Resource allocation with Time Series DEA applied to Brazilian Federal Saving banks

Thyago C. C. Nepomuceno
Sapienza University of Rome

Ana Paula C. S. Costa
Universidade Federal de Pernambuco, Department of Production Engineering

Abstract

One limitation in the economic analysis of efficiency and productivity is the impossibility to determine whether a service organization has reached their optimum output-to-input configuration, i.e. whether efficient units could be more efficient or whether inefficient units have reached their maximum potential and could not improve their performance. In this work, the usage of time series data instead of cross-sectional data from different DMUs is motivated to avoid this problematic of comparing units which might significantly differ in their internal structure (production technology) even presenting similar input/output levels. From the optimum output-to-input ratio, resource lacks (with respect to projected goals) and slacks can be determined for each decision unit evaluated individually. The case of Brazilian Federal Saving banks is presented as an empirical application of the methodology.
Abstract

One limitation in the economic analysis of efficiency and productivity is the impossibility to determine whether a service organization has reached their optimum output-to-input configuration, i.e. whether efficient units could be more efficient or whether inefficient units have reached their maximum potential and could not improve their performance. In this work, the use of time series data instead of data from different DMUs is motivated to avoid this problem of comparing units which might significantly differ in their internal structure (production technology) even presenting similar input/output levels. From the optimum output-to-input ratio, resource lacks (with respect to projected goals) and slacks can be determined for each decision unit evaluated individually. The case of Brazilian Federal Saving banks is presented as an empirical application of the methodology.
Introduction

Data Envelopment Analysis is a linear programming technique introduced by Charnes et al. (1978) and matured by Banker et al. (1984) to measure the performance of decision-making units (DMUs). From this perspective, the efficiency of a specific firm can be seen as a ratio of produced outputs from provided inputs. Therefore to increase efficiency, the decision unit must produce more output (e.g. sales) with the same level of inputs (e.g. the number of employees), or to produce the same level of outputs with fewer inputs (see Farrell 1957; Cooper et al. 2006; Daraio & Simar 2007). Since its introduction, an uncountable number of empirical applications in the service industry, especially in banking, transportation, and healthcare have raised robust comparisons among different decision units (see recent surveys on the field by Daraio et al. 2019A, Emrouznejad and Yang 2018 and Liu et al. 2013) with numerous tools to perform assessments (Daraio et al. 2019B).

The consistent contribution from the supplied resources for the efficient production may be underestimated in some configurations disregarding internal structures and environmental differences among the units along the time. Generally, it is not possible to determine the optimum efficient production (Sherman and Zhu 2006) and many occasions the resource allocation of potential slacks is made based on comparisons between “apples and oranges”. This is of crucial importance especially when we consider assessments of bank efficiency and the role of financial institutions to the economic development. Diallo (2016), for instance, evidences the mitigation of financial crises that efficient bank branches provide by supplying funnelled credit and funds to more externally dependent industries, and Nepomuceno et al. (2019) presents the importance of bank efficient practices to reduce the environmental impact.

The efficiency of Brazilian bank industry has been assessed by some important DEA approaches along the last decades, from which are worth to mention the work of Cava et al. (2016) exploring the relation of efficiency, business and risk rating, Perico et al. (2008, 2016) discussing the size effects, fusions and acquisitions on the overall efficiency classification of large retail banks and credit institutions and Staub et al. (2010) on the evolution of Brazilian banking (cost, technical and allocative) efficiency from 2000 to 2007. All these assessments (and similar works such as Tecles & Tabak 2010; Souza & Macedo 2009; Wanke & Barros 2014; and Wanke et al. 2015) share similar production technology: Cross-sectional pairwise evaluations of multiple DMUs at a macro level, with inputs such labour (employees), deposits (sometime as output) assets and expenses used to produce credit operations and revenue.

Some assessments and methodologies to measure the technical change of productivity over multiple periods have been proposed in the Productivity and Efficiency Analysis (PEA) literature (e.g. Asmild et al. 2004; Färe et al 1994; Hampf 2016) in order to meet some of the concerns in traditional efficiency analysis. This work aims to contribute in this discussion developing a Time Series Data Envelopment Analysis to produce robust time-benchmarks, slacks and to support the optimal number of (human) resources in a specific micro level organization based on internal envelopment instead of macro level peer-evaluation frontiers. Time series data are time-sorted observations provided to predict a certain outcome (Nepomuceno et al. 2017). In this paper, the application of Time Series DEA in the unique dataset of business
transaction from the Brazilian Federal Saving Bank, *Caixa Econômica Federal*, the largest state-owned financial institutions in the Latin America, brings some prospects for the management and levelling of financial institutions resources and results.

**Lacks and Slacks Determination**

Consider a set of \(d\) decision making units (DMU) where each unit \(j = 1, 2, 3\ldots, d\) uses \(i = 1, 2, \ldots, n\) inputs to produce \(r = 1, 2, \ldots, m\) outputs. The potential slacks in the resources \(s_i\) and in the results \(s_r\) are determined by the linear problem (Banker et al. 1984; Cooper et al. 2006; Zhu 2014):

\[
\max \sum_{i=1}^{n} s_i + \sum_{r=1}^{m} s_r
\]

Subject to:

\[
\begin{align*}
\sum_{j=1}^{d} \lambda_j x_{ij} - \theta^* x_{io} + s_i &= 0 \\
\sum_{j=1}^{d} \lambda_j y_{rj} - (y_{ro} + s_r) &= 0 \\
\sum_{j=1}^{d} \lambda_j &= 1 \\
\lambda_j &\geq 0
\end{align*}
\]

Where \(\theta^* = \frac{\sum_{i=1}^{m} y_{ri} y_{ro}}{\sum_{i=1}^{n} x_{io}}\) is the ratio scale that determines the efficiency measure of the decision unit \(o\) under evaluation. To include internal frontiers instead of peer evaluations of different units, consider the same decision unit \(j = 1, 2, 3\ldots, d\) evaluated within a specific period of time \(t = 1, 2, 3\ldots, p\), the model “(1)” is then modified to:

\[
\max \sum_{i=1}^{n} s_i + \sum_{r=1}^{m} s_r
\]

Subject to:

\[
\begin{align*}
\sum_{t=1}^{p} \beta_t x_{it} - \varphi^* x_{ig} + s_i &= 0 \\
\sum_{t=1}^{p} \beta_t y_{rt} - (y_{rg} + s_r) &= 0 \\
\sum_{t=1}^{p} \beta_t &= 1 \\
\lambda_j &\geq 0
\end{align*}
\]

Where \(\beta\) and \(\varphi^*\) applied in the self-evaluation model have the same interpretation as \(\lambda\) and \(\theta^*\) applied in the traditional efficiency measure of decision units. The model “(2)” requires the decision maker to project specific goals \(y_{rg}\) for the \(r = 1, 2, \ldots, m\) outputs, so that the current production set and resource levels may be compared with different production possibilities achieved by the decision unit in different moments. The projection of goal may induce the decision making unit to be enveloped by the
efficiency frontier (Figure 1), having resources that can be spared, or be outside (Figure 2), in which case the production set is not feasible without additional resources.

The distance between the inefficient pseudo-unit to their efficient benchmark (Figure 1) has a positive value and represent the slacks determined by the production dynamics of the decision unit in periods with similar input and outputs configuration (Sherman & Zhu 2006; Cooper et al. 2006). On the other hand, the distance between the unfeasible efficient pseudo-unit to their reachable efficient benchmark (Figure 2) has a negative value and represents the lack of resources that need to be supplied in order to achieve the projected goal, and it is determined by the production dynamics of the decision unit in periods with similar input and outputs configuration.

As result, metrics for the lack of and slack in the input are provided to support the decision making on allocation, planning and management of the organization resources.

**Assessing CEF Human Resource Levels**

*Caixa Econômica Federal* (CEF) is the largest state-owned financial institution in Latin America, working as commercial and investment bank with the monopoly on pledge operations, the administration of lottery retailers and responsible to collect and pay the severance indemnity fund designated for workers (FGTS). Having more than 98 thousand employees, revenue about US$ 3.8 billion and more than US$ 400 billion in assets, it is the government right-hand executor of public policies and plays a crucial role in the country's urban development and social justice, prioritizing sectors such as housing, basic sanitation, infrastructure and income transfer programs, contributing significantly to improving the lives of low-income people. 3288 bank agencies (units) are distributed across the country, and basically every city in Brazil has an agency, bank correspondent or partner to support CEF’s service provision. SR Recife is one of the many superintendence responsible to inspect, support and bring directions and guidelines to the bank units. This superintendence accounts for 54 units in 14 cities settled in the Brazilian northeast (the state of Pernambuco), having about half a million people looking for the banks’ services monthly.

Daily activities and results of the bank are divided into 3 main blocks: Business, Risk Assessment and Social Programs and Authentications. For the purpose and limitation of this work, only the results associated to the Business block (see Table 1) and 8 units from the 54 composing the SR Recife are selected for the evaluation. The decision units
were randomly chosen. For each unit we have 25 months data of inputs and outputs (from September 2015 through September 2017), a total of 200 month data, which are the units in the Time-series pairwise comparison. For each individual, corporate and government domains, the number of transaction is summed to be used as the output in this evaluation.

Table 1. Business Transactions and Products Associated to the Unit’s Results

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Individual</th>
<th>Corporate</th>
<th>Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saving Accounts</td>
<td>Individual and ‘Fácil’ Saving Accounts</td>
<td>Corporate Saving Account</td>
<td></td>
</tr>
<tr>
<td>Current account</td>
<td>Caixa Fácil Account, Individual Account</td>
<td>Corporate Account, Caixa Guaranteed Account</td>
<td>Gov. Account</td>
</tr>
<tr>
<td>Commercial Credit</td>
<td>Individual Credit/Loans, Payroll Consignation, Overdraft Facility, CDC</td>
<td>Working Capital, Corporate Revolving Credit, Giro Caixa, Special Corporate Credit</td>
<td></td>
</tr>
<tr>
<td>Financing and Investment</td>
<td>Personal Credit, Construcard, Caixa Fácil Credit, Real Estate Caixa</td>
<td>FINAME Producard</td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>Individual Financing</td>
<td>Corporate Financing</td>
<td></td>
</tr>
<tr>
<td>Credit Cards</td>
<td>Individual Credit Cards</td>
<td>Corporate Credit Cards</td>
<td></td>
</tr>
<tr>
<td>Insurance Products</td>
<td>Consortium, Capitalization</td>
<td>Bonds, Corporate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bonds, Private Pension, Life, Vehicle and House insurances</td>
<td>Private Pension,</td>
<td></td>
</tr>
</tbody>
</table>

The inputs considered for this assessment are the number of employees and monthly attendance in the unit under evaluation. Some outliers had to be removed, namely October 2015, April 2016 and September 2016, for not being representative (numbers that are extremely below the real panorama of the units) or due lack of data. Table 2 summaries the main statistics concerning the average transaction, attendance and number of employees. The model described by “(2)” requires the projection of a goal by the decision maker in order to derive the optimal resource, lack and slacks for every decision unit. In this assessment, the goal has been determined by the greatest outcome from all the months under evaluation in the two years dataset. Thereby, the projection of 1944 transactions for unit ‘A’, for instance, comes from the best result this specific unit obtained along the past 2 years, which is attributed to June 2017, when the bank agency had 34 employees and 7959 clients to attend. The last columns exhibit the main results coming from the application of this methodology.

Table 2. Summary Results and Basic Statistics

<table>
<thead>
<tr>
<th>Unit</th>
<th>Average Transaction</th>
<th>Average Attendance</th>
<th>Number of Employees*</th>
<th>Goal</th>
<th>Lack (-) and Slack (+)</th>
<th>Optimal Number of Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1171.875</td>
<td>8285.565</td>
<td>37</td>
<td>1944</td>
<td>1.5398</td>
<td>35</td>
</tr>
<tr>
<td>B</td>
<td>2143.083</td>
<td>14539.74</td>
<td>42</td>
<td>2858</td>
<td>-118.97</td>
<td>42</td>
</tr>
<tr>
<td>C</td>
<td>1924.833</td>
<td>13690.83</td>
<td>37</td>
<td>3055</td>
<td>-0.42401</td>
<td>37</td>
</tr>
<tr>
<td>D</td>
<td>1497.25</td>
<td>13556.7</td>
<td>30</td>
<td>2091</td>
<td>67.00</td>
<td>30</td>
</tr>
<tr>
<td>E</td>
<td>1433.583</td>
<td>14985.96</td>
<td>31</td>
<td>2652</td>
<td>-</td>
<td>31</td>
</tr>
<tr>
<td>F</td>
<td>1335.833</td>
<td>11374</td>
<td>29</td>
<td>2166</td>
<td>-0.67388</td>
<td>30</td>
</tr>
<tr>
<td>G</td>
<td>1395.583</td>
<td>13756.87</td>
<td>27</td>
<td>2365</td>
<td>-385.00</td>
<td>27</td>
</tr>
<tr>
<td>H</td>
<td>1578.417</td>
<td>11441.96</td>
<td>23</td>
<td>2215</td>
<td>-0.77123</td>
<td>24</td>
</tr>
</tbody>
</table>

*based on the last month of evaluation
Red highlights the most important units of which human resource must be levelled accordingly. Lacks and slacks values are rounded to keep the management of the resources close to what they must represent. The average attendance is considered as the input along with the last month number of employee, which means that the unit must guarantee at least those levels of resources for the efficiency computation. Unit ‘A’ is the only decision unit exhibiting (positive) slacks in their human resource structure (1.54 ≃ 2), which leads to the optimal value of 35 employees to obtain the number of 1944 transactions if we assume 8286 clients will be looking for the unit’s service in the month under evaluation. Following similar reasoning, units ‘F’ and ‘H’ must guarantee one more worker each (lacks of 0.68 and 0.77 ≃ 1) if we assume an average attendance to obtain the projected goal.

Another important outcome from this methodology provides a metric for lacks in the volume of clients as a potential demand to be stimulated. For instance, though unit ‘B’ has no lack of human resource to meet neither has slacks in their employees to be minimized; the attendance resource for the specific evaluation lacks about 119 clients in order for ‘B’ to achieve the proposed goal of 2365 transactions. Likewise ‘G’ will need additional 385 clients to achieve the projected goal. These estimative may present a relevant support for decision makers to yield marketing projects, financial portfolio advertisements and proactive publicity planning to increase the number of clients looking for the unit’s banking services. When more inputs can be accessed in the efficiency modelling of the financial institution, e.g. when the number of attendance can be discriminated into more specific client bases, such as individual account, corporate products, government programs, loans, credit or any sort of bank service, those metrics of lacks and slacks in the inputs can be further handful to managerial purposes for designating the specific demand that marketing campaigns should focus on.

Considering the other side of the attendance input, unit ‘D’ does not lack or slacks employees, nevertheless 67 clients can be dropped off the average monthly attendance (clients scheduling can be more restrictive) without affecting the projected goal or the overall efficiency of the decision making unit. A simple manner to reallocate the human resource so that all decision units may optimize their prospects is by direct transfers among the units. Thereby, unit ‘A’, which can spare 2 employees and still manage to obtain the projection of 1944 business transactions, transfers one employee for unit ‘F’ and one for unit ‘H’ whose lack one employee each to achieve the efficiency, considering each individual projected goal. The bank manager may, in addition, regulate employee vacation calendar, elaborate internal directives for the posting of workers, control day-offs and extra labour hours, since direct transfers many times may provoke different levels of discomfort.

Lastly, we want to provide some additional prospects comparing these results with the DEA individual evaluation. Table 3 analyses each unit technical efficiency with cross-sectional data to compare with the previous results. This superintendence accounts for 54 units. We have used some of the data from Nepomuceno et al. (2019) instead, disregarding small court units devoted mainly to judicial transactions and others. Four superintendence units are at their optimum (A, G, Y, Z), located at the border of the efficiency frontier. It is interesting to notice that Unit A, despite inefficient considering
their internal production technology, i.e. they present some slacks that can be reduced based on similar production possibilities in other months, this unit is presented as efficient in the cross-sectional evaluation. This specific situation highlights the main contribution by using Time Series DEA: the possibility to determine whether a service organization has reached their optimum output-to-input configuration based on long run time data analysis.

Table 3. Results for individual evaluations

<table>
<thead>
<tr>
<th>Unit</th>
<th>Slack Employees</th>
<th>Slack Attendance</th>
<th>Efficiency</th>
<th>Unit</th>
<th>Slack Employees</th>
<th>Slack Attendance</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.00</td>
<td>0.00</td>
<td>1.000</td>
<td>N</td>
<td>2.63</td>
<td>0.00</td>
<td>0.516</td>
</tr>
<tr>
<td>B</td>
<td>0.00</td>
<td>0.00</td>
<td>0.915</td>
<td>O</td>
<td>0.00</td>
<td>0.00</td>
<td>0.740</td>
</tr>
<tr>
<td>C</td>
<td>0.00</td>
<td>0.00</td>
<td>0.814</td>
<td>P</td>
<td>0.00</td>
<td>0.00</td>
<td>0.713</td>
</tr>
<tr>
<td>D</td>
<td>0.00</td>
<td>0.00</td>
<td>0.751</td>
<td>Q</td>
<td>6.54</td>
<td>0.00</td>
<td>0.667</td>
</tr>
<tr>
<td>E</td>
<td>13.69</td>
<td>0.00</td>
<td>0.889</td>
<td>R</td>
<td>0.00</td>
<td>0.00</td>
<td>0.918</td>
</tr>
<tr>
<td>F</td>
<td>0.00</td>
<td>0.00</td>
<td>0.748</td>
<td>S</td>
<td>0.00</td>
<td>504.42</td>
<td>0.918</td>
</tr>
<tr>
<td>G</td>
<td>0.00</td>
<td>0.00</td>
<td>1.000</td>
<td>T</td>
<td>0.00</td>
<td>753.48</td>
<td>0.640</td>
</tr>
<tr>
<td>H</td>
<td>0.00</td>
<td>0.00</td>
<td>0.933</td>
<td>U</td>
<td>0.00</td>
<td>15783.77</td>
<td>0.907</td>
</tr>
<tr>
<td>I</td>
<td>3.13</td>
<td>0.00</td>
<td>0.665</td>
<td>V</td>
<td>3.35</td>
<td>0.00</td>
<td>0.830</td>
</tr>
<tr>
<td>J</td>
<td>0.00</td>
<td>0.00</td>
<td>0.584</td>
<td>W</td>
<td>0.00</td>
<td>2812.38</td>
<td>0.669</td>
</tr>
<tr>
<td>K</td>
<td>0.00</td>
<td>0.00</td>
<td>0.734</td>
<td>X</td>
<td>0.00</td>
<td>26144.70</td>
<td>0.990</td>
</tr>
<tr>
<td>L</td>
<td>0.15</td>
<td>0.00</td>
<td>0.597</td>
<td>Y</td>
<td>0.00</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>M</td>
<td>3.42</td>
<td>0.00</td>
<td>0.736</td>
<td>Z</td>
<td>0.00</td>
<td>460.00</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Time Series Data Envelopment Analysis can be a powerful tool to perform efficiency and productivity analysis when we want to consider internal frontier to determine the optimum output-to-input ratio for decision units that can vary in technology though consider constant, or for efficiency analysis in monopolized markets presenting only one or few decision making units. In addition, from the results of the optimization process, many allocation methods to plan, assign and schedule human resources may provide systematic ways to manage the lacks and slacks of employees among the evaluated financial units.

References


