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Substitutability in Human Capital Formation and Education Inequality

Lin Zhang
Kindai University

Abstract

This paper shows that substitutability in human capital formation could influence inequality in education. By incorporating a model of self-selection in education where individuals are heterogeneous in initial wealth and innate ability, we show that any increase in the substitutability of innate ability for initial wealth reduces educational inequality.

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1. Introduction

Human capital formation involves various input factors such as family background, socioeconomic status, monetary payment, and innate abilities, and they could substitute for each other (see, e.g., Gyimah-Brempong and Gyapong 1992 and Cunha et al. 2006). This study aims to show that substitutability in human capital formation could affect education inequality.

This study incorporates a simple model of self-selection in education with heterogeneous individuals. In line with Fershtman et al. (1996) and Charlot and Decreuse (2010), heterogeneity comes from two aspects of educational costs: monetary and effort. The effort cost inversely reflects an individual's innate ability. An individual's monetary cost is inversely related with her financial recourse or capacity to invest in education when the credit market is imperfect. Note that in this model we focus on the *long-run* credit constraints in education decision (see Cunha et al. 2006). A series of empirical studies show that long-run family resources play an important role in explaining educational choices, while the short-run borrowing constraints only a minor one (see Cameron and Heckman 1998, 2001, and Carneiro and Heckman 2002, 2003). When the long-run credit constraint is binding, the monetary cost inversely reflects one's family background, socioeconomic status, and other environmental factors. Without loss of generality, we do not distinguish these environmental factors, and hereafter they are referred to as initial wealth.

On the basis of individuals' endogenous education decision, we obtain a threshold condition under which individuals are indifferent between getting educated and remaining uneducated. The threshold line could be linear between initial wealth and innate ability, as it coincides with the case shown by Fershtman et al. (1996) and Charlot and Decreuse (2010). However, beyond that, we show that the threshold curve could also be concave or convex, as it depends on the substitutability parameter. The parameter is exogenously given in this model and reflects substitutability of innate ability for initial wealth. By plotting human capital Lorenz curves, we show that inequality in educational attainment diminishes with substitutability of innate ability for initial wealth. This study contributes to the literature on education inequality by providing an alternative dimension that accounts for it.

2. The Model

Consider an economy populated by a continuum of individuals who live for one period. The population is normalized to one. Each individual inelastically supplies a unit of labor and receives wages for working.

Individuals are heterogeneous in cost of education as it involves two aspects: monetary cost (x) and effort cost (e). On the one hand, the long-run credit constraint is assumed to be binding. In this context, monetary cost of education could capture an individual's initial wealth. Denote the initial wealth as i and assume that it is uniformly distributed over the interval $[0,1]$. Hence, we have $x = 1 - i$ ($x \in [0,1]$). An individual with higher wealth i (or lower monetary cost x) could afford higher expenditure on education. On the other hand, the effort cost of education may capture an individual's innate ability. Denote innate ability as a and assume that it is uniformly distributed over the interval$[0,1]$. We have $e = 1 - a$ ($e \in [0,1]$). Therefore, individuals differ in initial wealth and innate ability. Note that initial wealth emphasizes people's family environmental factors while innate ability the endowment factors, and both of them affect individuals' human capital (see, e.g., Björklund and Salvanes 2011). There is limited consensus on the correlation between the two factors (see, e.g., Black and Devereux 2010). However, since we focus on the substitutability in human capital formation and for the sake of simplicity, we assume that initial wealth (i) and innate ability (a) are independent and have joint density $f(i, a) = 1$ over the entire population[1].

The economy consists of two sectors: skilled and unskilled. Educated individuals work in the skilled sector, and those who are uneducated work in the unskilled sector. The wage rate for the skilled jobs is w (w>0), and the wage rate for the unskilled jobs is normalized to 0. Individuals choose whether to receive education by taking the wage rate as given.

If an individual decides to become an educated worker receiving wage rate w, she would pay monetary cost x and effort cost e. For simplicity, assume that utility of consumption is linear, and disutility of effort is e^{θ} ($\theta > 0$). Hence, the utility of an educated worker is given as

$$w - x - e^{\theta}. \tag{1}$$

If an individual chooses not to invest in education and becomes an unskilled worker, she would not pay any cost of the education, and the utility of an uneducated worker is 0.

3. Self-selection in Education and Substitutability

By comparing the utility of working in the skilled/unskilled sectors, individuals make

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[1] The independence assumption simplifies model and relaxing it would not change the conclusion.

education decisions. An individual with monetary cost x (initial wealth $i = 1 - x$) and effort cost e (innate ability $a = 1 - e$) chooses to get educated if

$$w - x - e^{\theta} > 0. \tag{2}$$

A threshold condition exists under which individuals are indifferent to getting educated or not:

$$x = w - e^{\theta}. \tag{3}$$

By substituting $x = 1 - i$ and $e = 1 - a$ into equation (3), the threshold condition reads as

$$1 - i = w - (1 - a)^{\theta}. \tag{4}$$

Equation (4) indicates substitution between innate ability and initial wealth in the self-selection of education. An individual with lower abilities but wealthy enough to afford to invest more in education becomes a skilled worker. A poorer but more able individual can also have access to education.

Without loss of generality, assume that $w = 1$. The threshold condition reduces to

$$i = (1 - a)^{\theta}. \tag{5}$$

Recall that $e = 1 - a$ and substitute it into equation (5). The threshold condition becomes to that

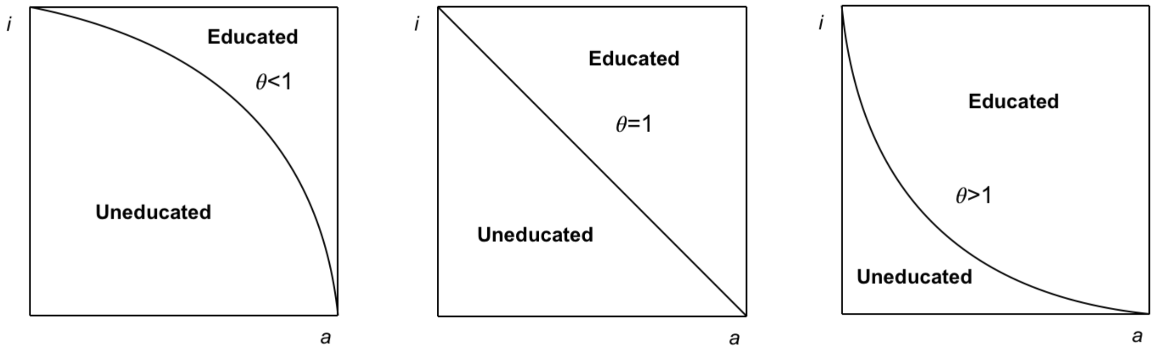
$$i = e^{\theta}. \tag{6}$$

Denote the effort cost elasticity of wealth as ρ, which is given by

$$\rho = \frac{di}{de} \cdot \frac{e}{i} = \theta e^{\theta - 1} \cdot \frac{e}{i} = \theta. \tag{7}$$

Equation (7) implies that under the threshold condition, for an individual whose effort cost e gets 1% higher (innate ability 1% lower), she has to be θ% wealthier to get educated. Thus, θ captures the substitutability of innate ability for initial wealth.

We plot a threshold curve from equation (5) on the (i, a) plane, as shown in Figure 1. In Figure 1, individuals above the threshold curve choose to invest in education, while the area below the threshold curve represents uneducated population.



(a) When $\theta < 1$ (b) When $\theta = 1$ (c) When $\theta > 1$

Figure 1. Threshold curve on the (i, a) plane

Note that the slope of the threshold curve depends on substitutability parameter θ. Denote the marginal substitution rate of ability for wealth on the threshold curve as MRS_{ai} and define $MRS_{ai} = -\frac{di}{da}$. We have

$$MRS_{ai} = -\frac{di}{da} = \theta(1-a)^{\theta-1}. \tag{8}$$

By taking the second derivative of i with a, we have

$$\frac{d^2 i}{da^2} = \theta(\theta-1)(1-a)^{\theta-2}. \tag{9}$$

When $\theta > 1$, $\frac{d^2 i}{da^2} > 0$. For a given level of θ, the marginal substitution rate of ability for wealth MRS_{ai} decreases with ability a. For individuals with relatively low ability, even slightly higher ability could make up the disadvantage in wealth a lot. However, MRS_{ai} becomes quite low when ability is relatively high. In other words, for individuals with relatively high ability, this substitution effect fades

When $\theta = 1$, $MRS_{ai} = 1$ and the threshold curve becomes linear. This is the case discussed by Fershtman et al. (1996) and Charlot and Decreuse (2010).

When $0 < \theta < 1$, $\frac{d^2 i}{da^2} < 0$. For a given level of θ, MRS_{ai} increases with ability a. For individuals with relatively low ability, the substitution effect by which higher ability could make up for lower wealth is quite limited. However, for individuals with relatively high ability,

MRS_{ai} becomes bigger.

4. Inequality in Education

In this section, we investigate the distribution of human capital in the entire population and how it is affected by the substitutability of innate ability for initial wealth. Following Mejia and St-Pierre (2008), we incorporate the human capital Lorenz curve to assess education inequality.

4.1 Lorenz Curve with Innate Ability

First, we consider the human capital Lorenz curve with innate ability. Heterogeneous individuals are lined up in order of their innate ability (a) from the lowest to the highest. The human capital Lorenz curve shows the proportion of human capital possessed by any given percentage of the population. The proportion of human capital could be expressed as the proportion of educated workers and calculated from Figure 1.

Note that in Figure 1, area above the threshold curve shows the population of educated workers and the continuum of individuals has joint density $f(i,a) = 1$ over the entire population. Therefore, for the individuals with innate ability lower than a given level of ability $\tilde{a}$ ($\tilde{a} \in [0,1]$), the population of educated workers among them Φ_a is given by

$$\Phi_a = \int_0^{\tilde{a}} [1 - (1 - a)^{\theta}] da = \frac{\tilde{a}(\theta+1) + (1-\tilde{a})^{\theta+1} - 1}{\theta+1}. \tag{10}$$

When $\tilde{a} = 1$, we obtain the total human capital in the entire population as

$$\Omega = \frac{\theta}{\theta+1}. \tag{11}$$

Therefore, the proportion of human capital obtained by the bottom population with innate ability $a \leq \tilde{a}$ is given by

$$\frac{\Phi_a}{\Omega} = \frac{\tilde{a}(\theta+1) + (1-\tilde{a})^{\theta+1} - 1}{\theta}. \tag{12}$$

Note that in equation (12), the slope of human capital Lorenz curve depends on θ. Numerical examples enable us to plot the Lorenz curves, which are shown in Figure 2.

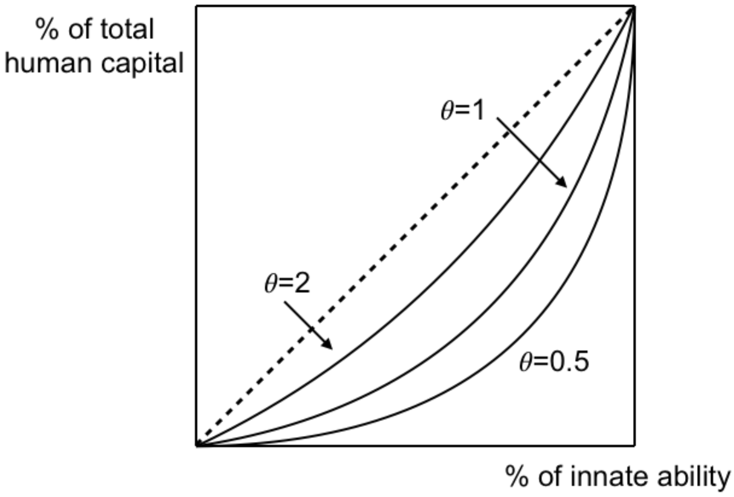


Figure 2. Human capital Lorenz curve with innate ability

As discussed earlier, this section shows how education inequality is influenced by the substitutability in human capital formation. However, to assess education inequality, we do not have to calculate the Gini coefficient, because the shape of human capital Lorenz curve provides a convenient and intuitive view. In a human capital Lorenz curve graph, the 45-degree line represents perfectly equal distribution of human capital, and a Lorenz curve less close to the line of perfect equality represents higher inequality in education. Numerical results in Figure 2 show that when θ increases, the human capital Lorenz curve moves closer to the line of perfect equality. Specifically, higher substitutability of innate ability for initial wealth raises equality in educational allocation.

Recall that the threshold condition for receiving education is $i = (1-a)^{\theta}$. It implies that for a given level of innate ability a, only individuals whose wealth is higher than $(1-a)^{\theta}$ could have access to education. Denote the wealth range which allows an individual to acquire education for a given level of ability a as π_a, and we have

$$\pi_a = 1 - (1-a)^{\theta}. \tag{13}$$

Differentiation result $\frac{d\pi_a}{d\theta} = -(1-a)^{\theta} \ln(1-a) > 0$ indicates that educated population increases with the θ. That is, for a given level of ability, it gets easier to be educated when θ increases. However, the increment in human capital is not equal for the entire population. By taking a partial derivative, we have

$$\frac{d^2\pi_a}{d\theta da} = (1-a)^{\theta}[\theta \ln(1-a) + 1]. \quad (14)$$

Equation (14) implies that when $a \lessgtr 1 - exp(-\frac{1}{\theta})$, $\frac{d^2\pi_a}{d\theta da} \gtrless 0$, where $exp\left(-\frac{1}{\theta}\right) \in (0,1)$. When ability is relatively low ($a < 1 - exp(-\frac{1}{\theta})$), the increment in human capital increases with ability. However, when ability is relatively high ($a > 1 - exp(-\frac{1}{\theta})$), the increment in human capital declines when ability increases. Particularly, increment reaches the top when $a = \hat{a} = 1 - \exp(-\frac{1}{\theta})$. Notice that this critical ability level $\hat{a}$ decreases with the substitutability parameter θ ($\frac{d\hat{a}}{d\theta} < 0$).

To summarize, if the substitutability of innate ability for initial wealth increases, the total educated population does increase. Meanwhile, the critical ability level $\hat{a}$ falls, that is, increased human capital is more intensively distributed over individuals with relatively low ability and this weakens education inequality.

4.2 Lorenz Curve with Initial Wealth

Similarly, we plot the human capital Lorenz curve with wealth. In this graph, the heterogeneous individuals are aligned in order of their wealth (i) from the lowest to the highest. For individuals with lower wealth than a given level of $\tilde{\imath}$ ($\tilde{\imath} \in [0,1]$), the population of educated workers among them is denoted as Φ_i.

$$\Phi_i = \int_0^{\tilde{\imath}} [i^{\frac{1}{\theta}}] di = \frac{\theta}{\theta+1} \tilde{\imath}^{\frac{1+\theta}{\theta}}. \quad (15)$$

The proportion of human capital obtained by the bottom population with wealth $i \leq \tilde{\imath}$ reads

$$\frac{\Phi_i}{\Omega} = i^{\frac{1+\theta}{\theta}}. \quad (16)$$

Figure 3 shows the human capital Lorenz curve from the numerical examples of equation (14).

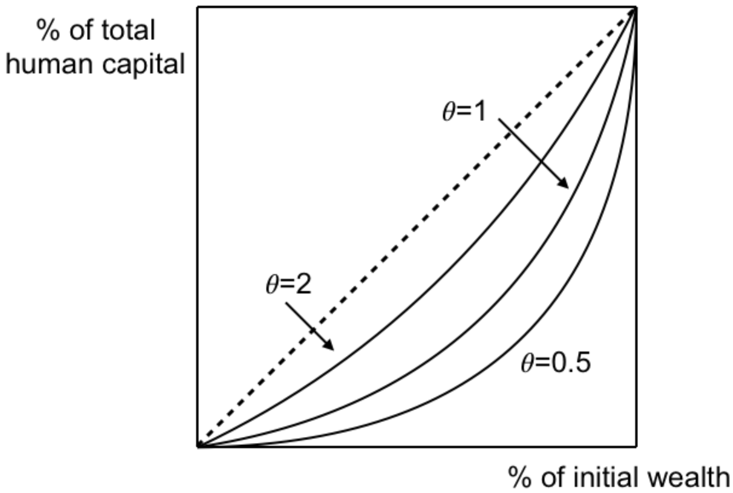


Figure 3. Human capital Lorenz curve with initial wealth

The numerical results in Figure 3 also shows that education inequality declines with the substitutability of innate ability for initial wealth. That is, when the substitutability parameter θ increases, a similar mechanism applies here.

5. Conclusions

By incorporating a simple partial equilibrium model of self-selection in education, we show how education inequality is influenced by the substitutability of innate ability for initial wealth, which is assumed to be exogenously given. Substitutability displays how different factors jointly contribute in the educational decisions and it could be determined by the labor market. Research on the general equilibrium model with endogenous substitutability and modifying other assumptions are left for the future work.

References

Björklund, A. and K. G. Salvanes (2011) "Education and Family Background: Mechanisms and Policies" *Handbook of the Economics of Education* **3**, 201-247.

Black, S. E. and P. J. Devereux (2010) "Recent Developments in Intergenerational Mobility" *Handbook of Labor Economics* **4(B)**, 1487-1541.

Cameron, S. V. and J. J. Heckman (1998) "Life Cycle Schooling and Dynamic Selection Bias: Models and Evidence for Five Cohorts of American Males" *Journal of Political economy*, **106(2)**, 262-333.

Cameron, S. V. and J. J. Heckman (2001) "The Dynamics of Educational Attainment for Black, Hispanic, and White Males" *Journal of political Economy* **109(3)**, 455-499.

Carneiro, P. and J. J. Heckman (2002) "The Evidence on Credit Constraints in Post-Secondary Schooling" *The Economic Journal* **112(482)**, 705-734.

Carneiro, P. M. and J. J. Heckman (2003) "Human Capital Policy" in *Inequality in America: What Role for Human Capital Policies?* by J.J. Heckman, A.B. Krueger, and B.M. Friedman, Eds., MIT Press: Cambridge.

Charlot, O. and B. Decreuse (2010) "Over-Education for the Rich, Under-Education for the Poor: A Search-Theoretic Microfoundation" *Labour Economics* **17(6)**, 886-896.

Cunha, F., J. J. Heckman, L. Lochner, and D. V. Masterov (2006) "Interpreting the Evidence on Life Cycle Skill Formation" *Handbook of the Economics of Education* **1**, 697-812.

Fershtman, C., K. M. Murphy, and K. M. Weiss (1996) "Social Status, Education, and Growth" *Journal of Political Economy* **104(1)**, 108-132.

Gyimah-Brempong, K. and A. O. Gyapong (1992) "Elasticities of Factor Substitution in the Production of Education" *Economics of Education Review* **11(3)**, 205-217.

Mejía, D. and M. St-Pierre (2008) "Unequal Opportunities and Human Capital Formation" *Journal of Development Economics* **86(2)**, 395-413.