Economics Bulletin

Volume 39, Issue 1

On the consistency of central banks´ interest rate forecasts

Jin-Kyu Jung
WHU - Otto Beisheim School of Management

Michael Frenkel
WHU - Otto Beisheim School of Management

Jan-Christoph Rülke
EBS Universitaet fuer Wirtschaft und Recht

Abstract

In this paper, we derive internal consistency restrictions on short-term and long-term interest rate forecasts as published by the central banks of the Czech Republic, New Zealand, Norway and Sweden. We find different degrees of forecast consistency across these countries and also document that consistency is more apparent among short-term forecasts compared to long-term forecasts. Our results are robust when taking a more complex lag structure and more consistency restrictions into account. These results offer interesting policy implications as central banks´ interest rate forecasts can be regarded as an important instrument of central bank communication.

We gratefully acknowledge the suggestions of the editor and anonymous referees on an earlier draft of this paper.
Citation: Jin-Kyu Jung and Michael Frenkel and Jan-Christoph Rülke, (2019) "On the consistency of central banks´ interest rate forecasts", *Economics Bulletin*, Volume 39, Issue 1, pages 701-716
Contact: Jin-Kyu Jung - jin-kyu.jung@whu.edu, Michael Frenkel - michael.frenkel@whu.edu, Jan-Christoph Rülke - jan-christoph.ruelke@ebs.edu.
Submitted: October 19, 2018. **Published:** March 28, 2019.

1 Introduction

The rational expectations hypothesis is a key assumption for most financial market models and equally serves as the basis of a large number of economic models. Assuming rational expectations of economic agents facilitates modeling their behavior and provides a certain ease of understanding of preferences and biases. However, many studies have questioned the validity of this hypothesis as empirical evidence suggests that most market participants and economic agents form expectations differently (Davidson 1982; Dominguez 1986; Frankel and Froot 1987a,b; Cavaglia et al. 1993, 1994; Elliott and Ito 1999; Menkhoff 1998, 2001). For example, only looking at the sheer daily turnover volume of the global OTC interest derivatives market of US $2.7 trillion or an average of US $5.1 trillion changing hands every day in the foreign exchange market (Bank for International Settlements, 2016) suggests that traders actually have very different expectations about the future development of interest rates and exchange rates.

Building on this, central banks as a key player in financial markets are also commonly assumed to be subject to the rational expectations hypothesis, but in reality they might be behaving quite differently as well. In this context, some studies have looked into central bank forecasts as these can be assumed to be a reflection of their expectations. Rosa and Verga (2007) examine consistency and effectiveness of forecasts by the European Central Bank (ECB) and find that the content communicated in the ECB's monthly press conferences enables a fairly reliable prediction of monetary policy. Rülke (2012) finds that growth and inflation forecasts made by 15 major central banks are largely unbiased and rational, however discrepancies emerge across different forecast variables with inflation forecasts showing more bias than growth forecasts. Frenkel et al. (2017) examine central bank interest rate forecast errors and find that central banks' forecast rationality can be partially restored by assuming an asymmetric loss function.

Central banks' own interest rate forecasts are especially interesting for the following reasons: (1) In theory, the forecast error could be zero as central bankers are forecasting the interest rate which they control (by virtue of their interest rate policy). A forecast error different from zero indicates that central bankers considered deviating from their forecast to be optimal. What these reasons might be is certainly a relevant question that future research should take on. (2) Interest rate forecasts predict the setting of one of the main instruments of monetary policy. A more transparent version of central bank communication is therefore hardly conceivable, thereby highlighting the significance of such forecasts. Blinder et al. (2008) even refer to central bank interest rate forecasts as "the new frontier" in central bank communication and transparency. In fact, central banks have gone through a remarkable transition from a secrecy-dominated communication stance advocating the release of as little information as possible to adopting open and active communication policies (Geraats 2002; Crowe and Meade 2008).

In this paper, we contribute to the exploration of this fairly new instrument of central bank communication by studying the internal consistency of short-term and long-term central bank interest rate forecasts. For this purpose, we first estimate a model governing central banks' forecasting behavior and subsequently derive internal consistency constraints conditional on the previously estimated model. While most forecast rationality tests require forecasts to follow a certain stochastic process that generates the actual value of the projected variable, testing for internal consistency of forecasts at different forecasting horizons is not subject to this condition. Put differently, while forecast rationality requires internal consistency, internal consistency in itself does not necessarily establish rationality of forecasts. Internal forecast consistency can therefore be considered a necessary, but not a sufficient condition for forecast rationality.

The remainder of this paper is structured as follows. In Section 2, we describe our central bank forecast data. In Section 3, we test for internal consistency of forecasts by assuming a distributed-lag model featuring one lag to capture how central banks form their interest rate forecasts. In Section 4, we extend the distributed lag model to include a more complex lag structure as a robustness test. In Section 5, we offer some conclusions.

2 Central Bank Interest Rate Forecast Data

We study interest rate forecasts by all central banks which publish their interest rate projections for a considerable long period of time.[1] In total, we use more than 1,100 interest rate forecasts as published by the central banks of the Czech Republic, New Zealand, Norway, and Sweden. Having started the publication of interest rate forecasts in different years, the observation periods for each central bank vary in both length and forecast horizons. Table 1 summarizes the observational details for each of the four central banks.

Table 1: Details on observational periods, frequencies and forecast horizons

Country	**Period**	**Frequency**	**Forecast horizon**	**Observations**
Czech Republic	2008Q1–2018Q4	Quarterly	3 − 18 months	44
New Zealand	1997Q4–2018Q4	Quarterly	3 − 24 months	85
Norway	2005Q1–2018Q4	Triannually	3 − 24 months	46
Sweden	2007Q1–2018Q4	Quarterly	3 − 24 months	50

Source: Česká Národní Banka, Norges Bank, Reserve Bank of New Zealand, Sveriges Riksbank. The available forecast horizons for New Zealand, Norway and Sweden differ across time and only forecast data for up to 24 months has been used in order to guarantee a consistent dataset. All central banks have implemented a managed floating exchange rate regime.

The central bank of the Czech Republic, Česká Národní Banka (CNB), is the last of the four central banks under consideration in this study to have started the publication of interest rate forecasts. The forecasted values have been published since 2008 and pertain to the 3-months Prague Interbank Offered Rate. The CNB thereby forecasts an interest rate which is not the key interest rate set directly by the policymakers themselves but rather a market reference rate.

The largest set of observations is offered by the Reserve Bank of New Zealand (RBNZ) which started to publish forecasts of the 90-day bank bill rates in 1997 on a quarterly

[1]Another candidate for this exercise would be Federal Reserve which has started publishing their interest rate projections some five years ago. However, the time period its too short and does not allow to derive robust results.

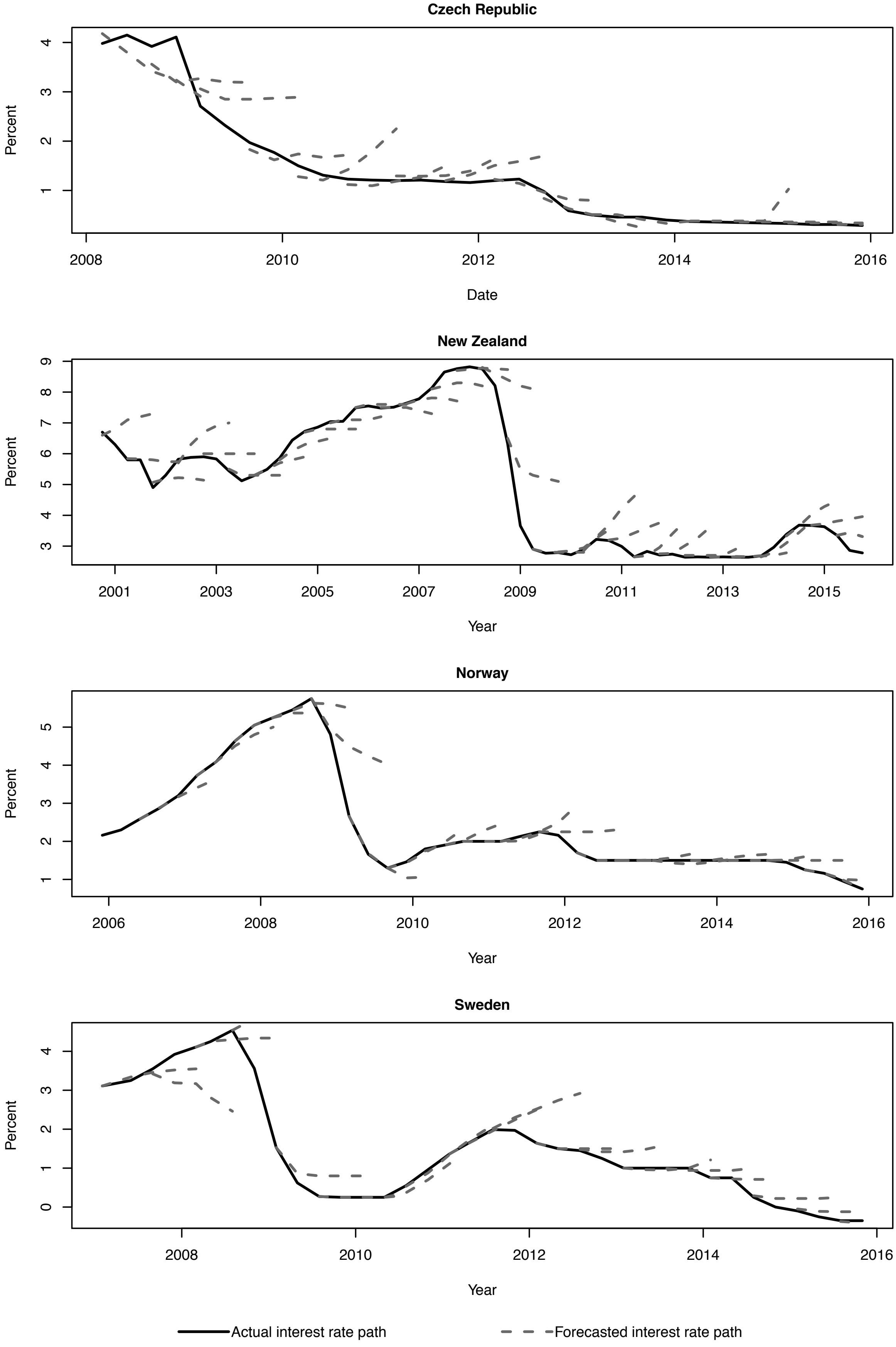
basis. The 90-day bank bill rate is an interbank interest rate and therefore not directly set by the RBNZ but rather to be considered a market interest rate. Nonetheless it shows a strong correlation with the Official Cash Rate (OCR) which is the key interest rate in New Zealand. Starting from 2016Q4 however the RBNZ discontinued the publication of forecasts of the 90-day bank bill rate and instead replaced them with forecasts of the OCR (Reserve Bank of New Zealand, 2016). The forecasts are smoothed quarterly averages and will continue to be published as part of the quarterly Monetary Policy Statements.

Norges Bank in Norway has started publishing forecasts of their key policy rate three years prior to the CNB and for forecast horizons of up to 36 months. The frequency of those forecast publications varies over time and amounted to three times per year until the end of 2012. After 2012 the Monetary Policy Reports containing these forecasts appeared on a quarterly basis and therefore increased the frequency thereof.

Sweden's Riksbank publishes forecasts of the repo rate and thereby also projects an interest rate set by policymakers rather than a market reference rate. The Riksbank has been publishing these forecasts since the beginning of 2007 as part of their Monetary Policy Reports on a quarterly basis and also includes confidence bands around the point forecast of the repo rate in each edition.

Due to differences in the forecast horizons of interest rate forecasts by New Zealand, Norway and Sweden we have decided to align the datasets and to consistently only consider forecasts up to 24 months in the future. Figure 1 shows the graphical representation of the projected interest rate paths of all four central banks and depicts the different periods covered by each. The respective interest rate forecast paths show no visually evident pattern on first sight, however the Reserve Bank of New Zealand offers the largest dataset due to the early publication of interest rate forecasts and shows both a period of high (during the run-up to the global financial crisis) and low (after the outbreak of the global financial crisis) interest rate environments. In fact, on closer examination, the actual interest rate paths of all four central banks consistently exhibit a steep decline in interest rates in 2008/2009 reflecting the global response of central banks to the unfolding financial crisis. While the first years of the new millennium after the burst of the dot-com bubble were characterized by strong economic growth driven by an extraordinary accumulation of debt particularly in the United States, the collapse of Lehman Brothers in September 2008 and the breakdown of the sub-prime mortgage sector triggered a recession on a global scale (Sanders, 2008). The virtually unanimous response of central banks across the globe was to establish exceptionally low interest rate environments and in some countries even introduce unconventional monetary policy measures. These measures and the generally heightened level of uncertainty amidst the unfolding crisis had significant ramifications on international bond markets and therefore on a broad range of interest rates beyond central bank key interest rates (Moro 2014; Gruppe et al. 2017). It is remarkable to observe the consistently downward pointing forecast paths especially between 2003 and 2008 and an indication of higher future interest rates after 2008 from figure 1. In this context, Alessi et al. (2014) find that overall forecast performances by central banks have been noticeably worse during the global financial crisis than before and OECD (2014) report similar findings for the performance of forecasts by the OECD. Kunze et al. (2015) and Kunze et al. (2017) specifically examine interest rate forecasts during the financial crisis and find evidence for increased forecast errors.

Figure 1: Actual and projected interest rate paths



Note: Figure 1 plots the actual interest rate (solid lines) and the projected interest rate (dotted lines). Data are taken from the websites of the Česká Národní Banka, the Reserve Bank of New Zealand, the Norges Bank, and Sweden's Riksbank.

3 Forecast Consistency in the Baseline Model with a Simple Lag Structure

To analyze the characteristics of how central banks forecast interest rates, we use the concept of consistency of short-term and long-term interest rate projections. While the former are a useful yardstick for future monetary policy, the latter might be important for guiding inflation expectations. To this end, we analyze whether these forecasts are consistent by applying a simple lag structure. Thereby, we test whether the short-term and long-term objective of four inflation-targeting central banks are consistent.

Following the approach developed by Froot and Ito (1989), we formulate a condition where consistency is present when interest rate forecasts made at the same time t for subsequent shorter forecast horizons result in the same value as a forecast for the entire forecast horizon. Put differently, if a forecast with a shorter forecast horizon is denoted by $E_t(r_{t+k})$ where k denotes the forecast horizon, this forecast iterated forward *until* period j must yield the same value as $E_t(r_{t+k+j})$, i.e. a forecast made *in* t for period $k+j$. This definition of internal forecast consistency allows us to formulate empirically testable hypotheses in terms of cross-equation constraints on coefficients of short-, medium-, and long-term forecasting equations. We follow the approach set out by Frankel and Froot (1987a,b) and assume that central bank forecasters derive their interest rate forecasts through an extrapolative forecasting equation. The academic research literature commonly refers to a Taylor or Taylor-type rule in such a context, and we indeed find the elements of a Taylor rule in the specification by Frankel and Froot (1987b) but are additionally able to obtain an extrapolative model suitable for the subsequent derivation of consistency restrictions[2]. The simplest form of such an extrapolative model can be expressed as a distributed-lag model as follows:

$$E_t[r_{t+k}] - r_t = \alpha_k + \beta_k(r_{t-1} - r_t) + \epsilon_t, \tag{1}$$

where r_t denotes the interest rate and $E_t[r_{t+k}]$ the interest rate forecast for period $t+k$ made in t, respectively. The actual change in the interest rate is given as $r_{t-1} - r_t$ rather than as $r_t - r_{t-1}$ in order to derive the consistency restrictions in an intuitively interpretable way. In this setting, a negative β_k indicates that in case of increasing (decreasing) interest rates, central bank forecasters would expect an even further increase (decrease) in the next period. This behavior could then be referred to as destabilizing and is also known as *bandwagon expectations*. In contrast, *contrarian expectations* prevail if β_k assumes a positive sign. In this case, forecasters predict an increase (decrease) in interest rates whenever the actual interest rate decreased (increased) right before a new forecasting cycle, thereby acting in a stabilizing manner.

By applying Equation (1) to both the short- and long-term forecasts across one and two periods, we can obtain

$$E_{t;i}[r_{t+1}] - r_t = \alpha_1 + \beta_1(r_{t-1} - r_t) + \epsilon_t, \tag{2}$$

[2]The advantage of the approach proposed by Froot and Ito (1989) is the flexibility as to which process of forecast formation is assumed. The imposed consistency restrictions can be tested on virtually any assumed forecast-generating process.

$$E_{t;i}[r_{t+2}] - r_t = \alpha_2 + \beta_2(r_{t-1} - r_t) + u_t. \tag{3}$$

Internal consistency of forecasts at different forecasting horizons as defined by Froot and Ito (1989) requires that by iterating short-term forecasts as given in Equation (2), we can achieve the same forecast at a longer forecast horizon as a long-term forecast as specified in Equation (3). As shown in Appendix A, by assuming this condition we can derive two internal consistency restrictions that internally consistent forecasts must be able to satisfy:

$(R1)$: $\alpha_2 = 2\alpha_1 - \beta_1\alpha_1,$

$(R2)$: $\beta_2 = \beta_1(1 - \beta_1).$

It is easy to see how restrictions $R1$ and $R2$ represent the consistency condition verbally laid out in the previous paragraphs. For instance, $R1$ implies that in case a forecaster expects the interest rate to increase by α_1 in the short term, he needs to forecast a total increase of $2\alpha_1$ less the intermediate change in interest rates $\beta_1\alpha_1$ over the long term in order for long- and short-term forecasts to be consistent (Pierdzioch and Rülke, 2014). Similarly, restriction $R2$ can be intuitively interpreted as a condition leaving forecasters who adjust their forecasts by β_1 in the short-term only with $1 - \beta_1$ for the subsequent period in order to be consistent over the entire time period. Consider an example with two short-term forecasts with a horizon of 3 months and a long-term forecast with a 6 month horizon. Additionally, the short-term β_1 is assumed to be 0.6 which according to $R2$ requires the long-term β_2 to be 0.24 in order for consistency to hold. In this case, if the interest rate were to increase by 10%, the forecaster would expect interest rates to decrease by 6% over the next 3 months whereas a 3.6% increase is expected for the subsequent 3 months. In sum, the forecaster projects interest rates to decrease by a total of 2.4% over the next 6 months which is why β_2 needs to equal 0.24 (Frenkel et al., 2011).

As a result, restriction $R2$ additionally implies that for $-1 \leq \beta_1 \wedge \beta_2 \leq 1$, the β-coefficients must have the same sign across different forecast horizons to ensure internal consistency. In other words, a forecaster who shows *bandwagon* (*contrarian*) behavior for short-term forecasts must continue to also make *bandwagon* (*contrarian*) forecasts in the longer-term in order to comply with consistency restriction $R2$. While it is possible to compare not only short- and long-term forecasts but for instance incorporate a "medium-term" forecast horizon in between (e.g. 3, 6, and 9 months), in the interest of brevity and lucidity we have limited our analysis to the comparison of two forecast horizons.

Estimating Equations (2) and (3) for short- and long-term interest rate forecasts yields the results summarized in Table 2 across different pairs of short- and long-term forecast horizons. For the central banks of the Czech Republic and Sweden most of the short- and long-term β-coefficients are not significantly different from zero. The central banks of the Czech Republic, New Zealand, and Norway show bandwagon forecasting behavior as indicated by the negative β-coefficients. However, this does not directly reflect consistent or inconsistent forecasts. For instance, for New Zealand β_1 for 3 (6) months forecasts is -0.2482 (-0.3872) and, hence, the theoretical β_2 according to R_2 ($\beta_2 = \beta_1(1 - \beta_1)$) should be -0.3098 (-0.5371) which is close to the estimated coefficient for 6 (12) months of -0.3872 (-0.5075) indicating that New Zealand's interest rate forecasts can be considered consistent. To empirically analyze whether the central bank forecasts are internally consistent, we apply all consistency restrictions to the results in Table 2.

Table 2: Expectation formation processes for central bank interest rate forecasts with one lag

Country	Forecast Horizon	**3 Months**	**6 Months**	**9 Months**	**12 Months**	**18 Months**	**24 Months**
Czech Republic	α	−0.0619	−0.0041	0.1206	0.3793***	1.1708***	no observations
		(0.0639)	(0.0687)	(0.0889)	(0.1286)	(0.2037)	
	β	−0.7586**	−0.7703**	−0.7275	−0.8304	−1.7649	
		(0.3423)	(0.3678)	(0.4759)	(0.6887)	(1.0905)	
	F	F(1,41) = 4.91	F(1,41) = 4.39	F(1,41) = 2.34	F(1,41) = 1.45	F(1,41) = 2.62	
	$Prob > F$	0.1070	0.0425	0.1340	0.2348	0.1132	
	No. of obs.	43	43	43	43	43	
New Zealand	α	0.0057	0.0242***	0.0441***	0.0658***	0.1105***	0.1407***
		(0.0056)	(0.0091)	(0.0139)	(0.0185)	(0.0281)	(0.0364)
	β	−0.2482***	−0.3872***	−0.4754***	−0.5075***	−0.4238*	−0.1657
		(0.0499)	(0.0812)	(0.1229)	(0.1654)	(0.2437)	(0.3110)
	F	F(1,73) = 24.72	F(1,73) = 22.74	F(1,72) = 14.96	F(1,73) = 9.42	F(1,69) = 3.02	F(1,65) = 0.28
	$Prob > F$	0.0000	0.0000	0.0002	0.0030	0.0865	0.5960
	No. of obs.	75	75	74	75	71	67
Norway	α	0.0027	0.0156	0.0574	0.1142**	0.2525***	0.4177***
		(0.0096)	(0.0262)	(0.0375)	(0.0488)	(0.0683)	(0.0885)
	β	−0.2026***	−0.3692***	−0.5504***	−0.6182***	−0.6058**	−0.5101
		(0.0350)	(0.0959)	(0.1373)	(0.1786)	(0.2500)	(0.3242)
	F	F(1,43) = 33.52	F(1,43) = 14.83	F(1,43) = 16.08	F(1,43) = 11.98	F(1,43) = 5.87	F(1,43) = 2.48
	$Prob > F$	0.0000	0.0004	0.0002	0.0012	0.0197	0.1230
	No. of obs.	45	45	45	45	45	45
Sweden	α	0.0762*	0.1514**	0.2428**	0.3488**	0.6123*	0.7975
		(0.0438)	(0.0720)	(0.1179)	(0.1735)	(0.3132)	(0.4839)
	β	0.0979	0.0105	−0.0086	0.0430	0.5685	1.7117**
		(0.0740)	(0.1218)	(0.1994)	(0.2933)	(0.5297)	(0.8274)
	F	F(1,44) = 1.75	F(1,44) = 0.01	F(1,44) = 0.00	F(1,44) = 0.00	F(1,44) = 1.15	F(1,44) = 4.28
	$Prob > F$	0.1926	0.9316	0.9659	0.8842	0.2890	0.0445
	No. of obs.	46	46	46	46	46	46

Notes: *** (**) and * indicate significance at the 1 (5) and 10 percent level, respectively.

Technically, the χ^2-statistic tests for all moment conditions whether the differences within the consistency restrictions $R1$ and $R2$ are normally distributed. Economically speaking, we test whether the results of the short-term and long-term expectation formation processes fit the consistency restrictions under a normal distribution. With regard to the internal consistency restrictions, Table 3 reports the results testing the internal consistency restrictions and shows that $R1$ ($R2$) can be rejected at the one percent level in 4 (5) out of 15 cases.

While forecast consistency can be rejected in most cases for the Czech Republic, New Zealand and Norway, for Sweden the consistency restrictions $R1$ and $R2$ together can only be rejected in one out of four cases indicating that for the Swedish Riksbank, short-term and long-term interest rate forecasts appear to be internally consistent. Interestingly, Sweden is commonly recognized to be one of the most transparent central banks (Dincer and Eichengreen, 2009) which might be a factor helping to understand the high degree of consistency in its central bank's forecasts.

Looking at the individual consistency restrictions, consistency restriction $R1$ ($R2$) can be rejected at the 10 percent level in 5 (7) out of 15 cases. Looking at the different forecast horizons, it seems that internal inconsistency of longer-term forecasts is higher. For instance, while only two out of eight individual restrictions involving 3 months forecasts are inconsistent, for 24 months forecasts four out of six consistency restrictions are violated. One possible explanation might be that longer-term forecasts are associated with a higher degree of uncertainty and hence, central banks are held less accountable for it. Another possible explanation for internal inconsistency could be events such as the global financial crisis that might have led to expectations about structural breaks, especially affecting the consistency of longer-term forecasts.

Table 3: Test of consistency restrictions ($R1$) and ($R2$)

		3 vs. 6 mth.	**6 vs. 12 mth.**	**9 vs. 18 mth.**	**12 vs. 24 mth.**
Czech Republic	χ^2	0.00	0.01	1.29	no observations
	$Prob > \chi^2$	(0.9609)	(0.9229)	(0.2625)	
	R1	1.77	5.22**	14.65***	
	$Prob > \chi^2$	(0.1833)	(0.0224)	(0.0001)	
	R2	0.87	0.39	0.24	
	$Prob > \chi^2$	(0.3510)	(0.5335)	(0.6238)	
	R1 & R2	3.70	8.56**	24.60***	
	$Prob > \chi^2$	(0.1569)	(0.0138)	(0.0000)	
New Zealand	χ^2	10.66***	1.45	0.15	2.64
	$Prob > \chi^2$	(0.0011)	(0.2321)	(0.7017)	(0.1092)
	R1	3.68**	0.66	0.00	0.00
	$Prob > \chi^2$	(0.0551)	(0.4163)	(0.9649)	(0.9511)
	R2	4.77**	0.16	9.50***	13.08***
	$Prob > \chi^2$	(0.0290)	(0.6882)	(0.0021)	(0.0003)
	R1 & R2	8.01	2.05	11.64***	15.38***
	$Prob > \chi^2$	(0.0182)	(0.3588)	(0.0030)	(0.0005)

Table 3: Test of consistency restrictions ($R1$) and ($R2$)

		3 vs. 6 mth.	**6 vs. 12 mth.**	**9 vs. 18 mth.**	**12 vs. 24 mth.**
Norway	χ^2	5.42**	6.39**	0.19	0.43
	$Prob > \chi^2$	(0.0199)	(0.0152)	(0.6685)	(0.5167)
	R1	0.38	9.01***	7.31***	4.24
	$Prob > \chi^2$	(0.5378)	(0.0027)	(0.0068)	(0.0394)
	R2	3.96**	2.61	6.84***	12.94***
	$Prob > \chi^2$	(0.0466)	(0.1064)	(0.0089)	(0.0003)
	R1 & R2	4.05	12.75***	29.46***	35.70
	$Prob > \chi^2$	(0.1319)	(0.0017)	(0.0000)	(0.0000)
Sweden	χ^2	1.69	0.03	2.48	7.34***
	$Prob > \chi^2$	(0.1933)	(0.8654)	(0.1224)	(0.0096)
	R1	0.03	0.40	0.64	0.14
	$Prob > \chi^2$	(0.0870)	(0.5294)	(0.4228)	(0.7046)
	R2	1.12	0.03	2.63	7.31***
	$Prob > \chi^2$	(0.2907)	(0.8626)	(0.1048)	(0.0069)
	R1 & R2	1.17	0.41	3.05	7.32**
	$Prob > \chi^2$	(0.5565)	(0.8143)	(0.2178)	(0.0257)

Note: Table 3 reports the test results for the internal consistency restrictions laid out in Section 3. The table summarizes F-values. The first row for each country reports the results of a test whether the β-coefficients for the short- and long-term as reported in table 2 are statistically significantly different from each other. The null hypothesis for χ^2 is that the estimates for the short- and long-term forecasts from table 2 are not statistically significantly different. The null hypothesis for the test of restrictions $R1$, $R2$ and $R1$ & $R2$ is that the internal consistency restrictions hold. *** (**) and * indicate statistical significance at the 1 (5) and 10 percent level.

4 Forecast Consistency in an Extended Model with a Distributed Lag Structure

To further analyze the robustness of our results we extend our initial analysis to a distributed lag model incorporating two lags to represent the forecast formation process of central banks. Such a specification allows for a so-called *twist* in the forecasts without compromising internal forecast consistency. More specifically, we now allow not only the change in the interest rate between $t-1$ and t but also the change between $t-2$ and $t-1$ to affect forecasts. Accordingly, we now consider the following short- and long-term forecast formation processes:

$$E_t[r_{t+1}] - r_t = \alpha_1 + \beta_1 r_t + \gamma_1 r_{t-1} + \delta_1 r_{t-2} + \phi_t \tag{4}$$

$$E_t[r_{t+2}] - r_t = \alpha_2 + \beta_2 r_t + \gamma_2 r_{t-1} + \delta_2 r_{t_2} + \chi_t \tag{5}$$

Appendix B shows a quick derivation of the resulting consistency restrictions which are summarized as follows:

$$
\begin{aligned}
&(R3): && \alpha_2 = 2\alpha_1 + \alpha_1\beta_1 \\
&(R4): && \beta_2 = 2\beta_1 + \beta_1^2 + \gamma_1 \\
&(R5): && \gamma_2 = \beta_1\gamma_1 + \gamma_1 + \delta_1 \\
&(R6): && \delta_2 = \delta_1 + \beta_1\delta_1
\end{aligned}
$$

As expressed by consistency restriction $R5$, internal forecast consistency in a distributed lag model with two lags may prevail even for short-term *bandwagon* and long-term *contrarian* forecasts, or vice versa. In other words, in this model different signs for the short- and long-term β-coefficients may still result in internal consistency, thereby allowing for the before-mentioned "twist" in the forecast formation process. Decreasing interest rates in the short-term in response to previously increasing interest rates may thus be followed by again increasing interest rates in the long-term (and vice versa) all by preserving forecast consistency.

Table 4 reports the estimation results for the above specified extended model with two lags applying the more relaxed consistency condition allowing for *twists*. The results largely corroborate our previous results. Again, internal consistency of interest rate forecasts is more pronounced for Sweden while for New Zealand and Norway most consistency restrictions are violated (see $R3$ to $R6$ in table 5). It is interesting that the Czech Republic's forecasts appear more consistent under the extended model with two-lags as compared to the baseline model. The more relaxed consistency restriction that now allows for a twist in forecasting behavior apparently helps to generate more forecast consistency of the Czech central bank. Overall, once again long-term forecasts seem to be more inconsistent compared to short-term forecasts which also confirms our observations made in section 3.

5 Conclusion

In this paper, we analyze more than 1,100 short-term and long-term interest rate forecasts to study whether the forecasts of the central bank of the Czech Republic, Norway, New Zealand, and Sweden are internally consistent. To this end, we derived internal consistency restrictions which are less restrictive compared to the rational expectations hypothesis which is usually applied to macroeconomic forecasts. The results suggest for the Czech Republic and Norway that interest rate forecasts are mostly inconsistent while consistency prevails for Sweden and New Zealand. We also document that consistency is more apparent among short-term forecasts compared to long-term forecasts which might be related to a higher long-term uncertainty, especially during the financial crisis. Moreover, we find that our results are robust when taking a more complex lag structure and more consistency restrictions into account. These results offer a number of policy implications.

Table 4: Expectation formation processes for central bank interest rate forecasts with two lags

	Parameter	3 Months	6 Months	9 Months	12 Months	18 Months	24 Months
Czech Republic	α_k	−0.1004	−0.0685	−0.0390	0.0920	0.4362***	no observations
		(0.696)	(0.0539)	(0.0751)	(0.0867)	(0.0807)	
	β_k	0.4936	0.4817	0.6802	0.5357	0.3645	
		(0.4592)	(0.3555)	(0.4909)	(0.5670)	(0.5277)	
	γ_k	−0.3883	−0.4167	−0.8109	−0.7950	−0.7435	
		(0.8069)	(0.6247)	(0.8662)	(1.0005)	(0.9312)	
	δ_k	−0.1552	−0.1824	−0.0144	−0.0089	−0.1376	
		(0.4582)	(0.3547)	(0.4940)	(0.5706)	(0.5311)	
	F	F(3,37) = 1.27	F(3,37) = 3.58	F(3,38) = 2.62	F(3,38) = 3.60	F(3,38) = 12.10	
	$Prob > F$	0.2986	0.0228	0.0648	0.0221	0.0000	
	No. of obs.	41	41	42	42	42	
New Zealand	α_k	−0.0314	−0.0029	0.0476	0.1012**	0.2188***	0.3073***
		(0.0174)	(0.0273)	(0.0391)	(0.0498)	(0.0668)	(0.0797)
	β_k	0.3182***	0.5052***	0.6218***	0.6439***	0.5144*	0.3044
		(0.0721)	(0.1133)	(0.1623)	(0.2063)	(0.2770)	(0.3475)
	γ_k	−0.2950**	−0.4987***	−0.6703**	−0.6864*	−0.6031	−0.6153
		(0.1264)	(0.1986)	(0.2844)	(0.3617)	(0.4876)	(0.6189)
	δ_k	0.0019	0.0103	0.0407	0.0101	−0.0027	0.1691
		(0.0729)	(0.1459)	(0.1641)	(0.2087)	(0.2823)	(0.3554)
	F	F(3,70) = 10.71	F(3,70) = 9.68	F(3,69) = 6.83	F(3,70) = 5.20	F(3,66) = 3.34	F(3,62) = 3.01
	$Prob > F$	0.0000	0.0000	0.0004	0.0027	0.0245	0.0368
	No. of obs.	74	74	73	74	70	66

Parameter		3 Months	6 Months	9 Months	12 Months	18 Months	24 Months
Norway	α_k	−0.0049	−0.0202	−0.0056	0.0291	0.1312**	0.2511
		(0.0115)	(0.0315)	(0.0418)	(0.0513)	(0.0641)	(0.0743)
	β_k	0.3138***	0.5625***	0.9243***	1.0591***	0.9750***	0.8077**
		(0.0555)	(0.1523)	(0.2022)	(0.2485)	(0.3103)	(0.3597)
	γ_k	−0.3902***	−0.5092***	−0.9183***	−1.1267***	−1.1427**	−1.0553*
		(0.0916)	(0.2515)	(0.3338)	(0.4102)	(0.5122)	(0.5938)
	δ_k	0.0719	−0.0313	0.0264	0.1006	0.1883	0.2390
		(0.0556)	(0.1527)	(0.2027)	(0.2491)	(0.3110)	(0.3605)
	F	F(3,40) = 11.92	F(3,40) = 5.96	F(3,40) = 8.46	F(3,40) = 6.99	F(3,40) = 3.58	F(3,40) = 1.83
	$Prob > F$	0.0000	0.0019	0.0002	0.0007	0.0219	0.1576
	No. of obs.	44	44	44	44	44	44
Sweden	α_k	0.0891**	0.1937***	0.3011***	0.4234***	0.7344***	0.9648***
		(0.0409)	(0.0513)	(0.0622)	(0.0667)	(0.0604)	(0.0467)
	β_k	−0.1943	−0.1542	−0.2134	−0.3138	−0.4651**	−0.6710***
		(0.1168)	(0.1466)	(0.1778)	(0.1906)	(0.1726)	(0.1337)
	γ_k	0.3425*	0.3421	0.4380	0.5311	0.2835	0.0462
		(0.1933)	(0.2425)	(0.2942)	(0.3153)	(0.2855)	(0.2211)
	δ_k	−0.2895**	−0.4428***	−0.6102***	−0.7288***	−0.5881***	−0.2609*
		(0.1201)	0.1507	(0.1828)	(0.1959)	(0.1774)	(0.1374)
	F	F(3,25) = 3.41	F(3,25) = 7.23	F(3,25) = 10.86	F(3,25) = 15.69	F(3,25) = 41.55	F(3,25) = 102.28
	$Prob > F$	0.0331	0.0012	0.0001	0.0000	0.0000	0.0000
	No. of obs.	29	29	29	29	29	29

Notes: *** (**) and * indicate significance at the 1 (5) and 10 percent level, respectively.

First, since central banks' interest rate forecasts have an effect on asset prices in the financial market (Detmers and Nautz, 2012) and might be regarded as an additional tool for forward guidance, internal (in)consistency of short-term (long-term) forecasts reflect that central banks react to shocks temporarily and might yield a loss in the forward guidance of financial market participants. Second, internal consistency of interest rate forecasts as such might also be a relevant consideration for central banks to communicate more effectively with market participants and to preserve their credibility. More specifically, central banks should be aware that substantial interest rate forecast errors might harm their credibility as they reflect a deviation from their yardstick. Third, for central banks exhibiting inconsistent forecasting behavior, the rational expectations hypothesis is challenged and poses risks to common financial models that operate on this assumption.

Certain limitations to our study apply as well which does not take into account that seemingly inconsistent forecasting behavior might in fact be intentional in nature due to changing economic realities and commensurate responses by monetary authorities. This potentially becomes a greater issue with increasing forecast horizons, as suggested by the results presented in sections 3 and 4. Moreover, the observed differences in forecast consistencies across central banks can not be easily explained without conducting an in-depth analysis of country specificities and possibly differing structural components across economies. The question of causal linkages between forecast consistency and country-specific idiosyncrasies leaves room for further research and should be explored in the future to fully understand the implications of consistent and inconsistent forecasting behavior.

Table 5: Test of consistency restrictions ($R3$) to ($R6$)

		3 vs. 6 mth.	**6 vs. 12 mth.**	**9 vs. 18 mth.**	**12 vs. 24 mth.**
Czech Republic	χ^2	0.00	0.01	0.45	no observations
	$Prob > \chi^2$	(0.9690)	(0.9266)	(0.5041)	
	R3	1.64	6.58**	10.48***	
	$Prob > \chi^2$	(0.1998)	(0.0145)	(0.0025)	
	R4	0.37	0.22	0.61	
	$Prob > \chi^2$	(0.5453)	(0.6408)	(0.4411)	
	R5	0.21	0.00	0.28	
	$Prob > \chi^2$	(0.6478)	(0.9765)	(0.5968)	
	R6	0.01	0.31	0.03	
	$Prob > \chi^2$	(0.9124)	(0.5821)	(0.8674)	
	R3 to R6	2.12	2.15*	3.37**	
	$Prob > \chi^2$	(0.7131)	(0.0941)	(0.0118)	

Table 5: Test of consistency restrictions ($R3$) to ($R6$)

		3 vs. 6 mth.	**6 vs. 12 mth.**	**9 vs. 18 mth.**	**12 vs. 24 mth.**
New Zealand	χ^2	11.25***	1.57	0.72	2.23
	$Prob > \chi^2$	(0.0013)	(0.2146)	(0.3989)	(0.1401)
	R3	13.03***	13.44***	4.11	1.33
	$Prob > \chi^2$	(0.0006)	(0.0005)	(0.0466)	(0.2527)
	R4	0.63	0.97	7.44***	5.74**
	$Prob > \chi^2$	(0.4317)	(0.3288)	(0.0082)	(0.0196)
	R5	1.03	0.09	4.99**	0.83
	$Prob > \chi^2$	(0.3137)	(0.7684)	(0.0289)	(0.3657)
	R6	0.03	0.00	0.80	2.61
	$Prob > \chi^2$	(0.8700)	(0.9478)	(0.3751)	(0.1115)
	R3 to R6	3.53**	3.87***	2.91**	3.23**
	$Prob > \chi^2$	(0.0110)	(0.0068)	(0.0278)	(0.0181)
Norway	χ^2	4.63**	13.13***	0.14	2.97*
	$Prob > \chi^2$	(0.0374)	(0.0008)	(0.7152)	0.0924
	R3	0.19	3.41*	5.06**	3.27*
	$Prob > \chi^2$	(0.6680)	(0.0723)	(0.0301)	(0.0780)
	R4	3.58*	0.46	7.64***	9.72***
	$Prob > \chi^2$	(0.0659)	(0.5032)	(0.0086)	(0.0034)
	R5	0.12	1.62	3.84*	6.57**
	$Prob > \chi^2$	(0.7273)	(0.2102)	(0.0571)	(0.0143)
	R6	1.38	1.49	0.90	0.02
	$Prob > \chi^2$	(0.2466)	(0.2294)	(0.3474)	(0.8827)
	R3 to R6	3.03**	1.43	3.76**	3.77**
	$Prob > \chi^2$	(0.0286)	(0.2404)	(0.0109)	(0.0108)
Sweden	χ^2	0.40	3.24*	4.18**	8.97***
	$Prob > \chi^2$	(0.5347)	(0.0840)	(0.0517)	(0.0061)
	R3	0.81	1.45	4.32**	4.70**
	$Prob > \chi^2$	(0.3763)	(0.2395)	(0.0481)	(0.0400)
	R4	0.83	3.52*	6.09**	10.23***
	$Prob > \chi^2$	(0.3696)	(0.0725)	(0.0207)	(0.0037)
	R5	2.33	5.39**	3.57*	2.38
	$Prob > \chi^2$	(0.1393)	(0.0287)	(0.0707)	(0.1352)
	R6	5.25**	7.02***	0.48	2.51
	$Prob > \chi^2$	(0.0307)	(0.0138)	(0.4931)	(0.1260)
	R3 to R6	1.93	1.82	2.81**	10.23***
	$Prob > \chi^2$	(0.1368)	(0.1560)	(0.0471)	(0.0000)

Note: Table 5 reports the test results for the internal consistency restrictions laid out in Section 4. The table summarizes F-values. The first row for each country reports the results of a test whether the β-coefficients for the short- and long-term as reported in table 2 are statistically significantly different from each other. The null hypothesis for χ^2 is that the estimates for the short- and long-term forecasts from table 2 are not statistically significantly different. The null hypothesis for the test of restrictions $R3$, $R4$, $R5$, $R6$, and $R3$ to $R6$ is that the internal consistency restrictions hold. *** (**) and * indicate statistical significance at the 1 (5) and 10 percent level.

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