Signaling through Public Antitrust Enforcement: An Extension

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**Abstract**

This note shows that the argument of Šaljanin(2017) [Šaljanin, 2017. “Signaling through public antitrust enforcement”, Economics Letters 169, 4 – 6] that public antitrust enforcement complements private investment is robust to allowing public investment in anti-trust enforcement to be productive. However, unlike as in the case of unproductive public investment, over investment in public antitrust enforcement does not necessarily signal that the government is pro-competition: in pooling equilibria either only the anti-competition government or both types of government over invests, whereas in the separating equilibrium only the pro-competition government over invests.

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1 Introduction

In a recent paper Šaljanin (2017) (henceforth SS in short) provides a new rationale for public antitrust enforcement. SS shows that, in the presence of asymmetric information regarding the government’s type – pro-competition or anti-competition, (a) the government’s motive behind over-investing in anti-trust enforcement is to signal her commitment to promote market competition and (b) public antitrust enforcement complements private investment. To demonstrate these results SS assumes that public investment in antitrust enforcement is unproductive. In a follow-up extension (Section 3.1, pp. 6) SS argues that, when public investment is productive in the sense that it enhances the probability of success of private investment projects, firms will be more willing to invest after observing higher public investment. However, it is not clear whether over investment in public antitrust enforcement will signal that the government is of pro-competition type. The reason is, if public enforcement increases the probability of success of private investment projects and the government attaches a positive value to successful private projects, the anti-competition government may have a greater incentive to invest in public anti-trust enforcement than that of the pro-competition government.

In this note we revisit the results of SS by explicitly analysing the implications of public investment in anti-trust enforcement to be productive and demonstrate the following. Under asymmetric information regarding the government’s type, three equilibria – a separating equilibrium and two pooling equilibria – exist depending on parametric configurations, a la SS. The separating equilibrium outcome is similar to that in SS: only the pro-competition government over invests in antitrust enforcement compared to that under symmetric information, which credibly signals the government’s true type, and private investment occurs only when there is pro-competition government. However, in contrast to SS, in pooling equilibria (a) either only the anti-competition government or both types of government over invests in public antitrust enforcement compared to that under symmetric information and (b) the private firm invests irrespective of the type of the government, though it cannot update its prior beliefs. We, therefore, can say that public investment in antitrust enforcement complements private investment regardless of whether public investment is productive or not. However, in contrast to SS, the rationale behind over investment in public antitrust enforcement is not necessarily that it is a signal of the pro-competition government’s support for competition, but it can as well be an outcome of the anti-competitive government’s desperation to masquerade her true identity.

2 The Model

The setup is exactly the same as in Šaljanin (2017), except that we allow for investment in public antitrust enforcement to be productive. There are two players, the government and a potential private investor (henceforth ‘firm’). The government can be either pro-competition type (henceforth ‘type-A’) or anti-competition type (henceforth ‘type-B’). The government’s type, which is exogenously determined, is her private information. The firm’s prior belief that the government is of type A is \( \alpha \in (0, 1) \), which is common knowledge. There are two stages of the game involved. In stage 1, the government decides the level of public investment \( e \geq 0 \) in public antitrust enforcement, which is publicly observable and verifiable. Next, in stage 2, the firm decides whether to invest the fixed amount \( I > 0 \) in a project or not, and payoffs are realized. The firm’s opportunity cost of fixed investment \( I \) is \( (1 + i)I \), where \( i > 0 \). The project may be successful or unsuccessful. Let \( p_j(e_j) \in (0, 1), j = A, B \), be the success probability of the project if the government is of type \( j \), where \( e_j \) denotes public investment made by type \( j \) government. In the case of success, the firm obtains the payoff \( (1 + r)I \) and the government receives benefit \( W \) regardless of her type, where \( r > i \) and
On the other hand, in the case of failure, each player’s payoff is assumed to be zero. These are common knowledge.

**Assumption 1:** (a) \( p_A(e_A) > p_B(e_B) > 0, \forall e_A = e_B \geq 0; \) (b) \( \frac{dp_j}{de_j} > 0 \) and \( \frac{d^2p_j}{de_j^2} < 0, \forall e_j \geq 0, j = A, B; \) and (c) \( \frac{dp_A}{de_A} = \frac{dp_B}{de_B} \) if \( e_A = e_B. \)

Assumption 1(a) states that, for any given level of public investment, the project is more likely to be successful under type-A government than under type-B government, which is in the same spirit as in Saljanin (2017). However, unlike as in Saljanin (2017), we consider that the success probability of the project increases at a decreasing rate in the government’s investment in public antitrust enforcement, regardless of her type (Assumption 1(b)). This means that there is decreasing returns to public investment in anti-trust enforcement. Assumption 1(c) states that the rate of change in the success probability, due to a change in public investment from any given level, does not depend on the type of the government implying that the independent effect of pro-competitiveness on success probability is constant at every level of public enforcement. This assumption is based on the intuition that public investment is used for implementation of the government policies which are already in place and, while the policies differ in pro-competitiveness across the two types of government, their implementation may be equally effective at the margin.

**Definition 1:** Let \( p_j(e_j^0)(1 + r)l = (1 + i)l, j = A, B. \)

From Assumption 1 and Definition 1, it follows that, in the case of symmetric information, the firm invests under type \( j \) government, if \( e_j \geq e_j^0; j = A, B. \) That is, \( e_j^0 \) is the minimum public investment that the type \( j \) government needs to commit in order to attract private investment. The underlying assumption is, the firm invests unless its expected gain from investing is strictly negative.

**Lemma 1:** Under symmetric information, the minimum public investment required to attract private investment is less for the government of type-A than of type-B: \( e_A^0 < e_B^0. \)

**Proof:** See Appendix

**Assumption 2:** \( e_A^0 \geq 0 \)

Lemma 1 and Assumption 2 together imply that \( e_B^0 > 0, \) i.e., in order to attract private investment the type-B government needs to make a strictly positive investment in public antitrust enforcement, while in the case of type-A government the firm may invest even when there is no public enforcement of antitrust (\( e_A^0 \geq 0 \)). We, thus, include the possibility that, in absence of any public antitrust enforcement (\( e_A = e_B = 0 \)), private investment occurs only if the government is of type-A, as in Saljanin (2017).

**Definition 2:** Let \( \tilde{e}_j = argmax_{e_j} [p_j(e_j)W - e_j], j = A, B. \)

**Lemma 2:** \( \tilde{e}_A = \tilde{e}_B \)

**Proof:** See Appendix

**Assumption 3:** \( p_j(\tilde{e}_j)W - \tilde{e}_j > 0, j = A, B. \)
Assumption 4: \( p_A(e_A^0)W - e_A^0 > 0 \) and \( p_B(e_B^0)W - e_B^0 < 0 \).

Assumption 5: \( e_A^0 < \hat{e}_A \) and \( e_B^0 < \hat{e}_B \)

Assumption 3 states that it is incentive compatible for the type-\( j \) (\( j = A, B \)) government to choose her unconstrained optimal level of public investment, \( \hat{e}_j \), if that attracts private investment, compared to choosing any other level of public investment. Assumption 4 implies that, under symmetric information, only type-A government would be better off by attracting private investment, compared to investing less than the minimum required amount. Assumption 5 implies that, if the government is of type-A (type-B), her unconstrained optimal choice of public investment satisfies (does not satisfy) the firm’s incentive compatibility constraint to invest: \( p_A(\hat{e}_A)(1 + r)I \geq (1 + i)I \), but \( p_B(\hat{e}_B)(1 + r)I < (1 + i)I \). Since type-B government is anti-competition type, it can be expected that the policies in place do not concur with competitive activities, hence the amount of public investment required to enhance the probability of success enough to induce private investment is sufficiently high. With assumptions 4 and 5 we consider the case when, given that the private firm is able to correctly identify the type of government, it is not optimal for the anti-competitive government to incur the minimum required amount of investment.

Lemma 2 and Assumptions 2 & 5 together imply the following.

\[
0 \leq e_A^0 < \hat{e}_A = \hat{e}_B < e_B^0
\]  

\textbf{Proposition 1:} Suppose that Assumptions 1 – 5 hold true. Then, the following strategies constitute the equilibrium under symmetric information, where superscript ‘S’ denotes the equilibrium under symmetric information.

(i) The type-A government chooses \( e_A^S = \hat{e}_A > e_A^0 \). This means that there is more than the minimum required level of public enforcement to attract private investment.

(ii) The type-B government chooses \( e_B^S = 0 \). This means that there is no public enforcement.

(iii) The firm invests only when the government is of type-A.

\textbf{Proof:} Consider that information is symmetric. Suppose the government is of type-\( j \), \( j = A, B \). Then, for any given \( e_j \), in stage 2 the firm invests, if \( p_j(e_j)(1 + r)I \geq (1 + i)I \) holds, i.e., if \( e_j \geq e_j^0 \) (by Definition 1); otherwise, the firm does not invest.

Now, in stage 1, the government can choose to invest or not to invest in public antitrust enforcement. If type-\( j \) government decides to invest \( e_j \in [0, e_j^0) \), firm does not invest and, thus, the government’s maximum payoff is zero. On the other hand, if the government decides to invest an amount so that the firm invests, her problem can be written as \( \max_{e_j} (p_j(e_j)W - e_j) \), subject to the constraint \( e_j \geq e_j^0 \). From Definition 2 and Assumption 5, it follows that the solution of this problem is given by \( \hat{e}_j = \begin{cases} \hat{e}_j, & \text{if } j = A \\ e_j^0, & \text{if } j = B \end{cases} \). However, \( p_j(\hat{e}_j)W - \hat{e}_j \geq 0 \), if \( j = A \), by Assumptions 3 and 4. Therefore, \( e_j^S = \begin{cases} \hat{e}_j, & \text{if } j = A \\ e_j^0, & \text{if } j = B \end{cases} \). 

[QED]
2.1 Asymmetric information

Under asymmetric information, given the public enforcement in antitrust \((e)\), the firm invests irrespective of the government’s type, if \(e\) satisfies (2).

\[
[\alpha p_A(e) + (1 - \alpha)p_B(e)](1 + r)l > (1 + i)l \tag{2}
\]

**Definition 3**: Let \(e^p: [\alpha p_A(e^p) + (1 - \alpha)p_B(e^p)](1 + r)l = (1 + i)l\).

**Lemma 3**: \(e^0_A < e^p < e^0_B\)

**Proof**: See Appendix.

That is, for any \(e \geq e^p\), given the firm’s prior beliefs, the firm invests.

**Lemma 4**: (a) For all \(e \in [0, \bar{e}_B]\), \(p_B(e)W - e > 0\) and (b) there exists a unique \(e = \bar{e}_B \in (\bar{e}_B, e^0_B)\) such that \(p_B(\bar{e}_B)W - \bar{e}_B = 0\).

**Proof**: See Appendix.

**Lemma 5**: \(p_A(\bar{e}_B)W - \bar{e}_B > 0\)

**Proof**: Follows directly from Assumption 1(a) and Lemma 4.

**Lemma 6**: (a) \(e^p < (\bar{e}_B\) and (b) \(e^p < (\bar{e}_B\).

**Proof**: See Appendix.

When the firm observes that the government has invested \(e \geq 0\) amount in public antitrust enforcement, it believes that the government is of type A with probability \(\mu(e)\) and is of type B with probability \(1 - \mu(e), \mu(e) \in [0, 1]\).

**Proposition 2**: Suppose that Assumptions 1 – 5 hold true and \(e^p \leq \bar{e}_B\), implying that condition (2) is satisfied at \(e = \bar{e}_B\). Then the following beliefs and strategies constitute a pooling perfect Bayesian equilibrium.

1. The firm’s beliefs are given by \(\mu(e) = \begin{cases} \alpha, & \text{if } e = \bar{e}_B, \\ 0, & \text{otherwise}. \end{cases}\)

2. The government chooses \(e^* = \bar{e}_B\), regardless of her type. This means that the type-A government chooses her symmetric information optimal level of public enforcement, whereas type-B government masquerades her true identity by overinvesting in public enforcement compared to that under symmetric information.

3. The firm invests.

**Proof**: First, consider the firm’s beliefs as given. Then it is optimal for type-A government to choose \(\bar{e}_B\). The reason is as follows. (a) Type-A government’s optimal choice under symmetric information \(\bar{e}_A = \bar{e}_B\) (by Proposition 1(i) and Lemma 2) and (b) the firm invests even if type-B government chooses \(\bar{e}_B\), since \(e^p < \bar{e}_B\) (by supposition). It is also optimal for type-B government to choose \(\bar{e}_B\), since (a) by choosing \(\bar{e}_B\) type-B government can masquerade her type, which induces the firm to invest \((e^p < \bar{e}_B\), by supposition), and (b) it is incentive compatible for type-B government to masquerade her true identity by choosing \(\bar{e}_B\) compared to choosing any \(e\) that reveals her true identity (when the firm invests, type-B government’s payoff is maximum at \(\bar{e}_B\) (by Definition 2) and that maximum payoff is strictly positive (by Assumption 3); whereas her payoff is non-positive if she chooses any \(e \neq \bar{e}_B\). Next, given the strategies of the firm and the government, beliefs are self-fulfilling.

[QED]
**Proposition 3:** Suppose that Assumptions 1–5 hold true and \( e_B < e^p < \bar{e}_B \), implying that condition (1) is not satisfied at \( e = \bar{e}_B \). Then the following beliefs and strategies constitute a pooling perfect Bayesian equilibrium.

1. The firm’s beliefs are given by \( \mu(e) = \begin{cases} \alpha, & \text{if } e = e^p, \\ 0, & \text{otherwise.} \end{cases} \)
2. The government chooses \( e^* = e^p \), regardless of her type.
3. The firm invests.

**Proof:** First, consider the firm’s beliefs as given. Then it is optimal for the government to choose \( e^p \), regardless of her type, due to following reasons. For any choice of type-A government \( e_A \in [0, \, \bar{e}_B) \), \( p_B(e_A)W - e_A > 0 \) (by Lemma 4 and Assumption 1(b)), i.e., type-B will mimic type-A’s behaviour if that attracts private investment. However, \( \forall \, e \in [0, \, e^p) \), the firm cannot update its belief and \( \{\alpha \, p_A(e) + (1 - \alpha)p_B(e)\}(1+r)I < (1+i)I \) (by Assumption 1 and Definition 3) and, thus, the firm does not invest. On the other hand, if type-A government chooses \( e_A \in [e^p, \, \bar{e}_B) \), the firm still cannot update its belief since the type-B government mimics type-A’s behaviour. But in this case the firm invests as the observed level of public investment is above the threshold level (by Assumption 1 and Definition 3). Type A government can also choose \( e_A = \bar{e}_B \), which credibly signals her true type (assuming that type-B mimics type-A only when type-B is strictly better off by doing so) and, thus, attracts private investment (since \( \bar{e}_B > e^p \)). However, \( \forall \, e > \bar{e}_B \), \( p_A(e)W - e \) is decreasing in \( e \) (by Assumption 1, Lemma 2 and Definition 2). Therefore, it is optimal for type-A government to choose \( e_A = e^p \), which makes it optimal for type-B government also to choose \( e^p \), and the firm invests. Next, take the strategies of the firm and the government as given. Then, beliefs are self-fulfilling.

[QED]

That is, when \( \bar{e}_B \leq e^p < \bar{e}_B \) holds, both type-A and type-B governments over invest in public enforcement compared to that under symmetric information. However, the firm is not able to identify the government’s type from the level of enforcement observed. Type-A government over invests in order to enhance private investment’s success probability sufficiently, so that it turns out to be incentive compatible for the firm to invest in absence of any additional information. Whereas, anti-competitive government over invests to masquerade her true type.

**Proposition 4:** Suppose that Assumptions 1–5 and \( \bar{e}_B \leq e^p \) hold true. Then the following beliefs and strategies constitute a separating perfect Bayesian equilibrium.

1. The firm’s beliefs are given by \( \mu(e) = \begin{cases} 1, & \text{if } e \geq \bar{e}_B, \\ 0, & \text{otherwise.} \end{cases} \)
2. Type-A government chooses \( e^* = \bar{e}_B \), whereas type-B government chooses \( e^* = 0 \).
3. The firm invests if \( e \geq \bar{e}_B \).

**Proof:** Take the beliefs of the firm as given. We have \( p_B(e)W - e \leq 0, \, \forall \, e \geq \bar{e}_B \) (by Lemma 4 and Assumption 1(b)), implying that type-B government will never choose any \( e \geq \bar{e}_B \). Thus, by choosing any \( e \in [\bar{e}_B, \, \bar{e}_A) \), where \( \bar{e}_A \) is given by \( p_A(\bar{e}_A)W - \bar{e}_A = 0 \) (by Assumption 1 and Lemma 4), type-A government can credibly signal her true type and it is incentive compatible for her to do so as compared to choosing \( e < \bar{e}_B \) (as \( \bar{e}_B \leq e^p \)). Since \( \bar{e}_A < \bar{e}_B < \bar{e}_A, \, e = \bar{e}_B \) calls for the lowest deviation from type-A’s symmetric information optimal choice and, thus, we can argue that type-A will choose \( e^* = \bar{e}_B \), and type-B will choose \( e^* = 0 \) in the equilibrium. The firm invests if \( e \geq \bar{e}_B \), since
only type-A can choose such an $e$ and $\bar{e}_B > e^0_A$. Now, take the strategies of the firm and the government as given. Then, beliefs are self-fulfilling.

[QED]

Propositions 2 and 3 show that, when (i) the minimum public investment required by type-B government to attract private investment is sufficiently high (such that it is unprofitable for her to invest under symmetric information) and (ii) the threshold level of public investment required for the firm to invest, even under uncertainty regarding government type, is profitably attainable by type-B government, the rationale behind over investment in public anti-trust enforcement can be the anti-competitive government’s attempt to masquerade her true type. However, Proposition 4 shows that when the threshold level of enforcement required is high (such that, it generates non-positive return to the type-B government), public investment in anti-trust enforcement acts as a signal of the pro-competitive government’s support for competition, which conforms to SS’s central result.

3. Conclusion

This note extends the analysis of SS by explicitly modelling productive public investment in antitrust enforcement. It shows that SS’s result of complementarity between public antitrust enforcement and private investment is robust. However, when public investment is productive, over investment by the government does not necessarily signal the government’s commitment for pro-competition behaviour, unlike as in the case of unproductive public investment in antitrust enforcement.

Our analysis can be further extended to show that the results hold under other possible specifications of the levels and technical relationships between the unconstrained optimal public enforcement and the level of investment defining de facto the outside option of the firm under both type of government. If the critical level of enforcement required by type-B government to attract private investment accrues positive returns to her (i.e. relaxing Assumption 4), then over investment, wherever applicable, takes place only by the type-A government. In this case, all resulting equilibria are pooling equilibria, and hence over investment is not a result of signalling but solely an outcome of the type-A government’s optimization exercise. Further, relaxing the constraint on the relationship between individual critical and threshold enforcement levels of each type of government (i.e. relaxing Assumption 5) it can be shown that, over-investment, wherever applicable, takes place by the pro-competition type government as a signal of its true type only when incurring the asymmetric information threshold enforcement (as specified by Definition 3) is not profitable for the anti-competition type government. But whenever this threshold is profitably attainable by the type-B government, the rationale behind over investment in public enforcement is exactly similar to that discussed for the pooling equilibria in our analysis.

Moreover, it can be shown that, for different degrees of productiveness of public enforcement across government types (i.e. relaxing Assumption 1(c)), the anti-competitive government is induced to pool with the procompetitive one under similar conditions as discussed above, provided the pro-competitive government’s unconstrained optimal does not signal its true type. Therefore, depending on nature of success probability functions, over investment in antitrust enforcement by a government may or may not signal the government’s commitment for pro-competition behaviour. A complete characterization of the equilibrium in a more general environment remains open for future research.
Appendix: Proofs of Lemmas

Proof of Lemma 1:
\[ p_j(e_j^0) = \frac{(1+i)l}{(1+r)l}, \quad j = A, B, \quad \text{by Definition 1.} \]
From Assumption 1, we have (a) \[ p_A(e) > p_B(e), \quad \forall e \geq 0 \] and (b) \[ \frac{dp_j}{de_j} > 0, \forall e_j \geq 0, \quad j = A, B. \]
It follows that \[ e_A^0 < e_B^0. \]
\[ \text{[QED]} \]

Proof of Lemma 2:
We have, from Definition 2, \[ \left. \frac{dp_j(e_j)}{de_j} \right|_{e_j = \tilde{e}_j} = \frac{1}{w}, \quad j = A, B. \]
It implies that \[ \tilde{e}_A = \tilde{e}_B, \quad \text{by Assumption 1(c).} \]
\[ \text{[QED]} \]

Proof of Lemma 3:
We have \[ e_A^0 < e_B^0 \quad \text{(by Lemma 1)} \] and \[ \frac{dp_j}{de_j} > 0, \quad \forall e_j \geq 0, \quad j = A, B \quad \text{(by Assumption 1).} \]
Then from Definition 1 it follows that (a) \[ [\alpha p_A(e_A^0) + (1 - \alpha)p_B(e_A^0)](1 + r)l < (1 + i)l \] and (b) \[ [\alpha p_A(e_B^0) + (1 - \alpha)p_B(e_B^0)](1 + r)l > (1 + i)l. \]
Since \[ [\alpha p_A(e) + (1 - \alpha)p_B(e)] \] is strictly increasing in \( e \geq 0 \), we must have \[ e_A^0 < e_B^0. \]
\[ \text{[QED]} \]

Proof of Lemma 4:
We have (i) \[ [p_B(e)W - e]_{e=0} = p_B(0)W > 0, \quad \text{since } p_B(e) \in (0, 1) \] \( \forall e \geq 0 \); (ii) \[ [p_B(e)W - e] \] is strictly concave (by Assumption 1(b)) and it is maximum at \( e = \tilde{e}_B \); (iii) \( \tilde{e}_B > 0 \) by Lemma 2, Assumption 2 and Assumption 5. It follows that (a) \( \forall e \in [0, \tilde{e}_B], \quad p_B(e)W - e > 0 \) and (b) \( \forall e > \tilde{e}_B, \quad p_B(e)W - e \) is strictly decreasing in \( e \). Further, we have \[ p_B(e_B^0)W - e_B^0 < 0 \] (by Assumption 4). Therefore, there exists a unique \( e = \tilde{e}_B \in (\tilde{e}_B, e_B^0) \) such that \[ p_B(\tilde{e}_B)W - \tilde{e}_B = 0. \]
\[ \text{[QED]} \]

Proof of Lemma 6:
We have \[ 0 \leq e_A^0 < \tilde{e}_A = \tilde{e}_B < e_B^0, \quad \text{by (1).} \]
It follows that \[ [p_A(\tilde{e}_B)](1 + r)l > (1 + i)l \] and \[ [p_B(\tilde{e}_B)](1 + r)l < (1 + i)l, \quad \text{by Definition 1.} \]
Thus, \[ [\alpha p_A(\tilde{e}_B) + (1 - \alpha)p_B(\tilde{e}_B)](1 + r)l < (1 + i)l. \]
Further, (i) by Assumption 1(b), \[ [\alpha p_A(e) + (1 - \alpha)p_B(e)] \] is strictly increasing in \( e \) and (ii) \[ [\alpha p_A(e^p) + (1 - \alpha)p_B(e^p)](1 + r)l = (1 + i)l \] (Definition 3), and \[ e_A^0 < e^p < e_B^0 \] (Lemma 3). Again, Inequality (1) and Lemma 4 and together imply that \[ [p_A(\tilde{e}_B)](1 + r)l > (1 + i)l \] and \[ [p_B(\tilde{e}_B)](1 + r)l < (1 + i)l. \]
Then, from Definition 3 and Assumption 1(b), it follows that \( e^p < (\tilde{e}_B) > \tilde{e}_B. \)
\[ \text{[QED]} \]