Economics Bulletin

Volume 39, Issue 2

Diverse Growth Patterns with External Habit Formation

Liutang Gong
Guanghua School of Management, Peking University

Wei Wang
University of International Business and Economics

Abstract

This paper investigates an external habit formation model with weakly concave utility functions. We show that there exist a continuum of history-dependent balanced growth path (BGP) in this model. This feature of our model can help understand a variety of cross-country growth patterns that are observed in the data.

We would like to thank the referee for helpful and constructive suggestions. We acknowledge the financial support from the National Natural Science Foundation of China (Grant No. 71703017), Beijing Natural Science Foundation (Grant No. 9184031), and the "Fundamental Research Funds for the Central Universities" in UIBE (Grant No. CXTD10-01). Corresponding author: Wei Wang.
Citation: Liutang Gong and Wei Wang, (2019) "Diverse Growth Patterns with External Habit Formation", *Economics Bulletin*, Volume 39, Issue 2, pages 1418-1423
Contact: Liutang Gong - ltgong@gsm.pku.edu.cn, Wei Wang - weiwangccer@gmail.com.
Submitted: May 15, 2019.   **Published:** June 15, 2019.

1 Introduction

Long-run growth patterns across countries observed from the data are highly heterogeneous (Acemoglu (2009); Jones (2016)). Some poor countries are catching-up while some rich one are stagnant or even shrinking. A few growth miracles spend only several decades to become developed economies. Growth theories with multiple balanced growth paths (BGPs) are useful to explain these substantially different growth experiences.[1] Meanwhile, there are countries share similar growth patterns with different underlying mechanisms or fundamentals.[2] In this paper, we explore the endogenous growth model with habit formation in Carroll, Overland and Weil (1997, 2000) to account for these growth patterns.

Carroll, Overland and Weil (1997, 2000) establish the existence and uniqueness of a saddle-path stable BGP in both internal and external habit formation models when the utility function is not concave.[3] Yang and Zhang (2018) derive a unique history-dependent BGP in the internal habit formation model when the utility function is weakly concave.[4] The history-dependence of a BGP refers to the growth rates along the BGP depend on the initial endowment.[5] We study the growth patterns in the external habit formation model of Carroll, Overland and Weil (1997) with the weakly concave utility function. We establish the necessary and sufficient condition for the existence of a BGP.

The history-dependent feature of the BGP in our external habit formation model has important implications on various cross-country growth patterns.[6] First, countries with different fundamentals can grow at the same rate, as long as they share the same habit-capital ratio. This could explain the identical growth experience of countries, such as East Asian growth miracles, that have remarkable differences in their fundamentals. Second, poor countries can catch up with and eventually surpass rich countries if the poor ones have lower initial habit-capital ratio. Hence, our model is useful in explaining growth miracles catching up with developed countries. Third, countries with similar initial fundamentals could have completely different growth patterns. This feature of our model can be used to explain growth miracles and disasters that were similar in terms of initial fundamentals.

The remainder of this paper is organized as follows. Section 2 describes the model. Section 3 establishes the existence and uniqueness of history-dependent BGPs. We also discuss implications of history-dependent BGPs on cross-country growth patterns. Section 4 concludes.

[1] Mulligan and i Martin (1993) and Benhabib and Perli (1994), among others, study the local transitional dynamics around the unique BGP in the Lucas (1988) model. Xie (1994) characterizes the global transitional dynamics in a special case of the same model.

[2] Young (1995) documents that the average annual GDP per capita growth rates of three East Asian miracles are 6.8 percent during 1966 to 1990.

[3] Hiraguchi (2011) proves the existence of the optimal balanced growth path even if the utility is not concave.

[4] The literature on growth models with habit formation is large. See Alonso-Carrera, Caballe and Raurich (2005); Alvarez-Cuadrado, Monteiro and Turnovsky (2004); Gomez (2014); Hiraguchi (2011); Ryder and Heal (1973), among others.

[5] The BGP is unique given any initial endowment ratio . Yang and Zhang (2018) consider this as multiple BGPs.

[6] See Yang and Zhang (2018) for an excellent discussion on the usefulness of history-dependent BGPs.

2 Model

We briefly present the endogenous growth model with external habit formation in Carroll, Overland and Weil (1997). Time is continuous and horizon is infinite.[7] The representative household's problem is to maximize the lifetime utility:

$$U=\int_0^{\infty} u(c,z)\,e^{-\theta t}dt, \tag{1}$$

subject to the budget constraint and the external habit formation:

$$\dot{z}=\rho\left(\bar{c}-z\right), \tag{2}$$

$$\dot{k}=rk-c, \tag{3}$$

where $r=A-\delta$ and the instantaneous utility is given by

$$u(c,z)=-\frac{z^{\beta}}{c^{\alpha}}. \tag{4}$$

$c>0$ and $z>0$ are the consumption and the habit, respectively. $\alpha>0$ and $\beta>0$ are the consumption and habit elasticity, respectively. The initial endowment of habit and capital are strictly positive $z_0>0$ and $k_0>0$, respectively. $\theta>0$ is the discount rate. $\delta>0$ is the capital depreciation rate and $A>\delta$ is the productivity parameter. $\rho>0$ is the weight of consumption at different times in the process of habit formation. Notice that $\bar{c}$ is taken as given when the household makes decisions and $\bar{c}=c$ in equilibrium.

3 Balanced Growth

In this section, we provide conditions for the existence of BGPs in the model described in Section 2. To ensure the optimality and uniqueness, we assume that the instantaneous utility function $u(\cdot,\cdot)$ is weakly concave. In particular, we assume $\beta=1+\alpha>1$. We begin our analysis by defining a BGP in equilibrium.

Definition 1. A balanced growth path (BGP) is a solution to the household's problem where consumption, capital and habit grow at constant rates.

Although the definition of a BGP allows different growth rates, c, z, and k grow at a common rate in our model. To see this, rewrite the law of motions (2) and (3) as follows:

$$\frac{\dot{z}}{z}=\rho\left(\frac{c}{z}-1\right), \tag{5}$$

$$\frac{\dot{k}}{k}=r-\frac{c}{k}. \tag{6}$$

Constant growth rates require both $\frac{c}{z}$ and $\frac{c}{k}$ remain constant over time, meaning z, c, and k grow at a common rate. Denote this common rate by g. We summarize this result in Proposition 2.

[7] The time index is omitted whenver it causes no confusion.

Proposition 2. *Along any BGP,* $\frac{\dot{c}}{c} = \frac{\dot{z}}{z} = \frac{\dot{k}}{k} = g$*. In addition,* $\frac{c}{z} = 1 + \frac{g}{\rho}$ *and* $\frac{c}{k} = r - g$*.*

The following condition ensures a bounded lifetime utility and a positive growth rate.

Condition 3. $0 < \frac{z_0}{k_0} < r$.

The first inequality ensures the boundedness of the lifetime utility and the second one guarantees a strictly positive common growth rate in equilibrium. We now derive the optimality conditions for the household's problem. The first-order condition and the Euler equations are

$$\frac{\alpha z^{\beta}}{c^{1+\alpha}} = \mu, \tag{7}$$

$$\dot{\mu} = (\theta - r)\,\mu, \tag{8}$$

where μ is the co-state variable associated with the law of motion of capital. In addition, the transversality condition is

$$\lim_{t\to\infty} e^{-\theta t}\mu k = 0. \tag{9}$$

The necessary conditions are equations (2), (3), (7), (8), and (9). Proposition 4 describes the unique BGP and the corresponding necessary and sufficient existence conditions.

Proposition 4. *Suppose* $\beta = \alpha + 1 > 1$*. Consider a unique BGP where* $z = z_0 e^{gt} > 0$*,* $k = k_0 e^{gt} > 0$*, and* $c = c_0 e^{gt} > 0$*. The common growth rate*

$$g = \frac{\rho\,(rk_0 - z_0)}{\rho k_0 + z_0} = \frac{\rho\left(r - \frac{z_0}{k_0}\right)}{\rho + \frac{z_0}{k_0}}.$$

g decreases with respect to the initial habit-capital ratio $\frac{z_0}{k_0}$*. The initial consumption* $c_0 = z_0\left(1 + \frac{g}{\rho}\right) = \left[\frac{(\rho+r)k_0}{\rho k_0 + z_0}\right] z_0 > z_0$ *and* $\frac{\partial c_0}{\partial z_0} > 0$*. The maximized lifetime utility is given by* $U = -\left(\frac{z_0^{\beta}}{c_0^{\alpha}}\right)(\theta - g(\beta - \alpha))^{-1}$*. If Condition 3 holds, then* $0 < g < r$ *and the lifetime utility U is bounded. In addition, the co-state varible* μ *is a constant over time:*

$$\mu = \mu_0 = \frac{\alpha z_0^{\beta}}{c_0^{1+\alpha}}. \tag{10}$$

Such a BGP exists if and only if $\theta = r$*.*

Proof. Notice that we have established in Proposition 2 that $\frac{\dot{c}}{c} = \frac{\dot{z}}{z} = \frac{\dot{k}}{k} = g$ along any BGP as well as $\frac{c}{z} = 1 + \frac{g}{\rho}$ and $\frac{c}{k} = r - g$. These are consistent with necessary conditions (2) and (3). We now show the existence and uniqueness of a history-dependent BGP, given the initial endowment. That is, the BGPs start from $t = 0$ and the common growth rate depends on the initial endowment ratio. Suppose there exists a unique history-dependent BGP. Along such BGP, we have $z = z_0 e^{gt}$, $k = k_0 e^{gt}$, $c = c_0 e^{gt}$, and

$$\frac{c}{z} = \frac{c_0}{z_0} = 1 + \frac{g}{\rho}, \quad \frac{c}{k} = \frac{c_0}{k_0} = r - g.$$

Solving these two equations for g and c_0 yields:

$$g = \frac{\rho\left(r - \frac{z_0}{k_0}\right)}{\rho + \frac{z_0}{k_0}} = \frac{\rho\left(rk_0 - z_0\right)}{\rho k_0 + z_0}, \quad c_0 = z_0\left(1 + \frac{g}{\rho}\right) = \frac{(\rho + r)\,k_0}{\rho k_0 + z_0}.$$

Notice that $g > 0$ if and only if the second part of Condition (3) holds. And hence $c_0 > z_0 > 0$ and $\frac{\partial c_0}{\partial z_0} = \rho > 0$. Notice that g is strictly decreasing with respect to $\frac{z_0}{k_0}$. This implies $g < r$ if and only if the first part of Condition 3 holds. The lifetime utility is

$$U = -\left(\frac{z_0^{\beta}}{c_0^{\alpha}}\right)(r-g)^{-1}.$$

The integral in the third line exists and is bounded since $g < r$. Hence, the lifetime utility is bounded.

Notice that our proposed BGP only satisfies necessary conditions (2) and (3). We still have to check other necessary conditions. Equation (10) implies the Euler equation (8) and the first-order conditon (7) at $t = 0$ are satisfied. For $t > 0$,

$$\frac{\alpha z^{\beta}}{c^{1+\alpha}} = \frac{\alpha\left(z_0 e^{gt}\right)^{\beta}}{\left(c_0 e^{gt}\right)^{1+\alpha}} = \mu_0 = \mu.$$

Finally, since $g < r = \theta$ and μ_0 is bounded, the transversality condition 9 is satisfied:

$$\lim_{t\to\infty} e^{-\theta t}\mu k = \lim_{t\to\infty} \mu_0 k_0 e^{-(\theta-g)t} = 0.$$

This completes the proof. □

Interestingly, the BGP and the associated condition of existence in our external habit formation model are essentially equivalent to the internal habit formation model in Yang and Zhang (2018). An economy with initial capital k_0 and habit z_0 jumps to the unique BGP that depends on the ratio $\frac{z_0}{k_0}$ at $t = 0$. The economy grows faster if the initial habit-capital ratio $\frac{z_0}{k_0}$ is lower. Intuitively, since consumption, capital and habit grow at the same rate on the BGP, Proposition 2 implies that initial consumption c_0 increases with respect to both the initial habit stock z_0 and initial capital stock k_0. Therefore, a higher k_0 implies a higher $\frac{c_0}{z_0}$ and hence a larger growth rate. Similarly, a higher z_0 implies a higher $\frac{c_0}{k_0}$ and Proposition 2 means a lower growth rate. The history-dependent feature of the BGP in our model has interesting implications on various cross-country growth patterns. Several examples are in order.

Firstly, countries with the same initial habit-capital ratio grow at the same rate. This implies countries with different initial capital stock and habit stock can grow at the same rate, as long as their ratio is the same. In this case, initially rich countries will be always richer than initially poor countries. Moreover, the initially rich countries will always consume more than initially poor countries. This case could explain the identical growth experience of countries, such as East Asian growth miracles, that have remarkable differences in their fundamentals.

Second, poor countries can catch up with and eventually surpass rich countries if the poor ones have lower initial habit-capital ratio. However, since initially poor countries have even lower initial habit stock z_0, they will still consume less for a while after they surpass the initially rich ones. They will eventually consume more because of the higher growth rate. Our model is useful in explaining growth miracles catching up with developed countries.

Third, countries with similar initial fundamentals could have completely different growth patterns. Countries with small habit stock can grow fast and become growth miracles while countries with large habit stock can become growth disasters. This case can be used to explain the pattern of growth miracles and disasters that are similar in their fundamentals initially. For example, consider Nigeria and South Korea. The two countries were very similar in terms of GDP per capita in 1950. South Korea becomes a developed country now but Nigeria has barely grown at all since then.

4 Conclusion

We study growth patterns in the external habit formation model with weakly concave utility functions. There exists a unique history-dependent balanced growth path (BGP) for any given initial habit z_0 and capital k_0. We establish the necessary and sufficient condition for the existence of the BGPs. We show that our model can explain a variety of cross-country growth patterns that are observed in the data.

References

Acemoglu, D. 2009. *Introduction to Modern Economic Growth.* Princeton University Press.

Alonso-Carrera, J., J. Caballe, and X. Raurich. 2005. "Growth, habit formation, and catching-up with the Joneses." *European Economic Review*, 49(6): 1665–1691.

Alvarez-Cuadrado, F., G. Monteiro, and S. J. Turnovsky. 2004. "Habit formation, catching up with the Joneses, and economic growth." *Journal of Economic Growth*.

Benhabib, J., and R. Perli. 1994. "Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth." *Journal of Economic Theory*, 63(1): 113–142.

Carroll, C., J. Overland, and D. Weil. 1997. "Comparison Utility in a Growth Model." *Journal of Economic Growth*, 2(12): 339–367.

Carroll, C., J. Overland, and D. Weil. 2000. "Saving and Growth with Habit Formation." *American Economic Review*, 90(3): 341–355.

Gomez, M. A. 2014. "Equilibrium dynamics in a class of one-sector endogenous growth models with external habits: an application of special functions." *Journal of Mathematical Economics*, 51: 50–51.

Hiraguchi, R. 2011. "A two sector endogenous growth model with habit formation." *Journal of Economic Dynamics and Control*, 35(4): 430–441.

Jones, C. 2016. "The Facts of Economic Growth." *Handbook of Macroeconomics*, 2A: 30–69.

Lucas, R. E. 1988. “On the mechanics of economic development.” *Journal of Monetary Economics*, 22(1): 3–42.

Mulligan, C., and X. Sala i Martin. 1993. “Transitional Dynamics in Two-Sector Models of Endogenous Growth.” *Quarterly Journal of Economics*, 108(3): 739–773.

Ryder, H. E., and G. M. Heal. 1973. “Optimal growth with intertemporally dependent preferences.” *Review of Economic Studies*, 40(1): 1–31.

Xie, D. 1994. “Divergence in Economic Performance: Transitional Dynamics with Multiple Equilibria.” *Journal of Economic Theory*, 63(1): 97–112.

Yang, Z., and R. Zhang. 2018. “On the diversity of growth patterns with habit formation.” *Economics Letters*, 170: 155–158.

Young, A. 1995. “The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience.” *Quarterly Journal of Economics*, 110(8): 641–680.