Economics Bulletin

Volume 39, Issue 2

A Joint Use Of The Mean and Median for Multi Criteria Decision Support: The 3MCD Method

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Abstract

The Mean-Median Compromise Method (MMCM) is a decision rule that lies between the Majority Judgment and the Majority Borda Count. In this note, we suggest an adaptation of the MMCM to multi-criteria decision problems. We call this extension, the Mean and Median for Multi-Criteria Decision (3MCD). We also examine some properties of this rule.

We thank the Associate Editor, Vicki Knoblauch, for all the relevant comments and suggestions.
Citation: Ruffin-benoit Mpoy Ngoie and Eric Kamwa and Berthold Ulungu, (2019) "A Joint Use Of The Mean and Median for Multi Criteria Decision Support: The 3MCD Method", *Economics Bulletin*, Volume 39, Issue 2, pages 1602-1611
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Submitted: April 27, 2019. **Published:** June 23, 2019.

1 Introduction

Recently introduced by Ngoie *et al.* (2015), Ngoie and Ulungu (2015), the *Mean-Median Compromise Method* (MMCM) is a decision rule based on a combined use of the mean and the median of the grades assigned by agents (decision-makers, voters, judges, etc.) to the options (alternatives, candidates, etc.) they have to assess. Under MMCM, agents grade alternatives on the basis of only one (weighted) criterion; then the rule proceeds by a division of the grades distribution of each candidate into intervals of same amplitude and it returns for each candidate a real number corresponding to the arithmetic mean of some selected grades. The operating mode of MMCM clearly defines it as a rule lying midway between the Majority Judgment (MJ) of Balinski and Laraki (2007, 2010, 2016) and the *Borda Majority Count* (BMC) of Zahid and De Swart (2015) which are also decision rules based on a single (weighted) criterion. In this note, we propose an extension of MMCM for decision problems based on more than one criterion. The decision rule we suggest, named the *Mean and Median for Multi-Criteria Decision* method (3MCD), simply consists in applying MMCM twice in a situation where judges grade alternatives on the basis of at least two criteria: once on the weights of the criterion and then on the grades. We also analyze some virtues and weakness of 3MCD. Prior a presentation of 3MCD, let us first introduce the MJ, BMC and MMCM.

1.1 The Majority Judgment

Under this decision rule, a finite set $J = \{1, 2, \dots, j, \dots, n\}$ of n ($n \geq 2$) voters have to grade elements of a finite set $A = \{a_1, a_2, \dots, a_i, \dots, a_m\}$ of m ($m \geq 2$) candidates using a common language or a well-defined grading system. The grading system can be made by a range of positive integers, a set of letters, words or phrases denoting the opinion or how a voter could evaluate a given candidate. Following Balinski and Laraki (2007, 2010), a *common language* $\Lambda = \{g_1, g_2, \dots, g_p\}$ is a set of strictly ordered grades, so $g_1 > g_2 > \dots > g_p$. A profile $\Phi(A, J)$ is an $m \times n$ matrix of the grades $\Phi(a_i, j) \in \Lambda$ assigned by each $j \in J$ to each of the candidates $a_i \in A$. The grading method is a function F which is defined as follows:

$$F: \quad \Lambda^{m\times n} \rightarrow \Lambda^m$$

$$\Phi(A,J) = \begin{pmatrix} g_{11} & \cdots & g_{1j} & \cdots & g_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{i1} & \cdots & g_{ij} & \cdots & g_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{m1} & \cdots & g_{mi} & \cdots & g_{mn} \end{pmatrix} \mapsto \big(f(g_{11}, g_{12}, \dots, g_{1n}), \dots, f(g_{m1}, g_{m2}, \dots, g_{mj}, \dots, g_{mn})\big)$$

with g_{ij} the grade given by judge j to candidate a_i, f is the aggregation function and $f(g_{i1}, \dots, g_{ij}, \dots, g_{in})$ the final grade of a_i. The majority-grade of $a_i \in A$ is given by

$$f^{\text{maj}}(a_i) = \begin{cases} f^{\frac{n+1}{2}}(g_{i1}, \dots, g_{ij}, \dots, g_{in}) & \text{if } n \text{ is odd} \\ \\ f^{\frac{n+2}{2}}(g_{i1}, \dots, g_{ij}, \dots, g_{in}) & \text{if } n \text{ is even} \end{cases}$$

where $f^k(.)$ is a function that rearranges its arguments in ascending order and returns the k^{th} highest grade.

The winner under MJ is the candidate with the highest median grade. Given two candidates a_i and a_s, if $f^{\text{maj}}(a_i) > f^{\text{maj}}(a_s)$ then $a_i \succ_{maj} a_s$. If $f^{\text{maj}}(a_i) = f^{\text{maj}}(a_s)$ then a_i and a_s tie, and one may resort to a tie-breaking mechanism. According to Balinski and Laraki (2007), in case of ties, the majority-grade is dropped from the grades of each of the candidates and the procedure is repeated. Balinski and Laraki (2010) admitted that this tie-breaking mechanism can become very arduous and they suggested another tie-breaking mechanism based on the concept of "*majority gauge*". For the formal definition of this concept, we refer to Balinski and Laraki (2010).

Balinski and Laraki (2007, 2010, 2016) showed that MJ meets a long list of desirable properties. Nonetheless, MJ is subject to many criticisms that raise both its normative and practical limits. For an overview of these criticisms, the reader may refer to the papers of Felsenthal (2012), Felsenthal and Machover (2008), Laslier (2019) and Zahid (2009).

1.2 The Borda Majority Count

The Borda Majority Count (BMC) is based on the well-known Borda rule. The Borda rule is a voting rule under which, given the preferences (rankings) of the voters, a candidate receives $m - p$ points each time he is ranked p^{th}; the Borda score of a candidate is the total number of points received and the winner is the one with the highest Borda score. Under BMC, the system of grades are converted into scores as under the Borda rule. Given $\widetilde{\Phi}(a_i, j)$ the ordered vector of $\Phi(a_i, j) \in \Lambda$, BMC defines $\Lambda^\star = \{g_1^\star, g_2^\star, \ldots, g_p^\star\}$ with $(g_1^\star > g_2^\star > \ldots > g_p^\star)$ the natural set associated to Λ such that $g_1^\star = p - 1$, $g_2^\star = p - 2$, …, $g_p^\star = 0$. Similarly $g_{ij}^\star$ is defined as the natural number associated with g_{ij} the grade given by judge j to candidate a_i. The Borda Majority Count of a_i is equal to the arithmetic mean of the natural numbers associated to g_{ij} given all the n judges. So, for $a_i \in A$ and $(g_{i1}, \ldots, g_{ij}, \ldots, g_{in})$, the Borda majority count of $a_i \in A$ denoted by $f^{\text{mean}}(a_i)$ is given by :

$$f^{\text{mean}}(a_i) = \frac{1}{n} \sum_{j=1}^{n} g_{ij}^\star$$

For two candidates a_i and a_s, if $f^{\text{mean}}(a_i) > f^{\text{mean}}(a_s)$ then $a_i \succ_{mean} a_s$. If $f^{\text{mean}}(a_i) = f^{\text{mean}}(a_s)$, we get a tie; in such a case the procedure is repeated step by step by dropping grades from lower to higher until a winner among a_i and a_s is found. Zahid and De Swart (2015) provided a list of properties satisfied or failed by BMC. They also provided a characterization of this rule. For more details, the reader may refer to their paper.

1.3 The Mean-Median Compromise Method

The Mean-Median Compromise Method (MMCM) is at midway between MJ and BMC in the sense that it is simultaneously based on the median and the average of the grades. The first task under MMCM is to divide the distribution of $\widetilde{\Phi}(a_i, j)$ into 2^k intervals of the same amplitude where $k \geq 2$ is an integer set in advance;[1] then, the real number $\varrho = \frac{n+1}{2^k}$, called the *amplitude of a division*, is computed. So, for a given candidate a_i and $\widetilde{\Phi}(a_i, j) \in \Lambda$, the quantity g'_{ij} is the *inter-median grade* for a_i if $\exists t \in \mathbb{N}$ such that $1 \leq t \leq 2^k - 1$ and $Rd(t \times \varrho) = j$; where $Rd(.)$ is the value rounded to the nearest integer.[2] It follows that for a given k, the *vector of the*

[1]The integer k is called the *degree of division*.

[2]It is generally admitted that $Rd(3.5) = Rd(3.9) = 4$ but $Rd(3.1) = Rd(3.4) = 3$.

non-redundant inter-median grades of a_i is:

$$\mathcal{M}_k(a_i) = \{g'_{i1}, \ldots, g'_{ij}, \ldots, g'_{it}\} = \{f^{Rd(1\times\varrho)}, f^{Rd(2\times\varrho)}, \ldots, f^{Rd((2^k-1)\times\varrho)}\}$$

The average majority compromise for a_i is given by

$$f^{\text{mm}}(a_i) = \frac{1}{t}\sum_{j=1}^{t} g'_{ij}$$

For two candidates a_i and a_s, if $f^{\text{mm}}(a_i) > f^{\text{mm}}(a_s)$ then $a_i \succ_{mm} a_s$. In the case of $f^{\text{mean}}(a_i) = f^{\text{mean}}(a_s)$, we need to repeat the process for $k+1$ and so on.

Let us notice that following Ngoie *et al.* (2015), Ngoie and Ulungu (2015), for $k = 1$, MMCM is equivalent to MJ, *i.e.* $f^{\text{mm}}(a_i) = f^{\text{maj}}(a_i)$ for all $a_i \in A$; when the value k is set at its maximum, MMCM is equivalent to BMC, *i.e.* $f^{\text{mm}}(a_i) = f^{\text{mean}}(a_i)$. So, MMCM appears as a decision rule lying midway between MJ (based on the highest median) and BMC (based on the highest mean). Ngoie and Ulungu (2015) provided a table in which given a set of normative properties, the reader can see how MJ, BMC and MMCM behave; it comes from this table that when MJ and BMC both meet a property, this is also the case for MMCM. They concluded that MMCM appears like a better compromise between MJ and BMC. Their analysis need to be generalized in order to draw more accurate results.

Let us use an example to illustrate how MMCM operates.

Example 1. Assume that 8 judges respectively grade a candidate a_i with $\Phi(a_i, j) = (9, 7, 3, 6, 5, 4, 5, 8)$ and it follows that $\widetilde{\Phi}(a_i, j) = (9, 8, 7, 6, 5, 5, 4, 3)$. If we assume that $k = 3$, we get $\varrho = \frac{8+1}{2^3} = 1.125$ and

$$\begin{aligned}
\mathcal{M}_3 &= \left(f^{Rd(1\times1.125)}, f^{Rd(2\times1.125)}, f^{Rd(3\times1.125)}, f^{Rd(4\times1.125)}, f^{Rd(5\times1.125)}, f^{Rd(6\times1.125)}, f^{Rd(7\times1.125)}\right)\\
&= \left(f^{Rd(1.125)}, f^{Rd(2.25)}, f^{Rd(3.375)}, f^{Rd(4.5)}, f^{Rd(5.625)}, f^{Rd(6.75)}, f^{Rd(7.875)}\right)\\
&= \left(f^1, f^2, f^3, f^5, f^6, f^7, f^8\right)\\
&= (9, 8, 7, 5, 5, 4, 3)
\end{aligned}$$

Then, $f^{\text{mm}}(a_i) = \frac{9+8+7+5+5+4+3}{7} = \frac{41}{7} = 5.8$

We can now introduce our adaptation of MMCM to multi-criteria problems.

2 Mean and Median for Multi-Criteria Decision

2.1 Description of the rule

In the real world, most decision problems involve several criteria; for example, it is obvious that even if the price of a car remains one of the determining factors in the purchase, at least it remains that the choice will depend on several other criteria such as the color, the shape of the chassis, etc. The adaptation of MMCM to multi-criteria problems, the Mean and Median for Multi-Criteria Decision (3MCD), that we suggest is related to a ranking problem; it boils down to applying MMCM twice: first to aggregate the weights of the criteria, then to aggregate the performance of the candidate on the criteria. The result of these two aggregations is a single decision table which makes it possible to calculate the final "scores" of the candidate. and to produce a complete ranking of the candidates. Our rule operates as described in Algorithm 1 and at the end of the process, a ranking is obtained on A.

Algorithm 1 3MCD

1: Apply the MMCM function on the weights of each criterion to find the overall weight;
2: Apply the MMCM function on the scores of each candidate on each criterion then find the global scores of the candidate on each of the selected criteria;
3: Apply the weighted sum to determine the overall performance of the candidates;
4: Stop the process when comparisons are made.

Let us provide an example illustrating the 3MCD method.

Example 2. Assume that five individuals (judges) have to take a decision over a set of four candidates (named H1, H2, H3 and H4) and that the choice is based on three criteria, let us say C1, C2 and C3. The judgment matrices of the voters are:

Judge 1

Weights	8	4	6
Criteria	C1	C2	C3
H1	6	5	7
H2	5	7	5
H3	6	8	4
H4	7	6	7

Judge 2

Weights	6	6	7
Criteria	C1	C2	C3
H1	7	6	5
H2	6	5	5
H3	7	7	6
H4	6	6	5

Judge 3

Weights	7	5	6
Criteria	C1	C2	C3
H1	8	10	6
H2	7	7	6
H3	6	6	7
H4	7	5	6

Judge 4

Weights	6	4	5
Criteria	C1	C2	C3
H1	7	10	8
H2	7	6	7
H3	8	7	7
H4	7	6	7

Judge 5

Weights	6	7	5
Criteria	C1	C2	C3
H1	7	7	6
H2	6	9	7
H3	8	8	7
H4	7	6	5

Let us assume that $k = 2$. First of all, we sort the weights of each criterion in descending order; so, for C1 we get $(8, 7, 6, 6, 6)$, for C2 we get $(7, 6, 5, 4, 4)$ and for C3 we get $(7, 6, 6, 5, 5)$. Then, we apply the MMCM function on the weights of each criterion to find the overall weight:

$$\begin{aligned} \text{overall weight of C1} &= \frac{7+6+6}{3} = 6.33 \\ \text{overall weight of C2} &= \frac{6+5+4}{3} = 5.00 \\ \text{overall weight of C3} &= \frac{6+6+5}{3} = 5.66 \end{aligned}$$

Following the other steps of 3MCD, we end with the following outcome:

Criteria	C1	C2	C3	**Score 3MCD**
Weights	6.33	5.00	5.66	
H1	7.00	7.66	6.66	120.3056
H2	6.00	6.33	6.00	103.5900
H3	7.00	7.33	6.66	118.6556
H4	6.66	5.66	6.00	104.4178

Let us explain how we get the figures in the outcome matrix. For example, to obtain the score of candidate H3 on the criterion C2, we proceed as follows: On criterion C2, the 5 decision-makers assign H3, respectively with grades 8, 7, 6, 7, 7. In descending order, we have: 8, 7, 7, 7, 6. If we apply MMCM to these sorted data, we see that the inter-median are the second, third and fifth data; namely 7, 7 and 6. Thus, the overall score of H3 on C2 is $\frac{7+7+6}{3} = 6.66$. The last column gives the 3MCD's scores of the candidates; it is obtained by calculating the weighted sum of the overall score of each candidate. For instance, the 3MCD's scores of candidate H4 is obtained by the calculation: $6.66 \times 6.33 + 5.66 \times 5.00 + 6.00 \times 5.66 = 104.4178$. Given the scores of the Candidates, we can conclude that, on the position, the committee will rank candidate H1 first, candidate H3 second, candidate H4 third and candidate H2 last.

2.2 Some properties

Here, we want to pay a particular attention to the following properties : the *neutrality condition*, the *anonymity condition*, the *condition of Independence of Irrelevant Alternatives*, the *majority condition*, the *Condorcet winner criterion*, the *Condorcet loser criterion*, the *homogeneity criterion*, the *reinforcement criterion*, the *participation criterion*, the *monotonicity criterion*, the *Pareto requirement*, the *Clone-resistance* and the *property of reversal symmetry.* Notice that we need to redefine the properties to fit clearly into 3MCD's context.

A decision rule meets the property of Anonymity if permuting the names of the voters do not have any impact on the final outcome. *Neutrality* indicates that permuting the names of the alternatives will result in the same permutation in the final outcome. It is obvious that 3MCD meets these two properties.

Proposition 1. 3MCD is neutral and anonymous.

According to the property of the Independence of Irrelevant Alternatives, if candidate a_i wins, then he would still win if another candidate is removed, ceteris paribus. By *Monotonicity*, if a candidate wins, he would still win if at least one of his grades is increased, ceteris paribus.

Proposition 2. 3MCD meets the condition of Independence of Irrelevant Alternatives.

Proof. The judges make their assessments based on the performance of the candidates independently of each other. Thus, if a judge i assigns a g_{ix} grade to the candidate x and g_{iy} to another candidate y such that $g_{ix} > g_{iy}$, regardless of the g_{iz} rank assigned to the z candidate, this has no impact on the order between x and y. □

A decision rule is Clone-resistant if whenever a clone of a losing alternative is introduced, this does not alter the original outcome.

Proposition 3. 3MCD is Clone-resistant.

Proof. As a consequence of the Independence of Irrelevant Alternatives, the 3MCD is Clone-resistant. □

A decision rule satisfies the monotonicity condition if a winning alternative is never harmed whenever some voters decide to lift up this alternative in their grades or rankings without changing anything else.

Proposition 4. 3MCD is monotonic.

Proof. To proof that the 3MCD is monotonic, we just need to show that a lifting up of a candidate over one or more criteria will not negatively impact its final score. Here, for simplicity, we will just assume a lifting up on one criteria; the case with more than one criteria is just an iteration of one-criteria cases. Assume that an alternative x is the final winner and that he benefits from a lifting up on a given criteria all other things remain unchanged. All other things remain unchanged directly implies that the intermediate grades $\mathcal{M}_k(y)$ of all the alternatives $y \in A \setminus \{x\}$ will remain unchanged as well as their final scores. Concerning x, the lifting up may imply three things: i) $\mathcal{M}_k(x)$ is unchanged, x remains the winner; ii) the new grade is now one element of $\mathcal{M}_k(x)$: in this case, the new MMCM score of x increases and this implies that x remains elected. iii)The new grade is not one of the inter-median but $\mathcal{M}_k(x)$ is modified. In this case, an inter-median grade is replaced by another by a shift of one rank on the left; the grades being ordered in a decreasing way, the new MMCM's score of x will be greater than the original one. □

A decision rule fulfils the Pareto condition if it never selects an alternative x while there is another alternative y that all the judges rank before x. In our framework, the Pareto condition will require that if all the judges award x with grades greater than that of y on each criterion, then y cannot scores greater than x.

Proposition 5. 3MCD meets the Pareto condition.

Proof. Let assume two alternatives x and y such that all the judges grade x higher that y on each criterion. This implies that for $k \geq 2$, each element in $\mathcal{M}_k(x)$ is greater than its correspondent in $\mathcal{M}_k(y)$. Then we get $f^{mm}(x) > f^{mm}(y)$. So, no matter what are the weights of the criterion, the 3MCD score of x will be greater than that of y. □

Definition 1. Given J the set of the judges and a criteria g, candidate x majority dominates candidate y on criterion g if $\sharp\{j \in J \backslash g_j(x) \geq g_j(y)\} > \frac{n}{2}$. We denote it by $xM_g y$. If for all the criteria, $xM_g y \quad \forall\, y \in A \setminus \{x\}$, x is called the *Condorcet winner*; if for all the criteria, $yM_g x \;\; \forall\, y \in A \setminus \{x\}$, x is called the *Condorcet loser*.

We will say that 3MCD satisfies the *majority condition* if $xM_g y$ implies that x scores better than y. 3MCD will meet the *Condorcet winner criterion* if it always select the Condorcet winner when it exists; it meets the *Condorcet loser criterion* if it never selects the Condorcet loser when it exists.

Proposition 6. 3MCD fails the majority requirement, the Condorcet winner criterion and the Condorcet loser criterion.

Proof. Let us consider the following judgement matrices of 3 judges on three candidates (named H1, H2 and H3) based on two criteria (C1) and (C2).

Judge 1

Weights		
Criteria	C1	C2
H1	2	2
H2	3	3
H3	4	4

Judge 2

Weights		
Criteria	C1	C2
H1	6	6
H2	0	1
H3	1	2

Judge 3

Weights		
Criteria	C1	C2
H1	2	1
H2	3	2
H3	4	3

The reader can easily check that H1 is the Condorcet loser while H3 is the Condorcet winner. By computing the scores with $k = 2$, we end with the following output:

Criteria	C1	C2
Weights		
H1	3.33	3
H2	2	2
H3	3	3

So, no matter what are the weights, H1 is ranked first followed by H3 and then H2. It follows that the 3MCD fails the majority requirement, the Condorcet winner criterion and the Condorcet loser criterion[3]. □

A decision rule is said to meet the Participation condition if when some voters supporting a winning candidate are added, this will not turn this candidate into a losing one.

Proposition 7. 3MCD fails the Participation condition.

Proof. Let us consider the following profile with five judges and two candidates (H1 and H2).

	Judges									
	J1		J2		J3		J4		J5	
Criteria	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
H1	3	7	2	8	2	6	6	7	6	7
H2	2	8	3	6	7	9	5	6	3	6

We assume that $k = 2$; no matter what are the weights of the criteria, the reader can check that the output matrix is as follows:

Criteria	C1	C2
H1	3.66	6.66
H2	3.33	6.66

Under this profile, H1 wins. Assume that we add two new judges (J6 and J7) with the following grades:

	J6		J7	
Criteria	C1	C2	C1	C2
H1	9	7	6	6
H2	7	6	5	5

The reader can check that the even though these two judges are in favor of H1, their arrival favors H2 who is the new winner as the new output matrix is:

Criteria	C1	C2
H1	4.66	6.66
H2	5	6.66

Thus, the 3MCD rule fails the Participation condition. □

[3]Example 2 can also be used to show the failure of the majority requirement and of the Condorcet winner criterion. In this example, the reader can check that on each of the criteria, H3 is the Condorcet winner and he is not chosen.

An aggregation function is homogeneous if given a preference profile and the corresponding outcome, replicating this profile λ times ($\lambda > 1$, $\lambda \in \mathbb{N}$) does not change the outcome. In our framework, we will say that the 3MCD is homogeneous if whenever we replicate λ times the original profile, the outcome remains unchanged. Notice that nothing is known about the MMCM concerning the homogeneity condition.

Proposition 8. 3MCD fails the homogeneity condition.

Proof. Let us consider the following profile with five judges and two candidates (H1 and H2).

	Judges									
	J1		J2		J3		J4		J5	
Criteria	C1	C2	C1	C2	C1	C2	C1	C2	C1	C2
H1	7	8	6	6	5	9	2	6	1	6
H2	6	7	5	8	5	6	5	6	1	7

We assume that $k = 2$ and that we replicate each of the judge two (or three) times. The reader can check that the output matrices for the original and the replicated profiles are as follows:

original profile

Criteria	C1	C2
H1	4	6.66
H2	3.66	6.66

replicated profile

Criteria	C1	C2
H1	4.33	6.66
H2	5	6.66

Under the original profile, H1 wins but under the replicated profile, H2 wins. So, by replicating the set of the judges, the outcome changes. Thus, the 3MCD fails the homogeneity condition. □

The reinforcement condition requires that when an electorate is divided into two groups of voters and the voting outcome is the same for both groups, this outcome will remain unchanged when both groups of voters are merged.

Proposition 9. 3MCD fails the reinforcement condition.

Proof. The proof follows directly from that of Proposition 7 or 8. □

A voting rule is said to fail the property of reversal symmetry if a candidate remains elected after the inversion of all individual preferences. With m competing candidates, inversion simply means that a candidate ranked j-th in the original profile, will be ranked $(m - j + 1)$-th in the reversal profile. We adapt this criterion to our framework. For a given profile, the marks given to the alternatives by the judges on the criteria imply a ranking among the alternative on each criterion; the reversal profile is obtained as follows: on each criterion, the mark of a candidate ranked j-th is replaced by that of the candidate ranked $(m - j + 1)$-th in the original profile.

Proposition 10. 3MCD does not meet the property of reversal symmetry.

Proof. Let us consider the following judgement matrices of 3 judges on three candidates (named H1, H2 and H3) based on two criteria (C1) and (C2).

Judge 1		
Weights		
Criteria	C1	C2
H1	0	1
H2	2	4
H3	3	5

Judge 2		
Weights		
Criteria	C1	C2
H1	0	2
H2	2	3
H3	3	4

Judge 3		
Weights		
Criteria	C1	C2
H1	6	6
H2	5	5
H3	0	0

By computing the 3MCD scores with $k = 2$, we end with the following output matrix:

Criteria	C1	C2
Weights		
H1	2	3
H2	3	4
H3	2	3

So, H2 is the winner (no matter what are the weights). The reader can easily check that H2 remains the winner for this profile after complete inversion of the preferences. □

3 Concluding remarks and discussion

The Majority Judgement, the Borda Majority Count and recently, the Mean-Median Compromise Method (MMCM) has been introduced in the literature for decision problems based on one criterion. The main objective of the paper was to suggest an extension of MCMM for multi-criteria decision problems: the Mean and Median for Multi-Criteria Decision (3MCD). We took also the opportunity to check whether 3MCD meets or fails some desirable properties of voting rules. It came out that it fulfills the neutrality condition, the anonymity condition, the monotonicity condition, the Pareto condition, the Independence of irrelevant Alternatives and clone-resistance condition. . Nonetheless, it fails the conditions of homogeneity, reinforcement, participation and does not fulfill either the majority requirement, the Condorcet winner criterion and the Condorcet loser criterion.

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