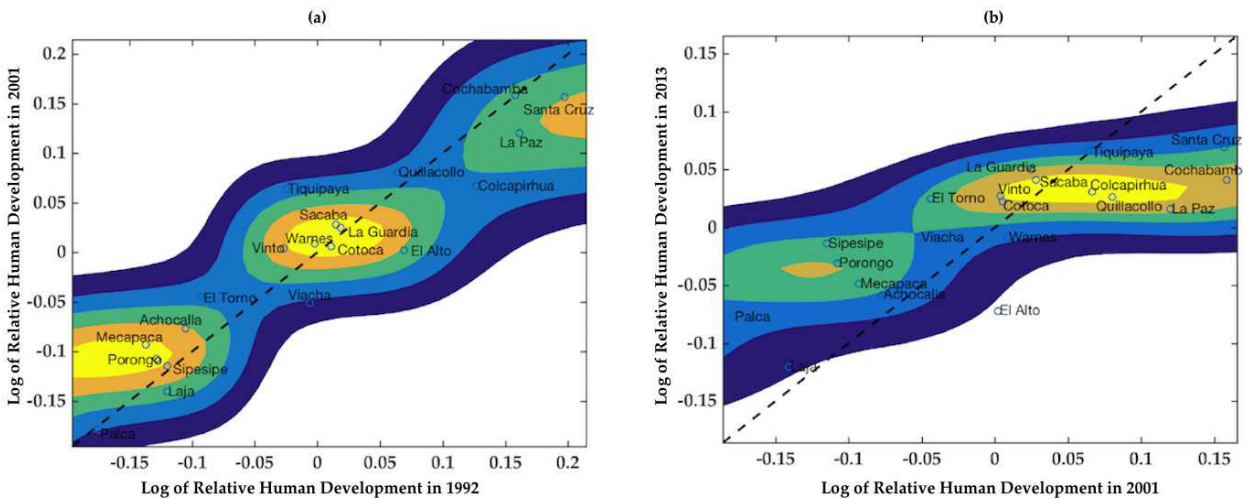


Figure 3: Stochastic Kernel: Contour Plots



For the 1992-2001 period, the stochastic kernel (Figure 2 and 3 , Panel a) points to three separate clusters of high density. Relative to the central cluster, located around the average human development level of the year 2001, there is a low human development cluster located at about 10 percent below average. On the other side of the distribution, there is a high human development cluster located at about 14 percent above average. In addition, note that in Figure

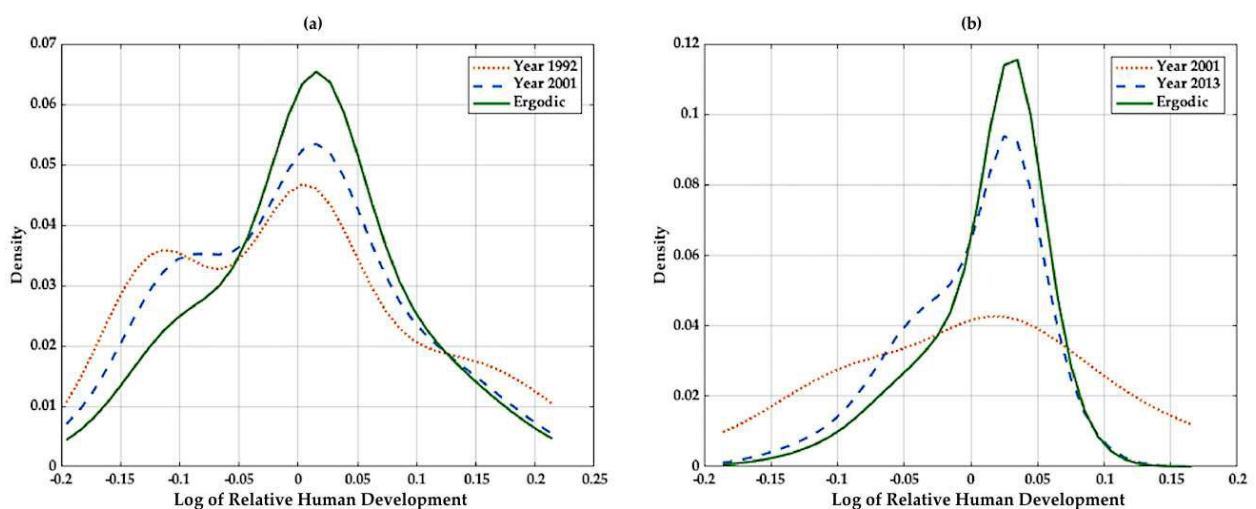
3 (Panel a) the low human development cluster is mostly located above the 45-degree line (that is, forward mobility) and the high human development cluster is mostly located below the 45-degree line (that is, backward mobility). Thus, over time, both clusters are moving closer to the central cluster.

For the 2001-2013 period, the stochastic kernel (Figure 2 and 3 , Panel b) highlights the merge (convergence) between the central cluster and the high-development cluster identified in the previous decade. The newly merged cluster is now located at about 3 percent above the human development average of the year 2013. The relatively low development cluster, on the other hand, is located at about 4 percent below the average of the same year. Overall, these transitional dynamics suggest that the convergence process arising from the bottom the distribution is much more sticky compared to that arising from the top of the distribution.

### 3.2 Long-Run Equilibrium

Figure 4 shows the long-run dynamics of convergence through the lens of the estimated ergodic distribution. The main purpose of an ergodic distribution analysis is to clarify and magnify the effects of the observed transitional dynamics.<sup>16</sup> Overall, Figure 4 shows a process of convergence characterized by the evolution of a trimodal distribution (year 1992) into a left-skewed unimodal distribution (ergodic estimation for the period 2001-2013). Moreover, similar to the transitional dynamics findings, the two periods of analysis show two largely different convergence dynamics in the long run.

Figure 4: Initial, Final, and Ergodic Distribution



Panel (a) of Figure 4 shows the marginal distributions for the years 1992 and 2001, and the long-run (ergodic) distribution associated to that time span. As expected, human development differences are smaller in the long run. However, the asymmetric and bumpy shape of the ergodic distribution may still suggest the existence of two convergence clubs. For one reason, it is clear that in the year 2001 the human development distribution shows two density peaks. And, to some extent consistent with this bimodality, the ergodic distribution still shows two density concentrations: one located at about 12 percent below average and the other at about 2 percent above average.

<sup>16</sup>Note that the estimation of a long-run distribution should not be considered as a forecast of what will happen in the future (Quah, 1997).



Panel (b) of Figure 4 shows the the long-run (ergodic) distribution given the transitional dynamics of the 2001-2013 period. Although there are no clear multiple density bumps in the long run, the shape of the ergodic distribution is still largely asymmetric. Indeed, the distance between the left tail and the mode of the distribution suggests that the least developed regions of the sample are still relatively<sup>17</sup> far from achieving convergence in the long run.

#### 4 Concluding Remarks

This article has documented the reduction of human development disparities (as measured by the United Nations' human development index) across the metropolitan regions of Bolivia over the 1992-2013 period. In particular, through the lens of a nonparametric density estimation framework, the process of regional convergence has been characterized in terms of its transitional dynamics and long-run equilibrium.

Overall, there is a tendency toward regional convergence that is driven by both the forward mobility of the less developed regions and the backward mobility of the more developed regions. However, the transitional dynamics analysis, via the estimated stochastic kernel, suggests that the formation of different convergence clusters is a salient feature of inequality reduction in human development. Furthermore, these clustering dynamics are notoriously different across the two decades of the analysis. While the 1992-2001 period appears to be characterized by the formation of three separate clusters, the 2001-2013 period highlights the convergence (merge) between the central cluster and the high-development cluster identified in the previous decade.

The long-run equilibrium analysis, via the estimated ergodic distribution (and the observed marginal distributions), suggests that the process of regional convergence is characterized by the transformation of a trimodal distribution (year 1992) into a left-skewed unimodal distribution (ergodic estimation). This unimodal transformation, however, largely depends on the continuation of the human development dynamics observed in the 2001-2013 period. If, for instance, the dynamics of the 1992-2001 period are taken as a more realistic input for the long run, then the human development distribution is more likely to be characterized by two convergence clubs. In any of these cases, it appears to be clear that the human development distribution is quite sticky at the bottom, and thus the least developed regions are still relatively far from achieving complete convergence in the long run.

Finally, further research on regional convergence at the municipal level in Bolivia seems promising in several fronts.<sup>18</sup> Perhaps the most critical front has to do with the effects of spatial dependence on the convergence process in general, and the distribution dynamics framework in particular. In this regard, the work of Rey and Janikas (2005) outlines a research agenda that focuses on the development of new empirical measures of space-time dynamics. Consistent with this agenda, an increasing number of authors (Basile, 2009; Fischer and Stumpner, 2008; Maza 2012; among others) have been extending the distribution dynamics framework with an emphasis on the treatment of spatial effects. Thus, in the Bolivian case, these kind of extensions may prove to be the most promising in future research endeavors.

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<sup>17</sup>Relative to the level of convergence experienced by the most developed regions in the sample.

<sup>18</sup>See, for instance, the topics discussed in Mendez-Guerra (2018).



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