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Versioning and advertising in social networks: uniform distributions of valuations

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Abstract

This note studies second-degree discrimination by a platform that sells a two-version (online) service to consumers engaged in a social network. Consumers choose between a premium version, which enhances network externalities, or a free version, which includes advertising about some other product. Under the assumptions that the consumers' valuations for the advertised product are uniformly distributed and that advertising has a signaling structure, we relate optimal pricing to the underlying degree distribution and the hazard rate of the random network. We derive close form expressions for the platform's profits in most prominent real-world social networks where online platforms operate. Platforms that operate in large and relatively sparse networks wish to provide only the free version, whereas platforms serving more densely connected networks prefer to provide both versions.

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1. Introduction

A group of consumers is engaged in a social network that enables positive externalities over the consumption of a service. A single platform provides the service through a two-version bundle: (1) a free version, with advertising about some (unrelated) product, and (2) a premium version, with no advertising, that allows for higher network externalities relative to the free version.¹ In these situations, there is a trade-off between the profits from the sales of the premium version and the profits from the advertising activity. The problem of the platform is to choose second-degree discrimination strategies, where consumers self-select themselves to adopt one version of the service or the other. Over the recent years, this type of discrimination schemes with advertising have become a prevalent practice.² This business model is known as *versioning* in the business literature.

Under fairly general assumptions, [Jimenez-Martinez \(2018\)](#) analyzes the incentives of the platform to pursue versioning, its optimal pricing policy, and the welfare implications. In that model, the results on optimal pricing, as well as the derived welfare implications, can be related to the degree distribution that underlines the social network. Given the generality of the model, a caveat of [Jimenez-Martinez \(2018\)](#)'s analysis, though, is that it does not allow for closed form expressions, neither for the platform's profit functions nor for the optimal pricing strategies. This makes it hard to map the implications of the model to sharp descriptions of how real-world platforms operate over social networks. This note aims at obtaining more detailed results from [Jimenez-Martinez \(2018\)](#)'s model on versioning by assuming that (1) the distributions of the product's valuations are uniform and (2) advertising has a signaling structure. Under these more restrictive assumptions, we are able to capture empirical regularities of second-degree discrimination with advertising in social networks. In particular, the two key applications of [Section 4](#) allow us to explore in detail the platform's profit functions for the two most prominent classes of degree distributions (according to empirical evidence): the power law pattern and the exponential pattern.³ Platforms that operate in large and

¹For example, think of Twitter or The Weather Channel platforms. Each of the two platforms offer advertising about automobiles, among many other products unrelated to their services.

²Platforms that follow this practice, with advertising, include Google and Yahoo (web search engine), Facebook, Twitter, Instagram, and Snapchat (social interaction), Whatsapp, Skype, and Line (communication) AirBnB (accommodation search), Amazon (retail, product search, and market matching), Waze (traffic and route forecasting), BBC, CNN, The New York Times (news), The Weather Channel (weather forecasting), Yelp and Foursquare (review and rating), YouTube, Vimeo, Apple Music, and Spotify (entertaining and information), Strava, MapmyRun, Nike Training Club, and Azumio (exercise and health tracking), Box (second-hand trade), or Tinder (dating).

³Power law distributions fit well the data on the nodes on the Internet ([Barabási and Albert, 1999](#)), the

relatively sparse networks—typically generated by power law distributions—have incentives to provide only the free version of the service, whereas platforms serving more densely connected networks—typically generated by exponential distributions—prefer to provide both versions.

This note is related to the literature on network externalities (Farrell and Saloner, 1985; Katz and Shapiro, 1985; Amir and Lazzati, 2011) and on second-degree discrimination (Mussa and Rosen, 1978; Maskin and Riley, 1984). Our notion of advertising builds upon the *informative advertising* notion proposed by Lewis and Sappington (1994) and Johnson and Myatt (2006). The technical side relies upon random networks and on the *configuration model* (Erdős and Renyi, 1959; Bender and Canfield, 1978; Bollobás, 2001; Newman et al., 2001; Jackson and Yariv, 2007; Galeotti and Goyal, 2009; Fainmesser and Galeotti, 2016).

2. Model

A unit mass of consumers, indexed by $i \in [0, 1]$, is embedded in a social network. A platform offers two unrelated commodities over the social network: (1) $z \geq 0$ units of a product, which it produces at a marginal cost $c \in (0, 1/2)$,⁴ and (2) any discrete quantities of a two-version online service bundle, which it produces at no cost. The network enables positive externalities for the consumption (only) of the service: the utility of each consumer from her consumption of the service increases with the number of her neighbors. The two-version service bundle consists of: (a) a *free version*, which is offered with advertising about the product, and (b) a *premium version*, which does not include advertising and, furthermore, allows consumers higher externalities relative to the free version. Notation $a = 0$ or $a = 1$ indicates that advertising is either absent or present, respectively. The price of each unit of the product is p and the price of the (premium version of the) service is q . There is no price discrimination for the advertised product: all consumers face the common price p , regardless of the version of the service they adopt.

Each consumer has a unit demand for the service. Let z_i be consumer i 's probability of purchasing the product and let x_i be the probability that consumer i adopts the premium

number of links in websites (Clauset et al., 2009), and the social commerce network through the Internet (Stephen and Toubia (2009)). On the other hand, Rosas-Calals et al. (2007), Ghoshal and Barabási (2011), and Scholz (2015) provide empirical evidence which suggests that some small and dense social networks adjust well to the pattern of exponential degree distributions.

⁴The production cost of the advertised product is naturally assumed to be lower than its prior expected valuation.

version of the service. Each consumer is willing to pay up to ω for a unit of the product and up to θ for a unit of the service. The consumers' valuations for the product ω are independently drawn from a uniform distribution $U[0, 1]$ and their valuations for the service θ are independently drawn from some interval $(\underline{\theta}, \bar{\theta})$, with $\underline{\theta} > 0$, according to some (common) distribution. The two commodities are totally unrelated and, therefore, the valuations ω and θ are assumed to be independent from each other. By construction, the analysis focuses on groups of consumers where all of them are willing to adopt at least the free version of the service.⁵

2.1. Informative Advertising

Advertising helps consumers to improve their knowledge of their own tastes for the product. Prior to their purchasing decisions, consumer do not know their valuation ω . Instead, they receive some (noisy) *public signal realization* $y \in (1/2, 1]$.⁶ Consumers then use the signal realization y to update their beliefs about their underlying valuations ω . The public signal yields either:

- (1) the actual realization of the valuation $y = \omega$, with probability $\frac{1+a}{2}$, or
- (2) some independent drawn from $U[0, 1]$, with probability $\frac{1-a}{2}$.

Given this signaling structure, each consumer i obtains the posterior expectation of her valuation for the product by applying Bayes rule:⁷

$$E[\omega | y, a] = \frac{1+a}{2}y + \frac{1-a}{2} \left(\frac{1}{2} \right). \quad (1)$$

When advertising is absent ($a = 0$), consumers place weights according to a uniform probability on either the public signal realization y or their prior $1/2$ being their true underlying valuations, $E[\omega | y, 0] = (1/2)y + (1/2)(1/2)$. When advertising is present ($a = 1$), consumers are able obtain additional information and turn to assign probability one to the

⁵This makes sense if we consider that such “minimally interested” consumers are indeed the ones who form the network targeted by the platform.

⁶The signal realization is naturally assumed to be higher than the expected prior of the advertised product's valuation. The signal realization y can be interpreted as the public observation of some posted information about the product quality (such as the one obtained from existing commercials, marketing samples, or from some other selling activities).

⁷In the expression in (1), the unconditional prior is given by $E[\omega] = 1/2$ since $\omega \sim U[0, 1]$.

valuation $\omega = y$ and probability zero to the prior: consumers learn their true valuations, $E[\omega | y, 1] = y$.⁸ Use $P_a(z)$ to denote the inverse demand of the product if *all* consumers receive an advertising level $a \in \{0, 1\}$ and suppose that the platform offers a quantity z of the product. Since the product's valuation is drawn from a $U[0, 1]$ distribution, the platform sells the product to those consumers receiving a public signal greater than $y = 1 - z$. The platform must therefore set prices according to the linear rules:

$$\begin{aligned} P_0(z) &= (3/4) - (1/2)z; \\ P_1(z) &= 1 - z, \end{aligned} \tag{2}$$

which gives us the expressions for the induced inverse demands of the product, conditioned on each advertising level $a \in \{0, 1\}$.

2.2. The Social Network and Preferences

The social network is stochastic.⁹ The platform and the consumers are uncertain about the specific architecture of the network but they commonly know the stochastic process that generates it. There is a set $[\underline{n}, \bar{n}] \subseteq \mathbb{R}_+$ of possible neighborhood sizes, or *degrees in the social network*.¹⁰ Let $n_i \in [\underline{n}, \bar{n}]$ be a possible *degree for consumer i* . The *degree distribution of the social network* is given by a twice continuously differentiable distribution $H(n)$, with strictly positive density $h(n)$, over the support $[\underline{n}, \bar{n}]$.¹¹

Let $x(q)$ be the fraction of premium version adopters at price q , conditioned on the degree distribution $H(n)$. Since each consumer is assumed to adopt one of the two versions of the service (which is ensured by $\underline{\theta} > 0$), the average consumption of the service of a consumer's

⁸This simple formulation builds on the settings proposed by [Lewis and Sappington \(1994\)](#) and [Johnson and Myatt \(2006\)](#) to analyze informative advertising. In particular, our specification of signaling ads induces the type of fairly general clockwise demand rotations which were proposed by [Johnson and Myatt \(2006\)](#) and subsequently used by [Jimenez-Martinez \(2018\)](#).

⁹The links facilitated by the platform are included in the targeted social network but the network can in principle be a broader graph. Conceivably, the network can include also family, friendship, working relations links, or links provided by other platforms. Then, one would naturally consider that the platform cannot create or remove family, friendship or other links which are external to the platform's provision of its online service bundle. While considering a random network makes the analysis tractable, it allows for interesting dynamic interpretations of how neighborhoods' sizes evolve over time, which can formally be related to the optimal pricing policy of the platform.

¹⁰For some applications, one might consider $\underline{n} = 0$. Also, the set $[\underline{n}, \bar{n}]$ could be unbounded as well and, in particular, \bar{n} could tend to infinity in some cases.

¹¹In other words, the degree of a fraction $H(n) = \int_{\underline{n}}^n h(m)dm$ of consumers does not exceed n .

neighbor equals one.¹² Therefore, in a quite convenient way, the number n_i captures in our setup the average consumption of the service of agent i 's neighbors, conditioned on consumer i having degree n_i .

The network allows consumers to interact *locally* with respect to their consumptions (only) of the service. The consumption of the service exhibits a local (positive) network effect: a consumer's utility from—any version of—the service increases as her neighbors increase their consumptions¹³—regardless of the version of the service that the neighbors adopt. A consumer's utility raises by an amount of 1, when he adopts the free version of the service, or by an amount $1 + \beta$, where $\beta > 0$, when he adopts the premium version, for each unit of the service consumed by her neighbors. The term β describes an *externality premium*.¹⁴

To close the model, we consider that consumers process their information about their neighbors according the *degree independence assumption*. This assumption requires that each member of the network regards her links as independently chosen from the random network¹⁵ and guarantees that her degree is the only relevant information about the network that each consumer needs to consider.

Under the assumptions above, the expected utility of a consumer i , conditioned on having degree n_i , is given by

$$u(z_i, x_i | n_i) \equiv \underbrace{z_i(\omega - p)}_{\text{product}} + \underbrace{x_i[\theta - q + (1 + \beta)n_i]}_{\text{premium version of the service}} + \underbrace{(1 - x_i)[\theta + n_i]}_{\text{free version of the service}}. \quad (3)$$

¹²Formally, under the assumptions of the *configuration model*, the expected consumption of some version of the service of a consumer i 's neighbor is derived as

$$\frac{\int_{\underline{n}}^{\bar{n}} mh_s(m) [\mathbb{P}(z_i = 1 | n_i = m) + \mathbb{P}(z_i = 0 | n_i = m)] dm}{\int_{\underline{n}}^{\bar{n}} nh_s(n) dn} = 1$$

since, given the assumption $\underline{\theta} > 0$, we directly obtain $\mathbb{P}(z_i = 1 | n_i = m) + \mathbb{P}(z_i = 0 | n_i = m) = 1$ for each degree $m \in [\underline{n}, \bar{n}]$.

¹³In practice, such externalities take the form of benefits from being able to interact with a higher number of people (e.g. social interaction, second-hand trading, exercise tracking, or dating services), informational gains (e.g., weather forecast, traffic monitoring, news services, or review and rating services), or collaborative gains (e.g., online gaming or collaborative projects).

¹⁴This feature seems to be the consistent with most real-world online services where the premium version allows consumers to enjoy the network externalities to a greater extent, compared to the free version.

¹⁵This is a very common assumption in the literature on random networks and has been used, among others, by Jackson and Yariv (2007), Galeotti et al. (2010), Fainmesser and Galeotti (2016), and Shin (2016).

It follows from the specification in (3) that the fraction of free version adopters at price q , conditional on the degree distribution $H(n)$, is

$$\begin{aligned} 1 - x(q) &= \mathbb{P}(x_i = 0) = \mathbb{P}(\theta - q + (1 + \beta)n_i \leq \theta + n_i) \\ &= \mathbb{P}(n_i \leq q/\beta) = H(q/\beta). \end{aligned} \tag{4}$$

For price q , let $n(q) \equiv q/\beta$ be the cutoff degree such that $x_i = 0$ if $n_i \leq n(q)$ whereas $x_i = 1$ if $n_i > n(q)$. The sensitivity of the demand $x(q)$ of the premium version with respect to its price depends on the random process that generates the network and on the externality premium: $x'(q) = -(1/\beta)h(n(q))$. Note that all consumers purchase the free version of the service (i.e., $x(q) = 0$) when its price satisfies $q \geq \bar{q}$, where the upper bound \bar{q} on the service price is given by $\bar{q} \equiv \beta\bar{n}$. Since we are interested in restricting attention to optimal policies such that $q \in (\underline{\theta}, \bar{\theta})$, we will accordingly consider that $\bar{\theta} \geq \bar{q} \equiv \beta\bar{n}$.

Hazard rate analysis is useful in to capture key features of how the random network evolves in dynamic interpretations. The *hazard rate function* of the random network with distribution degree $H(n)$ is the function $r : [\underline{n}, \bar{n}] \rightarrow \mathbb{R}_+$ defined as¹⁶

$$r(n) \equiv \frac{h(n)}{1 - H(n)}. \tag{5}$$

3. Main Result

Use $z_a(p)$ and $\pi_a(p)$ to denote, respectively, the demand of the product and the profits to the platform from the product's sales, for price p , conditioned on advertising level a . In the absence of advertising ($a = 0$), we derive (using the expression in (2)): $z_0(p) = 3/2 - 2p$ for $p \leq 3/4$, which in turn leads to the profits

$$\pi_0(p) = \begin{cases} (p - c) [3/2 - 2p] & \text{if } p \in (0, 3/4] \\ 0 & \text{if } p \in (3/4, 1]. \end{cases} \tag{6}$$

¹⁶For dynamic interpretations where the network evolves along several periods, the function $r(n)$ gives us the probability that a randomly selected consumer has approximately n neighbors in a subsequent period, conditioned on her current neighborhood size being no less than n . In addition, note that the hazard rate of the degree distribution underlying the random network gives us key information about the sensitivity of the demand for the premium version of the service: $r(n(q)) = -\beta x'(q)/x(q)$.

In the presence of adverting ($a = 1$), we derive (again, using the expression in (2)): $z_1(p) = 1 - p$ for $p \leq 1$, which leads to the profits

$$\pi_1(p) = (p - c)[1 - p] \quad \text{if } p \in (0, 1]. \quad (7)$$

The platform's optimal pricing follows backwards induction through a two-stage process. Given a degree distribution $H(n)$, that underlines the random network, and a price $q \in (\underline{\theta}, \bar{\theta})$ of the service, the platform's goal is first to set a price $p^* \in (0, 1)$ for the product so as to maximize the profits from the product's sales

$$\pi(p, q) \equiv x(q)\pi_0(p) + [1 - x(q)]\pi_1(p). \quad (8)$$

Since advertising affects (some of) the consumers' willingness to pay for the product, the profits from the product's sales naturally depends on the fraction $x(q)$ of premium version adopters. Then, given an optimal choice $p^* \in (0, 1)$ for the price of the product, the platform wishes to choose a price $q^* \in (\underline{\theta}, \bar{\theta})$ for the service so as to maximize its overall profits

$$\Pi(p^*, q) \equiv qx(q) + \pi(p^*, q). \quad (9)$$

Existence of equilibrium is guaranteed since the profits specified in (8) and (9) are continuous functions on compact convex sets.

Proposition 1 provides the key necessary conditions that must satisfy the platform's optimal pricing strategy for interior prices in our model.

Proposition 1. *Consider a random social network with degree distribution $H(n)$ and hazard rate function $r(n)$. Suppose that the platform optimally chooses interior prices $p^* \in (0, 1)$, for the advertised product, and $q^* \in (\underline{\theta}, \bar{\theta})$, for the service, with an equilibrium fraction of premium version adopters $x(q^*) \in (0, 1)$. Then, such optimal prices must satisfy:*

$$p^* = \frac{1}{2} \left(\left[\frac{1 + x(q^*)/2}{1 + 2x(q^*)} \right] + c \right) \quad (10)$$

and

$$q^* + \Phi(q^*) = \frac{\beta}{r(n(q^*))}, \quad (11)$$

where $\Phi(q)$ is a function that decreases strictly in q if $x(q) > 1/7$ (equivalently, if $H(n(q)) < 6/7$) and increases strictly if $x(q) < 1/7$ (equivalently, if $H(n(q)) > 6/7$).

Since the function $\Phi(q)$ identified in the equilibrium condition (11) of [Proposition 1](#) is not monotone in q , uniqueness of equilibrium cannot be guaranteed in general even when the hazard rate $r(n(q))$ is monotone in q . Nevertheless in our two main applications, which capture most real-world social networks targeted by online platforms, equilibrium is unique.

4. Applications

The proof of [Proposition 1](#) provided in the Appendix shows that the platform's profits, for interior optimal prices p^* , are given by

$$\Pi(p^*, q) = qx + \frac{1}{4} [\eta(x) - c] [1 + [1 - \eta(x) - c][1 + x]], \quad (12)$$

where x stands for short-hand notation of the fraction of premium version adopters at price q (i.e., $x = x(q)$), and $\eta(x)$ denotes the function $\eta(x) \equiv [1+x/2]/[1+2x]$. The expression above for the platform's profits $\Pi(p^*, q)$ will be used in our two key applications. Also, the proof of [Proposition 1](#) gives us the expression of function the $\Phi(q)$ identified in the proposition. Function $\Phi(q)$ will be used in the applications to compute optimal prices for the service, by using the composite function $\phi(x)$ specified as

$$\phi(x) \equiv \frac{1}{4} \left[\eta'(x) [1 + [1 + x] [1 - 2\eta(x)]] + [\eta(x) - c] [1 - \eta(x) - c] \right],$$

where $\phi(x(q)) \equiv \Phi(q)$. In addition, for the next two applications, consider a cost $c = 1/4$ and a value $\beta = 1$ for the externality premium, which implies $n(q) = q$.

4.1. Power Law Degree Distribution

Empirical evidence suggests that most real-world online and Internet-based social networks are *scale-free* and, therefore, that the corresponding degree distribution must follow a *power law*.¹⁷ For a degree support $[\underline{n}, +\infty)$, with $\underline{n} > 0$ a *power law degree distribution with parameter $\sigma > 1$* , is given by

$$H(n) = 1 - \underline{n}^{\sigma-1} n^{-(\sigma-1)}.$$

Most empirical estimates propose values for the parameter σ that lie in the interval (2, 3). The corresponding hazard rate function is $r(n) = (\sigma - 1)/n$, which decreases in n . Let us

¹⁷ Also, power law degree distributions are particularly suitable to model the formation of networks that follow a preferential attachment pattern.

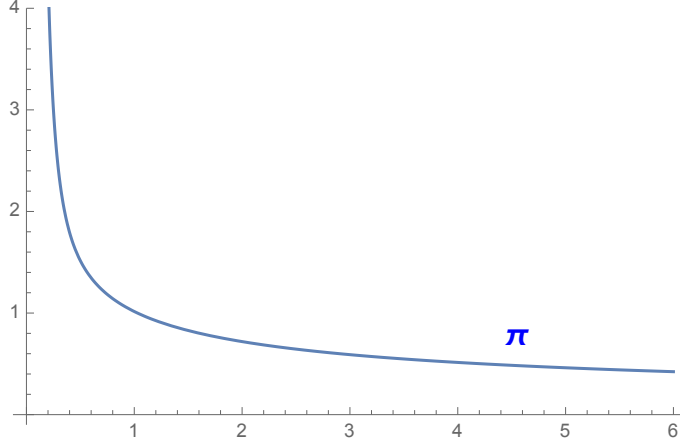


Figure 1 – Platform’s profits for the power law degree distribution.

consider $\sigma = 5/2$ and $\underline{n} = 1$. Then, the demand of the premium version of the service is given by $x(q) = q^{-3/2}$ so that $\eta(x) = [2 + q^{-3/2}]/[2 + 4q^{-3/2}]$. From the expression provided in (12), we obtain that the platform’s profit function $\Pi(p^*, q)$ takes the form

$$\Pi(p^*, q) = q^{-1/2} + \frac{1}{4} \left(\frac{2 + q^{-3/2}}{2 + 4q^{-3/2}} - \frac{1}{4} \right) \left(1 + \left[\frac{3}{4} - \frac{2 + q^{-3/2}}{2 + 4q^{-3/2}} \right] [1 + q^{-3/2}] \right).$$

The graph of such a profit function is depicted in [Figure 1](#).

The necessary condition provided by [Proposition 1](#) in (11) is not satisfied in this application. We observe that the platform’s optimal choice entails a corner solution. The platform’s profits are always decreasing in the price of the (premium version of) the service. It then follows that providing the premium version is not profitable and the platform chooses optimally to serve only the free version of the service.

4.2. Exponential Degree Distribution

A typical degree distribution with constant hazard rate function is the *exponential degree distribution*. At a theoretical level, exponential degree distributions are often used to capture the formation of links according to uniform randomness in growing random networks.¹⁸ For a degree support $[0, +\infty)$, an *exponential degree distribution with parameter $\sigma > 0$* is given by

$$H(n) = 1 - e^{-n/\sigma}.$$

¹⁸See, e.g., [Jackson \(2008\)](#), Chapter 5, for an insightful description of the use of the exponential degree distribution in growing random networks.

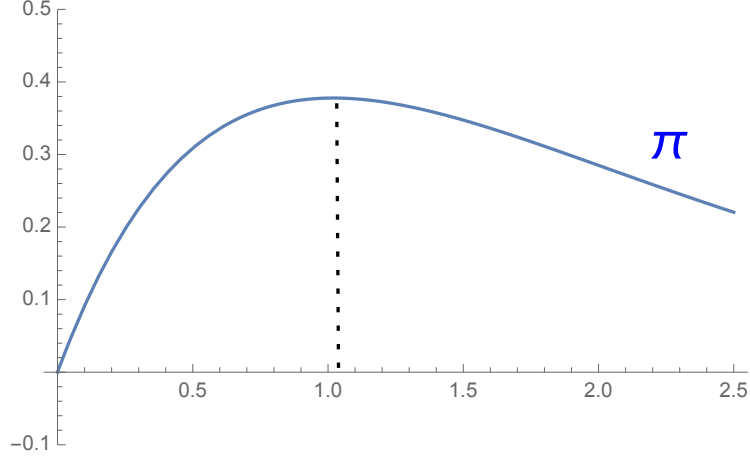


Figure 2 – Platform’s profits for the exponential degree distribution.

The corresponding hazard rate function is $r(n) = 1/\sigma$. Let us consider $\sigma = 1$. Then, the demand of the premium version of the service is given by $x(q) = e^{-q}$ so that $\eta(x) = [2 + e^{-q}]/[2 + 4e^{-q}]$. From the expression provided in (12), we obtain that the platform’s profit function $\Pi(p^*, q)$ takes the form

$$\Pi(p^*, q) = qe^{-q} + \frac{1}{4} \left(\frac{2 + e^{-q}}{2 + 4e^{-q}} - \frac{1}{4} \right) \left(1 + \left[\frac{3}{4} - \frac{2 + e^{-q}}{2 + 4e^{-q}} \right] [1 + e^{-q}] \right).$$

The graph of such a profit function is depicted in [Figure 2](#).

The necessary condition provided by [Proposition 1](#) in (11) yields the interior optimal price $q^* \approx 1.056$. In this application, the platform thus optimally chooses to pursue versioning by offering both versions of the service.

5. Concluding Remarks

We have obtained that platforms operating in social networks that feature power law patterns have strong incentives to offer only the free version of their services, whereas social networks governed by exponential patterns give platforms incentives to rely on both versions of their services.

In practice, Internet-based platforms such as Google, Facebook, and Twitter only serve the free version with advertising. On the other hand, casual observation suggests that Application-based platforms such as Nike Training Club, Strava, or MapmyRun serve to local networks that are certainly much smaller than the networks where Google or Twitter

operate. Furthermore, in these Application-based social networks, each user can conceivably access the information posted by most of their local users.

Appendix

Proof of Proposition 1. Consider a random network with degree distribution $H(n)$ and hazard rate function $r(n)$. Using the expressions derived in (6)–(9) for the platform’s profits, it follows that

$$\Pi(p, q) = \begin{cases} qx(q) + (p - c) [x(q) (\frac{3}{2} - 2p) + [1 - x(q)] (1 - p)] & \text{if } p \in [0, \frac{3}{4}] \\ (p - c) [1 - p] & \text{if } p \in (\frac{3}{4}, 1]. \end{cases}$$

Take given a price $q \in (\underline{\theta}, \bar{\theta})$ of the service. Application of the first-order conditions for interior solutions $\partial\Pi(p, q)/\partial p = 0$ to both steps of the profit function above yields:

(a) for candidates to interior maximum $p^* \in (0, \frac{3}{4})$, we have

$$x(q) (3/2 - 2p^*) + [1 - x(q)] (1 - p^*) - (p^* - c) [1 + x(q)] = 0 \Leftrightarrow p^* = \frac{\left[\frac{1+x(q)/2}{1+2x(q)} \right] + c}{2};$$

(b) for candidates to interior maximum $p^{**} \in (\frac{3}{4}, 1)$, we have

$$1 - p^{**} - (p^{**} - c) = 0 \Leftrightarrow p^{**} = \frac{1 + c}{2}.$$

For $x \in [0, 1]$, let us specify the function $\eta(x) \equiv [1 + x/2]/[1 + 2x]$, where x is used as shorthand notation for the proportion of premium version adopters at price q , i.e., $x \equiv x(q)$. Observe that the candidate p^* for interior optimum obtained above can be rewritten as $p^* = [\eta(x) + c]/2$ for $x \in [0, 1]$ and that p^* tends to $p^{**} = (1 + c)/2$ as x tends to zero (that is, as all consumers become free version adopters). Therefore, using the function $\eta(x)$, any candidate to interior optimum can be described by $p^* = [\eta(x) + c]/2$ for $x \in [0, 1]$. Note that $\eta'(x) = -3/2(1 + 2x)^2 < 0$, so that $\eta(x)$ always decreases in $x \in [0, 1]$, with $\eta(0) = 1$ and $\eta(1) = 1/2$. Thus, $\eta(x) \in [1/2, 1]$ for each $x \in [0, 1]$. Also, using the function $\eta(x)$ together with the expressions for candidates to interior optima derived above, it can be verified that while $0 < c < 3/2 - \eta(x)$ is required for $p^* \in (0, 3/4)$ to be an interior optimum, $c > 1/2$ is necessary for $p^{**} \in (3/4, 1)$ to be an interior optimum. Therefore, since the production cost is assumed to satisfy $0 < c < 1/2$, we obtain that any interior optimum p^* must

satisfy $p^* = [\eta(x) + c]/2$, with $x \in (0, 1)$ and $\eta(x) \in (1/2, 1)$. In addition, the second-order condition for the price choice p is automatically satisfied since $\Pi(p, q)$ is strictly convex in p . It follows that any interior optimal choice by the platform implies that it pursues versioning by offering both versions of its service, with $x(q^*) \in (0, 1)$ and $(1/2 + c)/2 < p^* < (1 + c)/2$. Since $c \in (0, 1/2)$, it follows that if p^* is an interior optimal choice by the platform for the advertised product, then $p^* \in (1/4, 3/4)$.

By plugging the expression $p^* = [\eta(x) + c]/2$ for the interior optimal price of the advertised product into the platform's profit function, it follows that

$$\Pi(p^*, q) = qx(q) + \frac{1}{4} [\eta(x) - c] [1 + [1 - \eta(x) - c][1 + x]]. \quad (13)$$

Given the optimal choice $p^* \in (1/4, 3/4)$ for the advertised product, the first-order condition for interior solutions with respect to the price $q \in (\underline{\theta}, \bar{\theta})$ of the service then leads to

$$\begin{aligned} \frac{\partial \Pi(p^*, q^*)}{\partial q} = & x(q^*) + q^* x'(q^*) + \frac{\eta'(x)x'(q^*)}{4} [1 + [1 - \eta(x) - c][1 + x]] \\ & + \frac{\eta(x) - c}{4} [-\eta'(x)[x + 1] + [1 - \eta(x) - c]] x'(q^*) = 0. \end{aligned} \quad (14)$$

Since the density $h(n)$ is strictly positive over the degree support, we have that

$$x'(q^*) = -(1/\beta)h(n(q^*)) \neq 0.$$

Hence, by dividing over $x'(q^*)$ the expression obtained in (14) above, we derive the necessary condition

$$\begin{aligned} \frac{x(q^*)}{x'(q^*)} + q^* + \frac{\eta'(x)}{4} [1 + [1 - \eta(x) - c][1 + x]] \\ + \frac{\eta(x) - c}{4} [-\eta'(x)[x + 1] + [1 - \eta(x) - c]] = 0. \end{aligned}$$

In turn, by rearranging the expression above, we obtain

$$q^* + \frac{1}{4} \left[\eta'(x) [1 + [1 + x][1 - 2\eta(x)]] + [\eta(x) - c] [1 - \eta(x) - c] \right] = -\frac{x(q^*)}{x'(q^*)}. \quad (15)$$

Let $\Phi(q)$ be the composite function specified as

$$\Phi(q) = \phi((x(q))) \equiv \frac{1}{4} \left[\eta'(x) [1 + [1 + x][1 - 2\eta(x)]] + [\eta(x) - c] [1 - \eta(x) - c] \right].$$

Application of the chain rule yields $\Phi'(q) = -(1/\beta)h(n(q))\phi'(x)$. Furthermore, by computing the required derivative, we obtain

$$\phi'(x) = \frac{1}{4} \left[\eta''(x) \left[1 + [1+x][1-2\eta(x)] \right] + \eta'(x) \left[2(1-2\eta(x)) - 2\eta'(x)[1+x] \right] \right].$$

Using the specification of the function $\eta(x)$, we can derive the useful identities:

$$\begin{aligned} \eta'(x) &= -\frac{3}{2(1+2x)^2}, \quad \eta''(x) = \frac{6}{(1+2x)^3}, \\ 1-2\eta(x) &= \frac{x-1}{1+2x}, \quad 1+[1+x][1-2\eta(x)] = \frac{x(2+x)}{1+2x}. \end{aligned}$$

By plugging such calculations into the expression of the derivative $\phi'(x)$ above, it follows that:

$$\phi'(x(q)) = \frac{3(7x(q)-1)}{8(1+2x(q))^4} > 0 \quad \Leftrightarrow \quad x(q) > 1/7.$$

Furthermore, it follows that:

$$\Phi'(q) = -\frac{h(n(q))}{\beta} \cdot \frac{3(6-7H(n(q)))}{8(3-2H(n(q)))^4} < 0 \quad \Leftrightarrow \quad H(n(q)) < 6/7 \text{ or, equivalently, } x(q) > 1/7.$$

Finally, by combining the implication that $r(n(q)) = -\beta x'(q)/x(q)$ and the condition derived in (15), we obtain the necessary condition for the platform's interior optimal choice:

$$q^* + \Phi(q^*) = \frac{\beta}{r(n(q^*))},$$

as stated. ■

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