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Retirement choice in a frictionless labor market

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Abstract

This paper investigates optimal consumption and retirement choices in an environment without labor market frictions but in which agents face liquidity constraints. To this end, we characterize the optimal savings strategy and the threshold level of assets associated with the decision to retire or not, which in turn yield the optimal consumption and work choices. In line with the literature, we show that wealth plays a crucial role since it determines whether or not it is optimal to save and then to retire. However, our results differ from the conventional view as we find that, for those workers with low levels of assets, retirement is never optimal regardless of income uncertainty.

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1 Introduction

The retirement decision is one of the most important decisions that a worker can make during her life cycle. This decision has been broadly studied under the assumption of uncertainty about future income. In this paper, we analyze the optimal consumption and retirement choices of an individual in a frictionless labor market, where the wage is fixed and there are no unemployment shocks, but in which agents face liquidity constraints. We show, in line with the existing literature, that wealth plays a crucial role since it determines whether or not it is optimal to accumulate savings and then to retire. However, our results differ from the conventional view as we find that, regardless of income uncertainty, retirement is never optimal for those workers with low levels of assets.

Traditionally, the retirement decision has been examined assuming that workers face uncertainty about their income and/or wealth. Some studies consider environments with constant wage earnings but in which agents hold risky securities, thus focusing on the effects of capital market shocks. In this vein, Farhi and Panageas (2007) assume that the labor supply decision is indivisible, whereas in Choi *et al.* (2008) the individual adjusts her supply of labor flexibly above a minimum work-hour. Further, some analyses deal with uninsurable labor income and uncertainty regarding asset returns and lifetime. As a matter of analogy, Dybvig and Liu (2010) assume inflexibility of labor supply before retirement, while in Chai *et al.* (2011) work hours decisions are endogenous. Notwithstanding these assumptions, all studies in this literature generally characterize the optimal retirement date as the first time when the individual's wealth reaches a certain threshold level.

In addition to income uncertainty, it has been well documented that borrowing constraints have a substantial effect on the optimal retirement decision (see Barucci and Marazzina 2012). In this sense, some studies have found that inability to borrow may lead individuals to postpone retirement, thus prolonging their working periods (Park and Jang 2014, Lim and Shin 2011, Chai *et al.* 2011). This is partly because borrowing-constrained agents are less resilient to shocks, so that a fall in income may cause a fall in consumption (Deaton 1991, Carroll *et al.* 1992, Carroll and Kimball 2001, Toche 2005, among others). Moreover, Zeldes (1989) argues that the presence of liquidity constraints induces individuals to save even when such constraints are not currently binding.

Unlike the conventional view, we study the optimal retirement decision assuming that there are no frictions in the labor market, so at any point in time the individual may earn flow wage as self-employment and there are no costs of being self-employed (see Albrecht *et al.* 2009). Notice that we do not consider participation in the formal or informal labor markets.[1] In our model, there is no uncertainty about future income given that the wage is fixed and workers do not bear any unemployment risk; however, they face liquidity constraints. As in the traditional literature, we show that the optimal retirement decision depends on the individual's wealth level. Furthermore, we find that those with very little assets (i.e., the poorest) optimally choose not to accumulate savings and do not intend to retire.

This paper contributes to the literature by showing that, even in a simple setting without uncertainty about future income, some workers do not find it optimal to save and then to

[1]Flórez (2017) presents an alternative framework with frictions in the labor market that distinguishes between the formal and informal sectors under savings.

retire. These results have strong policy implications for developing countries with a large number of poor workers, the majority of them working in the informal sector, as they suggest that these workers might never accumulate enough assets for their retirement. In this sense, our study calls for efforts towards more inclusive social protection systems.

The paper is divided into three sections, the first being this introduction. The second section presents a simple model of a frictionless labor market with liquidity-constrained workers. In this setup, individuals choose how much to consume and save, and whether and when to retire. We characterize the optimal savings strategy and the threshold level of assets associated with the retirement decision, which in turn yield the optimal consumption and work choices. Lastly, the third section summarizes the results and discuses some implications.

2 The model

Consider a model where time is continuous with $t \in [0, \infty)$. An agent earns wage w in the labor market and holds non-negative assets $A \geq 0$. The labor market has no frictions; so, at any point in time, the individual chooses between earning flow wage w by participating and earning nothing but enjoying flow utility u_B otherwise. The liquidity constraint $A \geq 0$ implies that asset-less agents cannot borrow from financial firms.

Workers die with probability $\mu > 0$, where μ also describes the inflow of new entrants. All of them have the same subjective discount rate $\rho > 0$. There is a perfectly competitive annuity market such that each worker enjoys the rate of return $r = \rho + \mu$ on savings and her wealth reverts to financial firms on death. Hence the individual indeed discounts the future at the market rate of return r, which 'augments' ρ to reflect the shorter expected horizon.

The agent chooses consumption $c \geq 0$. Flow utility $u(c)$ is described by an increasing, continuously differentiable, strictly concave function with $u(0) = 0$ satisfying the Inada conditions $u'(0) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$. Assume $u_B < u(w)$, which ensures the poorest (i.e., those with $A = 0$) always choose to work. There is no government in this framework.

Let $V(A)$ denote the agent's expected lifetime payoff using an optimal work and consumption strategy. The Bellman equation describing optimal behavior is

$$rV(A) = \max \begin{cases} \max_c \left[u(c) + u_B + \frac{dV}{dA}(rA - c)\right] \\ \max_c \left[u(c) + \frac{dV}{dA}(rA + w - c)\right] \end{cases}, \tag{1}$$

subject to $A \geq 0$.

The first line on the right side describes the maximized flow payoff of not participating in the labor market: the worker enjoys additional flow utility u_B but only receives income rA. Note that $\dot{A} = rA - c$, so $\frac{dV}{dA}(rA - c)$ describes the worker's capital gain through the optimal savings strategy. The second line is the maximized flow payoff from working, for which $\dot{A} = rA + w - c$ holds.

Note that non participation yields direct flow utility $u_B > 0$, while working yields added marginal value wdV/dA. It is immediate that the individual does not participate as long as

$$u_B > w\frac{dV}{dA},$$

where the marginal value of savings dV/dA is endogenously determined.

The optimal consumption choice, denoted by $c(A)$, solves the first order condition

$$u'(c) = \frac{dV(A)}{dA}.$$

From this condition, it follows that

$$\frac{d^2V(A)}{dA^2} = u''(c)\frac{dc}{dA}. \tag{2}$$

Let

$$\dot{c} = \frac{dc}{dA}\dot{A} \tag{3}$$

denote how consumption changes over time. Totally differentiating Equation (1) with respect to t and applying the envelope theorem, we get

$$r\frac{dV(A)}{dA} = \max\begin{cases} \frac{d^2V(A)}{dA^2}(rA-c) + r\frac{dV(A)}{dA} \\ \frac{d^2V(A)}{dA^2}(rA+w-c) + r\frac{dV(A)}{dA} \end{cases}. \tag{4}$$

Substituting Equation (2) into Equation (4), and after some algebraic manipulations, we get for both non participation and working that

$$u''(c)\frac{dc}{dA}\dot{A} = 0.$$

Equation (3) and strict concavity of $u(\cdot)$ establish the optimal strategy $\dot{c} = 0$, which implies that consumption does not change over time.

One possibly optimal strategy is to "work forever" and consume permanent income $c = rA + w$. The expected lifetime payoff to this strategy is

$$\Pi^E(A) = \frac{u(rA+w)}{r}.$$

Alternatively, the individual might quit into permanent non participation. As a permanently retired worker, he optimally consumes permanent income $c = rA$. This strategy instead yields the expected lifetime payoff

$$\Pi^R(A) = \frac{u(rA) + u_B}{r}.$$

The Inada condition $\lim_{c\to\infty} u'(c) = 0$ implies the "retire" strategy dominates the "work forever" strategy for A sufficiently large (e.g., for A satisfying $u'(rA) < u_B/w$). For $A = 0$, in contrast, the optimal choice depends on $\max\{u_B, u(w)\}$; and since, by assumption, $u_B < u(w)$, the dominant strategy in this case is to work forever.

Let us now consider the following (possibly optimal) savings plan: suppose an employed worker with asset level A consumes $c < rA + w$, and so wealth increases over time. Further, let us suppose the worker retires once assets reach the level $A = A^R$ and thereafter consumes $c^* = rA^R$. In any optimal savings plan, consumption smoothing implies that consumption does not change over time and thus the agent always consumes $c = c^*$. Note then that $A(\cdot)$ evolves according to

$$\dot{A} = rA + w - c^*.$$

The solution to this linear first-order differential equation is such that

$$A - A^R = -\frac{w}{r} + \frac{w}{r}e^{r(t-\tau)},$$

where $\tau \equiv \tau(A, A^R)$ denotes time to retirement; that is, given current asset level A, τ describes the time it takes to accumulate wealth A^R. To obtain τ, let $t = 0$ so the expression above becomes

$$w + r(A - A^R) = we^{-r\tau},$$

which implies that

$$\tau(A, A^R) = \frac{1}{r}\ln\left(\frac{w}{w + r(A - A^R)}\right).$$

The lifetime payoff yielded by this savings plan is thus

$$\Pi(A, A^R) = [1 - e^{-r\tau}]\frac{u(c^*)}{r} + e^{-r\tau}\frac{u(c^*) + u_B}{r},$$

where the first term on the right side describes the discounted utility obtained while saving for retirement and the second the discounted payoff from retiring in τ periods of time. As part of her optimal savings plan, the agent chooses A^R. Thus, the necessary condition for optimal A^R is

$$\frac{\partial \Pi(A, A^R)}{\partial A^R} = u'(c^*) - e^{-r\tau}u_B\frac{\partial \tau(A, A^R)}{\partial A^R} = 0.$$

Calculating $\partial\tau/\partial A^R$ and simplifying using the above conditions yields the necessary condition for optimality

$$u'(c^*) = \frac{u_B}{w}. \tag{5}$$

Equation (5) is a transversality condition. At the point along the optimal consumption path where the agent enters into retirement, the optimal strategy requires $wdV/dA = u_B$ so that the individual is indifferent between continuing to accumulate further assets and switching to non participation. Given c^*, let us define the asset region (A^H, A^R) satisfying[2]

$$A^H = \frac{c^* - w}{r}$$
$$A^R = \frac{c^*}{r}.$$

The rest of the paper establishes the following theorem, which describes the individual's optimal consumption and work choices.

Theorem. *Given parameters (u_B, w), the individual's optimal work and consumption choices are:*

[2]Note that the assumptions $u(w) > u_B$ and $u(\cdot)$ concave with $u(0) = 0$ guarantee $A^H > 0$.

(i) For $A \in [0, A^H]$, the agent chooses to work and consumes permanent income $c = w + rA < c^*$. This is an absorbing state (i.e., the individual never retires from work) for which $V(A) = \Pi^E(A)$.

(ii) For intermediate $A \in (A^H, A^R)$, the consumption plan $c = c^*$ is optimal and the expected lifetime payoff associated with it is $V(A) = \Pi^P(A)$.

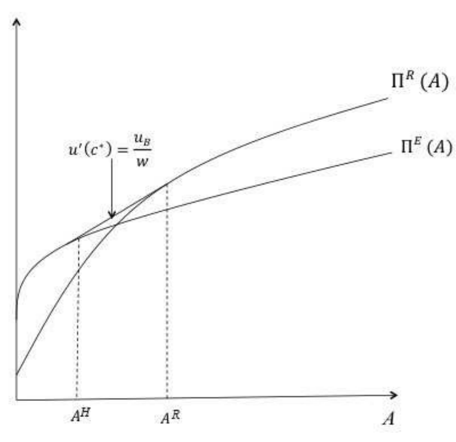
(iii) For $A \geq A^R$, the worker chooses to retire (never works again) and consumes permanent income $c = rA > c^*$. The lifetime payoff associated with these choices is $V(A) = \Pi^R(A)$.

Theorem (ii) identifies an ambiguity in the worker's optimal consumption strategy. One optimal option is to go to work and build up wealth to the point that $A = A^R$, whereupon he permanently retires from the labor market. But an equally optimal strategy is not to go to work in the short term, to run down assets to A^H and then to work indefinitely. Both strategies are optimal since $dV/dA = u'(c^*)$. By contrast, Theorem (i) states that the poorest (i.e., those with $A < A^H$) optimally choose not to accumulate savings and do not intend to retire, whereas Theorem (iii) posits that the very rich (those with $A > A^R$) permanently withdraw from the labor market.

The proof of the Theorem is by construction. Note that those who follow the "always work" strategy, as described in Theorem (i), enjoy lifetime value $\Pi^E(A)$, whereas those who follow the "permanent retirement" strategy in Theorem (iii) enjoy lifetime value $\Pi^R(A)$.

Figure 1 plots these two values as functions of A. The assumption $u(w) > u_B$ ensures that $\Pi^E(0) > \Pi^R(0)$, while $\lim_{c\to\infty} u'(c) = 0$ ensures that $\Pi^R(A) > \Pi^E(A)$ for A sufficiently large. The critical insight here is that, where these two functions intersect, the slope of $\Pi^R(\cdot)$ is strictly greater than the slope of $\Pi^E(\cdot)$. The upper envelope of these two functions, $\max\{\Pi^R(A), \Pi^E(A)\}$, is not a concave function.

Figure 1: Value function, $V(A)$



As depicted in Figure 1, the strategies described in the Theorem yield a value function $V(\cdot)$ which is the convex hull of the two functions $\Pi^R(A)$ and $\Pi^E(A)$. This is easily established by noting that at $A = A^H$ defined above, permanent consumption when always employed equals c^* and so $dV(A^H)/dA = u'(c^*)$. The tangent of $\Pi^E(\cdot)$ at $A = A^H$ is therefore

$$\Pi^E(A^H) + u'(c^*)[A - A^H] = \frac{u(w + rA^H)}{r} + u'(c^*)[A - A^H].$$

Similarly, the tangent of $V(\cdot)$ at $A = A^R$ is

$$\Pi^R(A^R) + u'(c^*)[A - A^R] = \frac{u(rA^R)}{r} + u'(c^*)[A - A^R].$$

As the definitions of A^H and A^R establish that

$$\frac{u(w + rA^H)}{r} - u'(c^*)A^H = \frac{u(rA^R)}{r} - u'(c^*)A^R,$$

these tangents are coincident as depicted in Figure 1.

Hence the lifetime payoff yielded by the savings plan $\Pi^P(A)$ is linear with slope $u'(c^*)$. Note that $\Pi^P(A) = \Pi^E(A)$ at A^H (as $\tau = \infty$) and $\Pi^P(A) = \Pi^R(A)$ at A^R (as $\tau = 0$).

As $V(\cdot)$ is concave, the optimal consumption and work decisions described in the Theorem satisfy the necessary conditions for optimality. Given that $V(\cdot) \geq 0$ is bounded below (as $u(c) \geq 0, u_B > 0$), the Principle of Unimprovability (see Kreps 1990) establishes the result.

3 Final remarks and policy implications

In this paper, we study optimal consumption and retirement choices in an environment without labor market frictions but wherein agents face liquidity constraints. As in the literature with uncertainty about future income, we show that there are some workers that save and decide to retire after reaching a certain level of assets. Yet we also find that, unlike the conventional view, there is a type of workers—the poorest—who optimally choose not to accumulate savings regardless of income uncertainty. For these workers, the optimal decision is to work indefinitely and never to retire.

While theoretical, our findings have important implications for developing countries, where a large share of the workforce is employed in the informal sector and does not accumulate enough wealth to retire. These countries are also known for lacking social safety net programs and attempts to overcome informality via deregulation and minimal state intervention (Williams and Padmore 2013). In this sense, our results call for policy designs providing protection to the oldest in need and are thus in line with a strand of the literature that suggests anti-poverty strategies do not conflict with reducing informality (see Perry *et al.* 2007, and Flórez 2019).

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