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Income inequality, utility, and optimal income taxation

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Abstract

Income inequality has increasingly become a concern for politicians, voters, and economists. Lively debates arise on how the tax and welfare system can reduce this income inequality. Economists studying optimal income tax design, however, search for the properties of a tax system that maximizes a social welfare function dependent upon individuals' utilities. This study analyzes policy differences and the social cost resulting from policymakers searching to reduce income inequality rather than utility inequality.

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1. Introduction

Rising U.S. income inequality has become a major political topic. Senator Bernie Sanders went far in the Democratic primaries warning that the top 1% get too much of national income. Policymakers may focus on income distribution, but optimal income tax studies focus on utility distribution. Mirlees (1971) began the modern approach to optimal income taxation by searching for the income tax system that would maximize social welfare, where social welfare depended upon the utility levels of people in that society. Utility depended upon goods directly purchased with money income as well as the labor necessary to generate that income. Numerous studies have continued his use of a Bergson-Samuelson individualistic social welfare function.

Policymakers and most government policy analysts focus on income inequality not utility inequality. Consider a simple society consisting of two people who each receive $48,000 in income from earnings, but one person faces a wage rate of $20/hour, and the other faces a wage rate of $60/hour. This society has no income inequality, but likely suffers significant utility inequality since one person must work three times as many hours as the other. Ryu and Slotje (2017) did compare current income inequality to utility inequality based on the Gini coefficient, but they also left out the pleasure from leisure in their utility function, and their comparison was not in an optimal tax framework. Their simple utility function depended only upon money income. We focus on how inequality in utility, including leisure, changes with the tax system, and how the tax system changes individual choices, including work effort.

We bring this income/utility inequality comparison into the optimal tax literature. As we shall see, where utilities depend heavily on the consumption of two items, but policymakers focus on the inequality of one item, we can expect differences in policy recommendations compared to optimal tax studies. We will also measure the social cost created by policymakers focusing on income inequality rather than utility inequality.

2. The Model

Simulations will be run using ten representative individuals to describe U.S. households. We assume these individuals have identical utility functions u(x,y), where x equals consumption, and y equals labor services. We also assume individuals have identical time endowments, $y_0 = 3120$ hours, available to divide between work and leisure. Individuals differ only in their skill levels and their unearned incomes. We order individuals by wage rates so that the lowest wage person earns w_1, and the highest wage person earns w_{10}. Unearned income A_i will also be included for each individual.

The utility function describing consumer tastes will be given by the CES utility function given in equation (1). Our simulations will assume $\beta = 0.995$ and an elasticity of substitution $\sigma = 1/(1-\rho) = 0.45$.[1]

[1] Simulations were run on values of β equal to 0.98, 0.99, 0.995, and 0.999 and $\sigma = 0.40$, 0.45, and 0.50. The pattern for welfare losses, tax rates, and guarantees were similar to those presented. These elasticities of substitution fall in line with those cited by Tuomala (2016). The values of $\beta = 0.995$ and $\sigma = 0.45$ chosen for presentation were among the simulations leading to reasonable labor supply results. Table 4 displays welfare losses for all of the parameters run.

$$u(x,y) = \left[\beta x^{\rho} + (1-\beta)\left(y_0 - y\right)^{\rho}\right]^{1/\rho} \tag{1}$$

The budget constraint facing individuals will be affected by society's tax function. In this study the government will be using a linear income tax (LIT) to finance transfers and to collect a fixed sum for nontransfer expenditures. This linear tax system can be described by the marginal tax rate t and the guaranteed income level G. The individual's budget constraint becomes: $x = (1 - t)[wy + A] + G$.

In determining the optimal guarantee and marginal tax rate economists typically maximize the Bergson-Samuelson social welfare function given in (2) subject to a government budget constraint that collects a fixed sum for nontransfer purposes. Here ε is an index of inequality aversion developed by Atkinson (1970). The greater is ε, the greater society's aversion to inequality. An ε equal to zero implies a simple sum of utilities, but as ε goes toward infinity, W moves to the Rawlsian maximin criterion.

$$W(u_1,u_2,...,u_N) = \frac{\sum_{i=1}^{N}\left[u(x_i,y_i)\right]^{1-\varepsilon}}{1-\varepsilon} \tag{2}$$

We will assume that policymakers, however, worry about the distribution of income not utility. They choose a tax system to maximize their view of welfare XW given in (3).

$$XW(x_1,x_2,...,x_N) = \frac{\sum_{i=1}^{N} x_i^{1-\varepsilon}}{1-\varepsilon} \tag{3}$$

To estimate the social welfare loss due to policymakers focusing on income inequality we first find the tax rate and guarantee that maximize XW subject to the government budget constraint but that actually achieves true welfare W_0. Then we find the tax rate and guarantee that maximize government revenue subject to the constraint that social welfare equal W_0. The increase in nontransfer revenue collected provides a dollar measure for the social welfare loss created by focusing on income inequality.

3. The Data

Data from the March 2016 Current Population Survey provided by NBER at nber.org were used to construct our representative skill distribution.[2] Only working householders were included. Wage rates were constructed using earnings and reported hours worked for the year 2015. These wage rates were then arranged in ascending order. These wage rates were placed in ten equal sized groups, and the mean wage rate for each group serves as the wage rate for that representative individual in our simulations. Table 1 presents these calculated wage rates, unearned income, and the current average hours worked for each of these representative workers.

Our government budget constraint assumes that the tax system must collect a fixed sum for nontransfer purposes. The true federal tax on income in 2015 will represent this sum after

[2] Construction of a representative skill distribution from CPS data has also been used in Moreland (2004) and Garfinkel, Moreland, and Sadka (1986).

adjusting for the householders' share of total income. A value of $8131 will be used to represent this fixed sum per person.

Table 1. Wage Rate Distribution

Representative Group	Average Hourly Wage Rate	Average Unearned Income	Average Annual Labor Supply
w_1	5.42	700	1718
w_2	10.04	589	1844
w_3	12.90	723	1978
w_4	15.64	920	2001
w_5	18.63	1155	2041
w_6	22.18	1737	2087
w_7	26.55	2040	2096
w_8	32.78	2852	2084
w_9	43.07	3624	2070
w_{10}	192.28	7075	1966

4. Results

4.1 Traditional Optimal Linear Income Tax

The optimal linear income tax approach searches for the t and G that maximize the social welfare function W subject to the government budget constraint accounting for individual reactions as tax changes alter individual budget constraints. Table 2 presents the optimal t and G for various societal attitudes toward inequality as well as the resulting mean income and labor supplies. The optimal tax rates and guarantees appear to be high even where society cares little about inequality in utility. The tax rate equals 65.2% for $\varepsilon = 0$ and rises to 85.2% for the maximin welfare function. The guarantee equals $29,425 for $\varepsilon = 0$ and rises to $34,873 for the

Table 2. Optimal Linear Income Tax Based on Inequality of Utility

	$\varepsilon = 0$	$\varepsilon = 0.4$	$\varepsilon = 0.75$	$\varepsilon = 1.4$	maximin
t	0.652	0.693	0.715	0.741	0.852
G	29,425	31,102	31,918	32,792	34,873
Mean Inc	57,601	56,613	56,013	55,227	50,475
y_1	938	730	609	456	0
y_2	1319	1177	1093	986	375
y_3	1425	1305	1235	1145	623
y_4	1489	1386	1325	1246	788
y_5	1535	1446	1393	1324	921
y_6	1568	1492	1446	1387	1035
y_7	1596	1531	1493	1443	1141
y_8	1612	1561	1531	1491	1244
y_9	1619	1584	1562	1534	1353
y_{10}	1385	1406	1418	1433	1507

maximin welfare function. The increased tax rates and guarantees that follow an increase in inequality aversion generally lead to a reduction in labor supplies and mean income.

4.2 The Policymaker Choice

We now assume policymakers search for the t and G that will maximize XW subject to the government budget constraint. Table 3 shows that t and G also increase as society's income inequality aversion increases, but there are significant differences between the values here and those for the traditional optimal LIT, particularly where society has little aversion to inequality. Where $\varepsilon = 0$ society does not care about income inequality, and the optimal t and G end up maximizing mean income with a zero tax rate and a negative guarantee that finances nontransfer expenditures with a lump-sum tax.

With a modest concern for income inequality illustrated by $\varepsilon = 0.4$ society wishes to transfer to low-income households by providing a $12,810 guarantee financed with a 33.7% tax rate. This tax rate and guarantee fall far below those for the same aversion to utility inequality found in Table 2, where t equals 69.3% and G equals $31,102.

As income inequality aversion increases however, tax rates and guarantees get closer to the utility inequality results. For the maximin welfare function the tax rate of 85.2% and guarantee of $34,873 equal the results for utility inequality shown in Table 2. In the maxi-min case the high tax rate and guarantee caused the least-skilled worker to choose not to work; i.e. leisure hit its upper limit. In this case the goal of either approach became to maximize the income of the least well-off person.

A similar pattern can be seen for the size of the social welfare loss created by policymakers focusing on income inequality rather than utility inequality. If society can be

Table 3. Optimal Linear Income Tax Based on Inequality of Income

	$\varepsilon = 0$	$\varepsilon = 0.4$	$\varepsilon = 0.75$	$\varepsilon = 1.4$	maximin
t	0	0.337	0.506	0.621	0.852
G	–8131	12,810	22,324	28,046	34,873
Mean Inc	64,501	62,139	60,187	58,256	50,475
y_1	2788	2048	1532	1080	0
y_2	2534	2061	1720	1416	375
y_3	2423	2039	1760	1506	623
y_4	2335	2014	1776	1559	788
y_5	2253	1985	1783	1596	921
y_6	2166	1947	1778	1620	1035
y_7	2081	1907	1770	1639	1141
y_8	1977	1851	1747	1646	1244
y_9	1846	1774	1709	1642	1353
y_{10}	1174	1261	1320	1370	1507
SWL	8654	5351	2804	1380	0

described by inequality aversion $\varepsilon = 0.4$, the social welfare loss (SWL) created by mistakenly focusing on income inequality will be a substantial $5351 per household as seen in Table 3.[3] If $\varepsilon = 1.4$ the loss is $1380 per household, and if society takes the maximin view of social welfare, the loss vanishes.

Examining Table 4 one can see the same pattern of social welfare loss for a wide range of utility function parameters. In a few cases under the maxi-min welfare function, the least-well off person worked a small number of hours resulting in a very small, but non-zero, social welfare loss.

Table 4
Social Welfare Loss in Dollars per Household

	$\varepsilon = 0$	$\varepsilon = 0.4$	$\varepsilon = 0.75$	$\varepsilon = 1.4$	Maxi-Min
$\beta = 0.98; \sigma = 0.40$	5891	8012	7044	3801	0
$\beta = 0.98; \sigma = 0.45$	7007	9851	5764	3001	0
$\beta = 0.98; \sigma = 0.50$	7858	8125	4474	2271	0
$\beta = 0.99; \sigma = 0.40$	6908	9495	5577	2884	0
$\beta = 0.99; \sigma = 0.45$	7957	7596	4140	2081	0
$\beta = 0.99; \sigma = 0.50$	8440	5620	2949	1466	0
$\beta = 0.995; \sigma = 0.40$	7885	7491	4114	2071	0
$\beta = 0.995; \sigma = 0.45$	8654	5351	2804	1380	0
$\beta = 0.995; \sigma = 0.50$	8541	3615	1842	897	0
$\beta = 0.999; \sigma = 0.40$	9416	3274	1673	810	65
$\beta = 0.999; \sigma = 0.45$	8736	1950	962	474	45
$\beta = 0.999; \sigma = 0.50$	6995	1106	540	264	37

5. Conclusions

This study uses simulations to find differences in the optimal linear income tax where society focuses on either income inequality or utility inequality. We found lower tax rates and smaller transfers where we judge the tax system by income inequality rather than utility inequality. Larger differences occur where aversion to inequality was low. A society extremely averse to inequality, however, will be sensitive to the source of any difference in inequality and more significant remedial actions will be needed whether income inequality or utility inequality guides policy recommendations.

We also wished to know whether society faces substantial losses when policymakers focus on income inequality where society's welfare truly depends upon utility inequality. We found that where society has a low aversion to inequality, policy recommendations differ greatly, and society does suffer substantial social welfare losses due to this misapprehension of

[3] Although the social welfare loss typically decreases substantially as ε increases, the loss can briefly rise as ε just begins to increase from zero; see Table 4. This result comes about because we restricted the marginal tax rate to be nonnegative.

policymakers. If society has a substantial aversion to inequality, then policy differences fade, and the social welfare loss will be low.

Assume that the U.S. has a modest aversion to inequality, say somewhere between 0.4 and 0.75, then the social welfare loss can be substantial if policymakers focus on income inequality. Although the U.S. does not have a linear income tax, the tax rates and guarantees recommended by the income-inequality model appear much closer to current policies than those from the utility-inequality model. In this scenario policymakers may not be so far from achieving their goal, but, assuming that the Bergson-Samuelson social welfare function better reflects society's well-being, then social welfare would be greatly increased with substantially higher tax rates and guarantees.

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