Education arms race, fertility rate and education inflation

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Abstract
There are many reasons behind the education inflation. I tried to explain it in two aspects. First, every individual family has an incentive to occupy a higher status in the education hierarchy for honor, which means there is a zero-sum education arms race game. Second, the low fertility rate intensifies the education arms race because it deprives parents of other options. The extreme case is China's one-child policy. I built a behavioral economic model based on Veblen's basic thoughts to explain how these two factors intertwine with each other and contribute to education inflation.
1. Introduction

The original motivation that drove me to build a model about education arms race and fertility rate is the obvious education inflation in China. Since the higher education expansion and education marketization from 1998, China’s graduates have increased greatly. And correspondingly, the unemployment and underpaying problem of graduates become increasingly serious (Wu, Bin and Zheng, Yongnian, 2008). In a word, there exist education inflation in China. So, I want to explain why parents continue to send their children to universities in spite of the decreased wage growth and rising youth unemployment.

People receive high education not only for higher income but also for honor. As Veblen (2017) pointed out in his masterpiece *The Theory of the Leisure Class*, high education is a way to demonstrate one’s social status. In an East Asian country with deep Confucian tradition such as China, receiving better education has always been a great honor. In fact, people’s decision often affected by others’ decision, one example is the famous “neighborhood effect”, a reference is made to interdependencies between individual decisions and the decisions and characteristics of others within a common neighborhood (Brock & Durlauf, 2002).

When we consider families as agents, however. Things become different. Parents usually compare with each other and gain some utility if they are better off than others in some aspects, such as education level of their children. But that don’t need to involve all members. As Veblen pointed out, some family members can do this for the whole family. For example, housewives usually need to perform “Vicarious Leisure” and “Vicarious Consumption” for the head of the household (Veblen, 2017).

The relationship between fertility rate and individual human capital has long been a hot topic. Qian (2009) argues that in rural China, increasing fertility rate leads to increased enrollment rates for the first-born child. Here, however, we focus on the “choosing” process. When the the family resource is limited, parents will choose one child that satisfies some standards and give that child more resource than others. In my paper, if a family has several children, it can choose the most brilliant one to perform “Vicarious Education” for the honor of the whole family.

When family size increases, more and more children free from that honor and can receive education according to their own ability, which is a good thing both for families and the whole society. But for those one-child families, they can’t choose. As a result, with lower fertility rate, especially under the one-child policy, more low-ability guys enter the college and caused a lower average ability and average productivity in college students, which leads to lower wages for college graduates.

In Section II, we present the basic model without education arms race. In section III, we add the education arms race into the model. In section IV, we consider the effects of fertility rate on education arms race. In Section V, we get several conclusions.

2. The basic model

Before we move on to models with education arms race and fertility rate, we need a basic model based on which we can make further analysis conveniently.
Consider a continuum of individuals of mass 1 characterized by heterogeneous ability $\theta$. Ability is distributed according to uniform distribution $F(\theta)$, whose density function is $f(\theta)$ over an interval $[\tilde{\theta}, \tilde{\theta}]$, where $1 \leq \theta < \tilde{\theta}$. And the critical ability level where individuals are indifferent in whether receiving higher education or not is $\theta^*$. 

Figure 1. The distribution of individuals’ ability and individuals’ choice

The probability density function is given by:

$$f(\theta) = \frac{1}{\tilde{\theta} - \theta}, \theta \in [\tilde{\theta}, \tilde{\theta}]$$  \hspace{1cm} (1)

The final productivity is influenced by education level and individual ability, $g$ represents graduate education and $u$ means the individuals are not graduates:

$$y(E, \theta) = \begin{cases} \theta, & \text{if } E = g \\ 1, & \text{if } E = u \end{cases}$$  \hspace{1cm} (2)

As a developing country, China’s education and labor market have serious asymmetry information problems. Firms can’t know individuals’ exact productivity and have to judge a worker’s potential ability and productivity according to their education level. Graduates get wage according to firms’ expectation on their productivity, which is also the average of all individuals with the same education level:

$$w(u, \theta) = y(u, \theta) = 1; w(g, \theta) = E[y(g, \theta)] = \frac{\theta^* + \tilde{\theta}}{2}$$  \hspace{1cm} (3)

As a result, for an individual, the productivity improvement and the wage improvement are not the same thing.

$$y(g, \theta) - y(u, \theta) = \theta - 1$$  \hspace{1cm} (4)

$$w(g, \theta) - w(u, \theta) = \frac{\theta^* + \tilde{\theta}}{2} - 1$$  \hspace{1cm} (5)

As Spence (1978) pointed out, as an efficient signaling tool, education cost should be inversely proportional to individual’s ability. So, we assume

$$c(\theta) = \mu \frac{\theta^{\text{average}}}{\theta}$$  \hspace{1cm} (6)

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1. The basic idea of this distribution comes from Ordine and Rose’s model (2017), I simplify it further by assuming it is a uniform distribution.

2. We can see the probability density is constant over $\theta$. This property will provide us with great simplicity in the following discussion. In other words, we use uniform distribution instead of the more realistic normal distribution for its unparallel convenience, without losing the essential stuffs in our analysis.
We assume the average ability is the critical point of whether higher education is socially optimal\(^1\). That is
\[
\theta_{\text{average}} - 1 = \mu
\] (7)

An individual will choose to get higher education when
\[
w(\theta, g) - w(\theta, u) > \mu \frac{\theta_{\text{average}}}{\theta}
\] (8)

And we know
\[
w(\theta, g) = \frac{\theta + \theta^*}{2}, w(\theta, u) = 1
\] (9)

That means an individual will choose to get high education if
\[
\frac{\theta + \theta^*}{2} - 1 \geq \mu \frac{\theta_{\text{average}}}{\theta}
\] (10)

As shown in Figure 2, in the critical point \(\theta^*\), the critical individual will be indifferent in receiving higher education because the wage increment equals the education cost.
\[
\frac{\theta + \theta^*}{2} - 1 = \mu \frac{\theta_{\text{average}}}{\theta^*}
\] (11)

Figure 2 also presents us the fraction of graduates in the population as \(g^p = \frac{\theta - \theta^*}{\theta - \overline{\theta}}\) and the social surplus of high education \(SSC = S_C = S_B - S_A\). In the following analysis in Section III, a decrease in critical ability \(\theta^*\) will lead to an increase in \(g^p\) and \(S_A\). As \(S_B\) remains constant, \(S_C\) will decrease.

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\(^1\) This is a very strong assumption, which will not necessarily be the case in real world. And we make it just for convenience. But theoretically, this critical ability point must exist somewhere and changing its positions will not alter the main conclusions in this paper.
3. The model with education arms race

As we have mentioned, high education is a symbol of social status (Veblen, 2017). Higher education usually provides individuals and their families more respect from others. But in this comparison game, the utility is determined by your relative position in a hierarchy. Because when you are respected, someone else will be despised. Under the one-child policy, each family has only one child and all children must take the responsibility of “Bring honor to the family.” We begin our analysis from one-child policy because it is the simplest case.

We assume the degree of respect for an individual is proportional to his height in the hierarchy. Here we adopt this assumption purely for simplicity. His disutility due to lower education is proportional to the proportion of people having higher education level than him. For simplicity, we assume there are only two layers in the hierarchy: graduates and undergraduates. In Figure 3, on the left there are very few graduates and many undergraduates. On the right there are many graduates and few undergraduates.

We use $r$ to measure the degree of credentialism, that is how much importance people attach to higher education in a society. If we assume the proportion of graduates is $a$, the undergraduates’ proportion will be $(1-a)$. For a graduate, he can get a utility of $(1-a)r$. Then the total utility of graduates will be $(1-a)ra$. Similarly, an undergraduate will get a disutility $-ar$, the total negative utility will be $-ar(1-a)$. We can find $(1-a)ra-ar(1-a)=0$. No matter what $a$ is, it is always a zero-sum game.

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$$U(E, \theta) = w(E, \theta) + R(E)$$

(12)

$$R(E) = \begin{cases} (1-a)r, & \text{if graduated} \\ -ar, & \text{if undergraduate} \end{cases}$$

(13)

When an individual wants to decide whether or not receiving higher education, he just needs to compare:

$$U(g, \theta) - U(u, \theta) = [w(g, \theta) + R(g)] - [w(u, \theta) + R(u)]$$

(14)

$$U(g, \theta) - U(u, \theta) = \frac{\theta+\theta^*}{2} - 1 + r$$

(15)

So, the honor of high education is just like a fixed number of bonus. When an individual find receiving higher education can give him higher utility, he will choose to receive high education. So, changes before and after the education arms race are shown in the following Table1 and
Table 1 Changes due to education arms race

<table>
<thead>
<tr>
<th></th>
<th>Critical ability</th>
<th>Wage for graduates</th>
<th>Total graduates’ number</th>
<th>Social surplus changes due to high education</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before</td>
<td>$\theta^{*o}$</td>
<td>$w_o$</td>
<td>$\bar{\theta} - \theta^{*o}$</td>
<td>$\bar{\theta} - \theta$</td>
</tr>
<tr>
<td>After</td>
<td>$\theta^{*1}$</td>
<td>$w_a$</td>
<td>$\bar{\theta} - \theta^{*1}$</td>
<td>$\bar{\theta} - \theta$</td>
</tr>
<tr>
<td>Change</td>
<td>Decrease</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
</tr>
</tbody>
</table>

Figure 4. Education arms race and social welfare loss

4. Vicarious education and education arms race

If we loosen the one-child policy and allow each family to have more children, things will be different. We assume a family’s education level is determined by its member with highest education level. In that situation, a family with more children will not need to support all the children’s college education. The more practical way is to support the most brilliant child if the pride and economic return can compensate the education cost, that is “vicarious education”, which is derived from Veblen’s two famous conception: “vicarious leisure” and “vicarious consumption” (Veblen, 2017). So, the utility of those participants in education arms race will
be affected by his family’s position in the education hierarchy, which is determined by his own education level.¹

Now let’s consider if one family has more than one child. Then what will happen? As I assumed above, families will only compare their children’s highest education level. So, families can choose to support the smartest one to receive education and enjoy the pride and economic outcome.

If each family has 2 children, then for an individual with ability \( \theta \), the probability that he is the smarter one is \( \frac{\theta - \bar{\theta}}{\bar{\theta} - \tilde{\theta}} \).² If each family has 3 children, then that possibility should be \( \left(\frac{\theta - \bar{\theta}}{\bar{\theta} - \tilde{\theta}}\right)^2 \).³ For the same reason, in a \( n \)-children family, the possibility that he is smartest should be \( \left(\frac{\theta - \bar{\theta}}{\bar{\theta} - \tilde{\theta}}\right)^{n-1} \). We define that possibility as \( g_n(\theta) \). Obviously, \( g_1(\theta) = 1 \), because one-child families don’t have other choices.

We can use \( f(\theta)g_n(\theta) \) to represent the possibility that an individual has \( \theta \) possibility and is the smartest one in his family. We can see the differences with different \( n \) in Figure 5.

We can find from \( f(\theta)g_1(\theta) \) to \( f(\theta)g_2(\theta), f(\theta)g_3(\theta), \) and \( f(\theta)g_4(\theta) \), the lower area will be smaller and smaller. What’s more, the left part of the lower area decreases much faster. This means much fewer low-ability individuals will participate in the education arms race as each family has more children. When \( n \) is large enough, the low-ability individuals in education arms race will be so few that they affect others very little.

![Figure 5 The participants in education arms race if each family has n children](image)

For the convenience of further discussion. We define \( \theta^{n*} \) as the critical ability in non-participants, \( \theta^{p*} \) as the critical ability in participants, \( g^p \) as the graduates’ number in participants, \( g^n \) as the graduates’ number in non-participants, \( g = g^p + g^n \) as total graduates’ number.⁴ “Participants” means participants in education arms race. “Non-participants” means individuals not in education arms race. SSC means social surplus change due to high education.

¹ Notice here we refer to the family’s position in the hierarchy, not the individuals’ position. This ensures the zero-sum game. If we assume the proportion of “graduate families” is \( a \), the “undergraduate families’ proportion” will be \( (1-a) \).

² Similarly, we can get \( (1-a)ra-ar(1-a)=0 \).

³ We can also see it as the possibility that the other one is not as smart as him.

⁴ That means the other two are not as smart as him.

⁴ As all individuals are assumed to be mass 1, \( g \) is also the proportion of graduates in all individuals.
As Figure 6 shows, when the degree of credentialism $r$ is different, the outcome can be different. The light-grey area represents graduates in participants. The deep-grey area represents graduates in non-participants. When $r$ is very large, all the participants in arms race will receive high education. And when $r$ is not so large, not all participants will receive high education. So, we need to discuss them separately. We assume each family has $n$ children.

Then we have

$$\theta^{\text{average}} = \frac{\bar{\theta} + \theta}{2}$$  \hspace{2cm} (16)

$$\mu = \frac{\theta^{\text{average}}}{2} - 1$$ \hspace{2cm} (17)

$$g^n = \int_{\theta^{n*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left[ 1 - \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} \right] d\theta$$ \hspace{2cm} (18)

$$w(g, \theta) = \frac{\int_{\theta^{p*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} \theta d\theta + \int_{\theta^{n*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left[ 1 - \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} \right] \theta d\theta}{g^n + g^p}$$ \hspace{2cm} (19)

$$\frac{\mu \theta^{\text{average}}}{\theta^{n*}} = w(g, \theta) - 1$$ \hspace{2cm} (20)

When all participants receive high education, we can insert $\theta^{p*} = \bar{\theta}, g^p = \frac{1}{n}$ into (19). Then from (16)(17)(18)(19)(20), we can get $\theta^{n*}, g^n$ and $w(g, \theta)$.

When $r$ is not so large and not all participants receive higher education,

$$g^p = \int_{\theta^{p*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} d\theta$$ \hspace{2cm} (21)

And we know, for the critical participant,

$$\frac{\mu \theta^{\text{average}}}{\theta^{p*}} = w(g, \theta) - 1 + r$$ \hspace{2cm} (22)

From (16)(17)(18)(19)(20)(21)(22), we can get $\theta^{p*}, g^p, \theta^{n*}, g^n$ and $w(g, \theta)$.

Finally, we can find the social surplus change due to high education:

$$\text{SSC} = \int_{\theta^{n*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left[ 1 - \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} \right] \left( \theta - 1 - \frac{\mu \theta^{\text{average}}}{\theta} \right) d\theta$$

$$+ \int_{\theta^{p*}}^{\bar{\theta}} \frac{1}{\bar{\theta} - \theta} \left( \frac{\theta - \theta}{\bar{\theta} - \theta} \right)^{n-1} \left( \theta - 1 - \frac{\mu \theta^{\text{average}}}{\theta} \right) d\theta$$ \hspace{2cm} (23)
Table 2 Calibration Results ($\bar{\theta} = 2, \tilde{\theta} = 6, \theta^{average} = 4, \mu = 3, c(\theta) = \frac{12}{\theta}$)

<table>
<thead>
<tr>
<th>r</th>
<th>NAR</th>
<th>0</th>
<th>n/a</th>
<th>3.292</th>
<th>4.646</th>
<th>n/a</th>
<th>0.677</th>
<th>0.677</th>
<th>0.668</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>n/a</td>
<td>4</td>
<td>1</td>
<td>n/a</td>
<td>1</td>
<td>-0.296</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>2</td>
<td>3.386</td>
<td>4.544</td>
<td>0.5</td>
<td>0.214</td>
<td>0.714</td>
<td>0.563</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1/3</td>
<td>2</td>
<td>3.309</td>
<td>4.626</td>
<td>0.333</td>
<td>0.351</td>
<td>0.684</td>
<td>0.649</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1/4</td>
<td>2</td>
<td>3.296</td>
<td>4.641</td>
<td>0.25</td>
<td>0.429</td>
<td>0.679</td>
<td>0.664</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2.325</td>
<td>4.163</td>
<td>n/a</td>
<td>0.919</td>
<td>n/a</td>
<td>0.919</td>
<td>0.061</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>2.164</td>
<td>3.384</td>
<td>4.546</td>
<td>0.499</td>
<td>0.214</td>
<td>0.713</td>
<td>0.566</td>
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<tr>
<td>3</td>
<td>1/3</td>
<td>2.133</td>
<td>3.309</td>
<td>4.627</td>
<td>0.333</td>
<td>0.351</td>
<td>0.684</td>
<td>0.649</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1/4</td>
<td>2.127</td>
<td>3.296</td>
<td>4.641</td>
<td>0.250</td>
<td>0.429</td>
<td>0.679</td>
<td>0.664</td>
<td></td>
</tr>
</tbody>
</table>

It will be difficult for us to compare directly, so we just need to set $\bar{\theta} = 2, \tilde{\theta} = 6^1$. Then we can get $\theta^{average} = 4, \mu = 3, c(\theta) = \frac{12}{\theta}$. Table 2 is the results. We choose $r = 2$ and $r = 4$ to establish the two cases in Figure 6. In this model, $r = 4$ is high enough because it forces all participants to receive high education, which is an extreme case in real world. And the no education arms race (NAR or $r=0$) is the benchmark, it represents the lowest level of education inflation and highest social surplus. Here $n$ is the fertility rate, or the number of children in each family. We just set $n$ value as 1,2,3,4 because those ferties are common and $n=4$ is high enough to explain our problem. PP means the participate proportion, or the fraction of children that take part in the education arms race. Other variables have already been explained.

We can find many interesting things from Table 2.

First, education arms race actually reduces social welfare by enhance the unnecessary education inflation. In table 2, under education arms race, no matter what value $n$ and $r$ is, we can find the SSC will be smaller than the NAR benchmark. So, although it is a zero-sum game by itself, education arms race costs real social resources and reduces social welfare. It does so by increasing education inflation. We can find the graduate proportion $g$ will be larger than the benchmark group. That means those low-ability individuals, who would not receive high education otherwise, are forced to receive it by education arms race.

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1 The value of $\bar{\theta}$ means that, if the worker with lowest ability receive high education, how many times of undergraduate productivity they can provide. Of course, this ratio is only a rough estimation. It varies from country to country and changes all the time. As we have assumed $\bar{\theta} > 1$ and a too large $\bar{\theta}$ will seem unrealistic, here we set that ratio as 2. This means, for example, in China, if an undergraduate worker with the lowest ability create 2000 RMB value per month, he will create 4000 RMB value per month if he receives high education. $\bar{\theta}$ can be interpreted similarly. According to our assumptions, if we set $\bar{\theta}$ and $\tilde{\theta}$, then $\theta^{average}, \mu, c(\theta)$ are also fixed.
Second, the impact of education arms race is greatly related to fertility rate. We can see that very clearly if we notice the case when \( n = 4 \). We can find when \( n=4 \), no matter \( r=2 \) or \( r=4 \), \( g \) values are the same (0.679) and SSC values are the same (0.664). And they are very close to the benchmark value (\( g=0.677; \) SSC=0.668). That means under sufficiently high fertility rate, education arms race for honor will only distort the outcome from NAR case only a little. In other words, under high fertility rate, education arms race is not a serious problem. But the real case is just the opposite, especially under China’s one-child policy, the distortion reaches its maximum level. We can find when \( n=1 \), \( g \) will be very large (When \( r=4 \), \( g=1 \); When \( r=2 \), \( g=0.919 \)) and SSC will be very small (When \( r=4 \), SSC=−0.296; When \( r=2 \), SSC=0.061).

Third, the fertility rate has “diminishing return”. That is, when we increase the fertility rate from a very low level (\( n=1 \)), the distortion effect reduce dramatically (When \( r=4 \), \( g_{n=2} - g_{n=1} = 0.714 - 1 = -0.286; \) \( SSC_{n=2} - SSC_{n=1} = 0.563 - (-0.296) = 0.859 \)); When \( r=2 \), \( g_{n=2} - g_{n=1} = 0.713 - 0.919 = -0.206; \) \( SSC_{n=2} - SSC_{n=1} = 0.566 - 0.061 = 0.505 \)). But if the fertility rate is already high (\( n=3 \)) and the distortion is already sufficiently low, increasing fertility rate will not make much difference any more (When \( r=4 \), \( g_{n=4} - g_{n=3} = 0.679 - 0.684 = -0.005; \) \( SSC_{n=4} - SSC_{n=3} = 0.664 - 0.649 = 0.015 \)); When \( r=2 \), \( g_{n=4} - g_{n=3} = 0.679 - 0.684 = -0.005; \) \( SSC_{n=4} - SSC_{n=3} = 0.015 \)). In a word, most distortion under one-child policy can be overcome by just one additional kid.

5. Conclusions

Through our analysis, we our main conclusions.

First, the education arms race, which is a zero-sum game itself, exacerbate the inefficient signaling problem of high education and lowers the average productivity of graduates. Of course, this means more social welfare loss and lower wage for graduates.

Second, higher fertility rate can reduce the distortion due to education arms race dramatically regardless the degree of credentialism. Even in a society with extremely high credentialism, if we allow each family to choose the smartest child as the “proxy” to receive “vicarious education”, then we can reduce the impact of education arms race to such an acceptable level. However, under the low fertility rate, especially under the one-child policy in China, too many students are involved in education arms race. The result is high education inflation, lower wage for graduates and a social welfare loss.

Third, to overcome the education arms race problem, we don’t need to increase the fertility rate too much. Under the “vicarious education” assumption, just two children each family can mitigate that problem greatly. In other words, that problem will remain moderate when the fertility rate is above 2. But if the fertility rate continues to decrease (which is also a big problem for most developed economies), that problem will become serious suddenly and in an increasing speed.

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References


