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Correlated shocks may reduce outcome correlations when outcomes are endogenous: a new paradox

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Abstract

Correlated shocks normally increase correlations between outcomes. This note shows that when goods are substitutes in supply or demand, price correlations may vary inversely with the correlation between their shocks. This new paradox is explained.

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1. Introduction

The correlated shocks model has greatly influenced economic theory and econometrics. The capital asset pricing model (Sharpe 1964) is based on the premise that asset price shocks are correlated because they share a common market risk factor, and arbitrage pricing theory (Ross 1976) assumes that the correlation between asset price shocks may be induced by multiple factors. Insurance pricing theory has been generalized to the case of correlated risks (Schroder, Zweifel and Eugster 2013) as has local labor market theory (Moretti 2011) and local housing market theory (Pryce 2013). In econometrics, the seemingly unrelated regression model (Zellner 1962) was based on the premise that disturbance terms might be correlated between equations, and Pesaran and Shin (1998) have drawn attention to the role of correlated shocks in vector autoregression models. The common correlated effects estimator proposed by Pesaran (2006) has proved to be a major development in allowing for correlated effects in microeconometrics. In short, almost every aspect of economic theory and econometric theory has been influenced by the correlated shocks model.

In purely statistical contexts, outcomes must be more correlated if their shocks are more correlated. For example, if two outcomes happen to depend on independent random shocks the correlation between these outcomes must be zero. Matters will be different, of course, if their shocks are correlated. In economic contexts in which the outcomes happen to be endogenous, outcomes tend to be correlated even if their shocks are uncorrelated. For example, if the outcomes refer to the prices of substitute goods, a demand shock to good 1 raises the price of good 1 directly and it raises the price of good 2 indirectly because good 2 is a substitute for good 1. If in addition, their demand shocks are positively correlated, we might expect their prices to be more correlated. We show that this expectation may be incorrect; it depends on the degree of endogeneity of the outcomes and the degree to which their shocks are correlated. Therefore, the greater the correlation between shocks, the less correlated are outcomes. An explanation for this paradox is provided.

Similar results apply more widely. For example, if local labor markets and housing markets are substitutes, the spatial correlation between wages and house prices will be positive in the absence of correlated shocks. These outcome correlations do not necessarily increase if their shocks are positively correlated. The correlation between asset prices does not necessarily increase if their shocks are more correlated. In multivariate econometric models, such as structural vector autoregressions or models estimated by three stage least squares, just because shocks happen to be more correlated does not necessarily mean that the state variables are more correlated. No doubt there are many other examples where the paradox might matter.

The paradox is introduced in terms of a simple model of supply and demand involving two goods. What matters is that their prices are endogenous because the goods are substitutes in supply and demand.

2. The Model

Let the log demand for good 1 be:

$$D_1 = -\beta_1 P_1 + \gamma_1 P_2 + d_1 \qquad \text{(1a)}$$

where d_1 denotes a mean zero iid demand shock for good 1, and $\beta_1 > \gamma_1$ since the own elasticity of demand exceeds the cross elasticity. Equation (1a) is assumed to apply in all time periods (t), as are equations (1b), (1c) and (1d). Subscript t is omitted for convenience. The log demand for good 2 is:

$$D_2 = \beta_2 P_1 - \gamma_2 P_2 + d_2 \qquad (1b)$$

where d_2 denotes a mean zero iid demand shock for good 2.

The log supply schedules are:

$$S_1 = \theta_1 P_1 - \phi_1 P_2 + s_1 \qquad (1c)$$

$$S_2 = -\theta_2 P_1 + \phi_2 P_2 + s_2 \qquad (1d)$$

where s_1 and s_2 denote zero mean iid supply shocks, and $\theta_1 > \phi_1$ if own elasticities of supply exceed cross elasticities.

The equilibrium solutions for prices are:

$$P_1 = \Omega_1(d_1 - s_1) + \Phi_1(d_2 - s_2) \qquad (2a)$$

$$P_2 = \Omega_2(d_1 - s_1) + \Phi_2(d_2 - s_2) \qquad (2b)$$

Where:

$$\Omega_1 = \frac{\gamma_2 + \phi_2}{\Psi}$$

$$\Omega_2 = \frac{\beta_2 + \theta_2}{\Psi}$$

$$\Phi_1 = \frac{\gamma_1 + \phi_1}{\Psi}$$

$$\Phi_2 = \frac{\beta_1 + \theta_1}{\Psi}$$

$$\Psi = (\beta_1 + \theta_1)(\gamma_2 + \phi_2) - (\beta_2 + \theta_2)(\gamma_1 + \phi_1) > 0$$

Notice that because the goods are substitutes in demand and supply, the price of good 1 depends on supply and demand shocks to good 2 via Φ_1 and the price of good 2 depends on shocks to good 1 via Ω_2.

3. The Relation between Outcome Correlations and Correlated Shocks

The standard deviations of $d_1 - s_1$ and $d_2 - s_2$ are denotes by σ_1 and σ_2. Supply and demand shocks may be dependent within markets. The correlation of d – s between markets is denoted by ρ, and their covariance by $\sigma_{12} = cov(d_1d_2) + cov(s_1s_2) - cov(s_1d_2) - cov(s_2d_1)$. Hence, ρ is zero when σ_{12} is zero. Notice that ρ may be zero even if the components of σ_{12} differ from zero.

The time series correlation between P_1 and P_2 is:

$$r = \frac{cov(P_1P_2)}{sd(P_1)sd(P_2)} \qquad (3a)$$

The variances of P_1 and P_2 and their covariance generated by equations (2) are:

$var(P_1) = \Omega_1^2\sigma_1^2 + \Phi_1^2\sigma_2^2 + 2\Omega_1\Phi_1\rho\sigma_1\sigma_2$ (3b)

$var(P_2) = \Omega_2^2\sigma_1^2 + \Phi_2^2\sigma_2^2 + 2\Omega_2\Phi_2\rho\sigma_1\sigma_2$ (3c)

$cov(P_1P_2) = \Omega_1\Omega_2\sigma_1^2 + \Phi_1\Phi_2\sigma_2^2 + (\Omega_1\Phi_2 + \Omega_2\Phi_1)\rho\sigma_1\sigma_2$ (3d)

Since the second moments of prices vary directly with ρ, there might be cases in which r varies inversely with ρ. Differentiating equation (3a) with respect to ρ shows that dr/dρ is negative when:

$$r\left(\Omega_1\Phi_1\frac{sd(P_2)}{sd(P_1}+\Omega_2\Phi_2\frac{sd(P_1)}{sd(P_2))}\right) > \Omega_1\Phi_2 + \Omega_2\Phi_1 \qquad (4a)$$

and is otherwise positive.

Assuming symmetry, $\Omega_1 = \Phi_2$, $\Omega_2 = \Phi_1$ and $\Omega_1 = a\Omega_2$ where $a = \frac{\gamma_2+\phi_2}{\gamma_1+\phi_1} > 1$ because own elasticities exceed cross elasticities. Setting $\sigma_1 = b\sigma_2$ and substituting these assumptions into equation (4a) re-expresses the inequality condition:

$$\frac{(1+b^2)^2}{(1+a^2b^2)(a^2+b^2)} > \frac{1}{a^2} \qquad (4b)$$

For example, if a = 1.3 the inequality holds when b exceeds 2.7 (approx). For example, when $\sigma_1 = 0.06$, $\sigma_2 = 0.2$, $\Omega_1 = 1.69$ and $\Omega_2 = 1$, r varies inversely with ρ (case 1 in table). If instead $\sigma_2 = 0.06$, r varies directly with ρ as in case 2 since b = 1. As suggested by equation (4a), r tends to vary inversely with ρ when r is large as in case 1. In cases 1 and 2 the relationship between r and ρ is monotonic. However, it need not be monotonic, since r may vary inversely with ρ, but it may vary directly with ρ subsequently as r falls below its critical value in equation (4a).

Table I Paradoxical Relation between Outcome Correlations and the Correlation between Shocks

ρ	Case 1: r	Case 2: r
0	0.9895	0.7795
0.1	0.9760	0.8576
0.5	0.9600	0.8647
1	0.9425	0.8696

4. Correlation and Substitution

The correlation for prices under symmetry is:

$$r = \frac{1+b^2}{\sqrt{(1+a^2b^2)(a^2+b^2)}} \qquad (5)$$

which tends to 1 as a tends to 1, i.e. when cross elasticities equal own elasticities, and tends to $\frac{2}{1+a^2}$ when $b = 1$ ($\sigma_1 = \sigma_2$). Notice that r does not depend upon the absolute values of the Ωs, only their relative values through a. This result is an artefact induced by symmetry.

Under asymmetry matters are different because absolute values affect r. As the two goods become perfect substitutes the sums of the supply and demand elasticities, such as $\gamma_2 + \phi_2 = S$

tend to infinity. Since the asymptotic order of Ψ is 2, i.e. $\Psi \sim O(S^2)$, the asymptotic order of Ω_1 is $O(S^{-1})$. The same applies to Ω_2, Φ_1 and Φ_2. Since the asymptotic order of products is the sum of their asymptotic orders, the asymptotic orders of the variances of P_1 and P_2 and their covariance are -2. Substituting these results into equation (4a) implies:

$$r = \frac{(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)O(S^{-2})}{\sqrt{(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)^2 O(S^{-4})}} = 1 \qquad (6)$$

Therefore, when goods 1 and 2 are perfect substitutes their prices are perfectly positively correlated regardless of ρ. However, the correlation tends to be large even when the elasticities are modest and in the vicinity of 1. For example, setting $\beta_1 + \theta_1 = 2.2$, $\gamma_2 + \phi_2 = 1.8$, $\beta_2 + \theta_2 = 1.3$, $\gamma_1 + \phi_1 = 1.6$, $\sigma_1 = 0.06$ and $\sigma_2 = 0.1$ generates r = 0.968 when $\rho = 0$. If the cross elasticities β_2, θ_2, γ_1 and ϕ_1 are zero, then r = 0, as expected.

5. Conclusion

Substitution induces correlation because prices depend on mutual shocks. This correlation varies directly with the degree of substitution, and it varies inversely with the difference between own and cross elasticities of supply and demand. As these elasticities tend to infinity under perfect substitution, the correlation tends to 1. However, the correlation tends to be high even when the elasticities are in the vicinity of 1.

The correlation between prices depends also on the correlation between supply and demand shocks. Provided the former correlation is not too high, the correlation in prices varies directly, as expected, with the correlation between shocks. Matters are reversed, however, when the correlation is sufficiently high. This apparent paradox is induced by the fact that the standard deviations of prices and their covariance vary directly with the correlation between shocks when goods are substitutes. When the correlation is large, the product of the standard deviations of prices increases by more than the covariance, so that the correlation decreases instead of increases.

How concerned should we be by this paradox? We should not be concerned when goods are poor substitutes, but as they become close substitutes matters are different. If the null hypothesis is that goods are close substitutes, correlated shocks may create the misleading impression that they are not, because the price correlation is reduced by the correlation between their shocks. More generally, the paradox is unlikely to be salient when endogenous variables are weakly related. If the null hypothesis is that they should be strongly related, positive correlation between shocks may create the misleading impression that they are weakly related.

References

Moretti, E. (2011) "Local Labor Markets" in *Handbook in Labor Economics*, Elsevier Vol 4, 1237 -1313.

Pesaran, M.H. (2006) "Estimation and inference in large heterogeneous panels with a multifactor error structure" *Econometrica* **74**, 967-1012.

Pesaran, M.H and Y. Shin (1998) “Generalized impulse response analysis in linear multivariate models” *Economics Letters* **58**, 17-29.

Pryce, G. (2013) “Housing submarkets and the lattice of substitution” *Urban Studies* **50**, 2682-2699.

Ross, S.A. (1976) “The arbitrage theory of capital asset pricing” *Journal of Economic Theory* **17,** 343-362.

Schoder, J., P. Zweifel and P. Eugster (2013) “Insurers, consumers and correlated risks” *Journal of Insurance Issues* **36**, 194-222.

Sharpe, W.F. (1964) “Capital asset prices: a theory of market equilibrium under conditions of risk” *Journal of Finance* **19**, 425-442.

Zellner, A. (1962) “An efficient method of estimating seemingly unrelated regression and tests for aggregation bias” *Journal of the American Statistical Association* **57**: 348-368.