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Present bias and endogenous fiscal deficits: Revisiting Woo (2005)

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Abstract

This study examines a dynamic game of governmental budgeting by introducing policymakers' dynamically inconsistent preferences with present bias (i.e., quasi-hyperbolic discounting) into the game considered by Woo (2005, "Social polarization, fiscal instability and growth," European Economic Review, 49, 1451-1477). Under a condition with a plausible economic interpretation, we show that our game has the same non-cooperative equilibrium as that of a discrete-time version of Woo (2005) in which two policymakers have dynamically consistent preferences (i.e., exponential discounting). This result suggests that when analyzing endogenous fiscal deficits, it is not too restrictive to assume that the policymakers' discounting is exponential.

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1. Introduction

Various studies have sought to explain why some governments generate excessive debts and deficits (see, e.g., Alesina and Passalacqua [2016]). Among them, Woo (2005) extends Velasco's (1999) dynamic game to show that excessive fiscal debts and deficits arise from the polarized preferences of policymakers regarding the composition of government spending, and, due to political instability, the fiscal situation becomes more volatile when policymakers are less patient. These results are broadly consistent with Woo's (2003a, 2003b) empirical findings.

Numerous studies in macroeconomic theory, including Woo (2005), deal with intertemporal optimization, assuming that optimizing agents have exponential (or geometric) discounting functions; that is, people discount future utility at a constant rate. However, experimental and field research on intertemporal choices suggests that discounting is *hyperbolic* rather than exponential; that is, people tend to discount gains in the more distant future at a lower rate.[1] A serious problem triggered by hyperbolic discounting is *dynamic* (or *time*) *inconsistency*: If no commitment is enforceable, people with hyperbolic discounting tend to revise plans they previously made (e.g., saving for the future) for immediate gratification.

This article revisits Woo (2005) and assumes that self-interested policymakers (hereafter, "ministers") have dynamically inconsistent preferences with present bias. Specifically, we present a dynamic game by modifying that of Woo (2005) to include two policymakers with *quasi-hyperbolic* (or *quasi-geometric*) discounting.[2] As a result, we prove that under a condition that has a plausible economic interpretation, a non-cooperative equilibrium of our game is identical to that of a discrete-time version of Woo (2005) with exponential discounting. Therefore, the propositions shown by Woo (2005) continue to hold even in the present game, with no essential change. This result suggests that the discounting method used by policymakers is not necessarily relevant to endogenous fiscal deficits.

In the literature on the equivalence between exponential and hyperbolic discounting, Strulik (2015) finds that, under a plausible restriction, the two discounting methods give the same equilibrium growth rate in a standard Ak model of endogenous growth (see also Barro [1999] and Cabo, *et.al* [2015], among others). However, unlike the current research, the studies that focus on economic growth do not deal with strategic interdependence among optimizing agents or interpersonal externality. This study contributes to the literature by proving that a *strong equivalence* as defined by Strulik (2015) can hold even in our dynamic game.

[1] See, for example, Loewenstein and Thaler (1989), Ainslie (2001), Ikeda (2016).

[2] In the literature, quasi-hyperbolic discounting has frequently been used as a simplification of hyperbolic discounting. See, for example, Laibson (1997) and O'Donoghue and Rabin (1999).

The rest of this article is organized as follows: Section 2 sets up our model based on Woo (2005). Section 3 describes our solution concept and solves the model analytically. Section 4 provides the main result, and Section 5 concludes. Mathematical details are explained in appendices.

2. Model

Following Velasco (1999) and Woo (2005), we suppose that two ministers "share" the government budget, from which each can choose to fund their preferred items, and both ministers have an infinite time horizon. Time is discrete and denoted by $t \in \{0, 1, 2, \ldots, \infty\}$.

2.1. Governmental budget equation

The government provides public good A (e.g., public education) and public good B (e.g., national defense) using lump-sum tax revenue and issuance of public bonds. Public goods are assumed to be non-storable. Let b_t denote the outstanding stock of public bonds at the beginning of period t. The initial value, b_0, is exogenously given. The evolution of b_t follows

$$b_{t+1} = Rb_t + g_{A,t} + g_{B,t} - T, \tag{1}$$

where $g_{i,t}$ (> 0) and T (> 0) are government spending in period t for providing public goods $i \in \{A, B\}$ and the lump-sum tax revenue, respectively. As in Woo (2005), we suppose that T remains constant over time. In Eq. (1), $R \in (1, 2)$ denotes the gross rate of interest, which is assumed to be exogenous.

We suppose that in period t, $g_{i,t}$ is determined by minister i who takes b_t, R, and T as given.

2.2. Preference of minister i

We assume that minister i's instantaneous utility function is given by

$$u_{i,t} = \alpha_i \ln g_{i,t} + (1 - \alpha_i) \ln g_{j,t},$$

where $i, j \in \{A, B\}$, $j \neq i$, and $\alpha_i \in [0, 1]$ is a constant. As in Velasco (1999) and Woo (2005), the logarithmic utility is assumed for obtaining an explicit solution. We suppose that $\alpha_i \geq 1/2$ for each i; that is, minister i prefers public good i to public good j. We allow for $\alpha_A \neq \alpha_B$; that is, the preferences of the two ministers can be asymmetric.

We define $\theta := \alpha_i - (1 - \alpha_j)$, which reflects the degree of social polarization of preferences for the public goods. Note that $0 \leq \theta \leq 1$. If $\theta = 1$ (i.e., $\alpha_i = 1$ for each i), minister i does not value public good j at all, whereas no conflict exists between the two ministers if $\theta = 0$ (i.e., $\alpha_i = 1/2$ for each i).

In considering the dynamic optimization for minister i, we assume that the objective function that minister i seeks to maximize in period t, $U_{i,t}$, is given by

$$U_{i,t} = u_{i,t} + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} u_{i,t+\tau}.$$

where $\delta \in (0,1)$ is a long-run discount *factor* and $\beta \in (0,1)$ denotes the degree of *present bias*. We suppose that the two ministers are homogeneous with respect to time discounting, following Woo (2005).

We notice that when $\beta \in (0,1)$, the preference of each minister is dynamically inconsistent and exhibits present bias: In period t, the discount factor between $t+1$ and $t+2$ is δ; however, in period $t+1$, it changes from δ to $\beta\delta$ ($< \delta$). Therefore, in period $t+1$, each minister is urged to revise the spending schedule made in period t for immediate gratification. Given this, we assume the following:

Assumption 1: (i) No commitment is enforceable. (ii) None of the ministers is "naïve," and both are "sophisticated" decision makers; that is, each minister takes into account that the "future selves" would revise the spending schedule made by the "current selves."

Clearly, if $\beta = 1$ (i.e., exponential discounting), as assumed in Woo (2005), the discount factor between t and $t+1$ is δ for every t; that is, both ministers have dynamically consistent preferences.

2.3. Governmental net wealth

For the analysis in the following sections, we define

$$w_t := \frac{T}{R-1} - b_t,$$

where $T/(R-1)$ denotes the discounted sum of future tax revenue at the beginning of period t; that is, $\sum_{\tau=1}^{\infty} TR^{-\tau}$. In this note, we refer to w_t as *governmental net wealth* at the beginning of period t. We assume that the government is always subject to a solvency constraint (or debt limit); that is, $w_t \geq 0$ for each t.[3]

With the definition of w_t, we can rewrite Eq. (1) as

$$w_{t+1} = Rw_t - g_{A,t} - g_{B,t}. \tag{2}$$

[3]We suppose the following: If $w_t = 0$ at a certain time $t = \bar{t} < \infty$, then $g_{A,t} = g_{B,t} = 0$ for all $t > \bar{t}$. Then, because $u_{i,t} \to -\infty$ as $g_{i,t} \to 0$, the net wealth of the government, w, is never "exhausted" in finite time, as long as $w_0 > 0$. This will be verified in Section 3.

3. Solving the model

Regarding the decision rule (or strategy) of the ministers, we assume the following:

Assumption 2: Minister i employs a feedback (or Markov) strategy, $g_{i,t} = \phi_i(w_t)$; in other words, the government spending in period t for providing the public good, $g_{i,t}$, is conditioned only on the current value of governmental net wealth, w_t.

We note that sophisticated minister i takes into account the decisions of his/her future selves, $\phi_i(\cdot)$, as well as those of the opponent (including opponent's future selves), $\phi_j(\cdot)$. Therefore, the optimization problem for minister i is formulated as follows:

$$V_i(\phi_A(w), \phi_B(w)) = \max_{g_i}[\alpha_i \ln g_i + (1-\alpha_i)\ln \phi_j(w) + \beta\delta W_i(w')], \tag{3}$$

where

$$w' = Rw - g_i - \phi_j(w).$$

Let $\hat{\phi}_i(w)$ denote the solution of this problem. The value function, $W_i(\cdot)$, in Eq. (3), satisfies:

$$W_i(w) = \alpha_i \ln \phi_i(w) + (1-\alpha_i)\ln \phi_j(w) + \delta W_i(\tilde{w}'), \tag{4}$$

where

$$\tilde{w}' = Rw - \phi_i(w) - \phi_j(w).$$

The non-cooperative equilibrium of the present game is defined as follows:

Definition 1: A pair of feedback strategies, $\{\phi_A^*(w), \phi_B^*(w)\}$, constitutes a *feedback Nash equilibrium* of the present game if and only if: (i) For each i and for every possible w, $\phi_i^*(w) = \hat{\phi}_i(w)$, and (ii) For each i and every possible w, minister i cannot become better off by deviating from $\phi_i^*(\cdot)$ to any other $\phi_i(\cdot)$, taking $\phi_j^*(\cdot)$ as given;

$$V_i(\phi_i^*(w), \phi_j^*(w)) \geq V_i(\phi_i(w), \phi_j^*(w)),$$

where $i, j \in \{A, B\}$ and $j \neq i$.

With this definition, we can show the following:

Proposition 1: Suppose that for each i, $\phi_i(w_t)$ is linear in w_t. Then, there exists a symmetric feedback Nash equilibrium, $\{\phi_A^*(w_t), \phi_B^*(w_t)\}$, where

$$\phi_i^*(w_t) = \frac{\alpha_i R(1-\delta)}{(1+\theta)(1-\delta)+\beta\delta} w_t, \quad i \in \{A, B\}. \tag{5}$$

Proof. See Appendix 1. ‖

To simplify the mathematical expression, we define the long-term subjective discount *rate* of each minister, ρ, as

$$\rho := \frac{1}{\delta} - 1, \tag{6}$$

and assume that $\rho \geq R - 1$.

Using Eq. (6), we rewrite the equilibrium strategy of minister i, Eq. (5), as

$$g_{i,t} = \frac{\alpha_i R \rho}{(1+\theta)\rho + \beta} w_t. \tag{7}$$

By plugging Eq. (7) into Eq, (2), we can find that the equilibrium dynamics of w_t follow

$$w_{t+1} = \frac{\beta R}{(1+\theta)\rho + \beta} w_t. \tag{8}$$

From Eq. (8), it follows that for any t,

$$0 < \frac{w_{t+1}}{w_t} = \frac{\beta R}{(1+\theta)\rho + \beta} < 1,$$

where we use $\beta \in (0,1)$, $\rho \geq R - 1$, and $1 + \theta \geq 1$. Therefore, irrespective of the initial value of governmental net wealth, w_0, we have $w_t \to 0$ (i.e., $b_t \to T/(R-1)$) as $t \to \infty$, which implies the persistent accumulation of public debts.

4. An equivalence result

We note that if $\beta = 1$ (i.e., exponential discounting) and $\rho = \varrho := R - 1 + p$, then our model is essentially the same as that of Woo (2005) without the private sector, where $p \in [0,1)$ is a constant reflecting the political risk of the ministers being removed from office.

In that case, the equilibrium linear strategy of minister i is derived as

$$g_{i,t} = \frac{\alpha_i R \varrho}{(1+\theta)\varrho + 1} w_t. \tag{9}$$

See Appendix 2 for details. Therefore, from Eqs. (2) and (9), the change in governmental net wealth, w_t, is given by

$$w_{t+1} = \frac{R}{(1+\theta)\varrho + 1} w_t. \tag{10}$$

From Eq. (10), we can verify that unless $(\theta, p) = (0, 0)$, $w_{t+1} < w_t$ for any t, which implies that $w_t \to 0$ (i.e., $b_t \to T/(R-1)$) as $t \to \infty$, irrespective of the w_0 given initially; that is, persistent fiscal deficits arise endogenously. As noted by Woo (2005, p.1462), if there is no political risk of losing office (i.e., $p = 0$), then the disagreement between ministers about the ideal composition of government spending (i.e., $\theta > 0$ in the present model) is crucial for endogenous fiscal deficits.

Hereafter, we focus on the pairs (β, ρ) such that:

Assumption 3: Two discounting methods (i.e., exponential and quasi-hyperbolic discounting) yield the same present value for any infinite stream of constant gains; that is,

$$\sum_{\tau=1}^{\infty} \frac{1}{(1+\varrho)^{\tau}} = \sum_{\tau=1}^{\infty} \frac{\beta}{(1+\rho)^{\tau}}. \tag{11}$$

Following Strulik (2015), we define the concept of strong equivalence as:

Definition 2: In the present model, exponential and quasi-hyperbolic discounting are *strongly equivalent*, if both give identical equilibrium paths of governmental net wealth, w, under Assumption 3.

From Eq. (11), we have

$$\varrho = \frac{\rho}{\beta}. \tag{12}$$

Plugging Eq. (12) into Eqs. (9) and (10) replicates Eqs. (7) and (8), respectively; that is, under Assumption 3 or Eq. (12), Eqs. (7) and (8) are equivalent to Eqs. (9) and (10), respectively.

Therefore, we have:

Proposition 2: Under Assumption 3, even if the two ministers have dynamically inconsistent preferences (i.e., $0 < \beta < 1$), the present game has the same feedback Nash equilibrium as that of a discrete-time version of Woo (2005) in which each minister has dynamically consistent preferences (i.e., $\beta = 1$).

That is, Proposition 2 provides a strong equivalence between exponential and quasi-hyperbolic discounting.

Furthermore, when $\rho = R - 1$, we have

$$\varrho = \frac{\rho}{\beta} \Leftrightarrow R - 1 + p = \frac{R-1}{\beta} \Leftrightarrow p = \left(\frac{1}{\beta} - 1\right)(R-1), \tag{13}$$

which states that the "political uncertainty" in Woo's (2005) model can be understood as policymakers' present-biased preferences: Under Eq. (13), the greater risk that ministers will lose office (i.e., the larger p) in Woo's (2005) model corresponds with the stronger present bias (i.e., the smaller β) in our model, for a given interest rate, $R-1$.

5. Concluding remarks

This article revisits Woo (2005) using a dynamic game of governmental budgeting in which, contrary to Woo (2005), policymakers have dynamically inconsistent preferences with present bias. As a result, we proved that under Eq. (11) (or Eq. (12)), the present game has the same non-cooperative equilibrium as that of a discrete-time version of Woo's (2005) game. In this sense, Woo (2005) has already considered the case in which policymakers' preferences are dynamically inconsistent, probably without intending to do so. Our result suggests that when analyzing endogenous fiscal deficits, it is not too restrictive to assume that the policymakers' discounting is exponential, given the strong equivalence between exponential and quasi-hyperbolic discounting.

In future research, we will examine the robustness of our present result because it was obtained under certain specific conditions (e.g., logarithmic utility). It would be interesting to extend our dynamic game to include heterogeneity of ministers' time preferences. Although we assumed that policymakers are "sophisticated," it is also worthwhile to consider "naïve" policymakers who mistakenly believe that they can commit to a plan made at the present time.

Appendix 1

We derive the equilibrium strategy of minister i, Eq (5), following Futagami and Nakabo (2018).

The first-order condition for the maximization appearing in Eq. (3) is

$$\frac{\alpha_i}{g_i} + \beta\delta \frac{\mathrm{d}}{\mathrm{d}g_i} W_i(w') = 0. \tag{1.1}$$

We use a "guess-and-verify" method to find a solution (see, for example, Adda and Cooper [2003, Ch.2]). For each $i \in \{A, B\}$, we guess that the strategy is linear in w, and that

$$W_i(w) = p_i + q_i \ln w, \tag{1.2}$$

where we need to solve for the unknown constants p_i and q_i. From Eq. (1.1), together with Eq. (1.2), we have

$$\frac{\alpha_A}{g_A} = \frac{\beta\delta q_A}{Rw - g_A - g_B} \tag{1.3}$$

for minister A, while

$$\frac{\alpha_B}{g_B} = \frac{\beta\delta q_B}{Rw - g_A - g_B} \tag{1.4}$$

for minister B. Solving Eqs. (1.3) and (1.4) simultaneously for g_A and g_B yields

$$g_i = \frac{\alpha_i R\beta\delta q_j}{(\beta\delta q_i + \alpha_i)(\beta\delta q_j + \alpha_j) - \alpha_i\alpha_j} w,$$

where $i, j \in \{A, B\}$ and $i \neq j$. Namely, the strategies take the form of $\phi_i(w) = \sigma_i w$ where σ_i is a constant to be determined;

$$\sigma_i := \frac{\alpha_i R\beta\delta q_j}{(\beta\delta q_i + \alpha_i)(\beta\delta q_j + \alpha_j) - \alpha_i\alpha_j}. \tag{1.5}$$

For each i, by substituting Eq. (1.2) and $\phi_i(w) = \sigma_i w$ into Eq. (4), we obtain

$$p_i + q_i \ln w = \alpha_i \ln\sigma_i + (1-\alpha_i)\ln\sigma_j + \delta p_i + \delta q_i \ln(R - \sigma_i - \sigma_j) + (1 + \delta q_i)\ln w. \tag{1.6}$$

Because Eq. (1.6) should hold for every w, by comparing the coefficients of $\ln w$ on both sides of Eq. (1.6), we have

$$q_i = \frac{1}{1-\delta} \tag{1.7}$$

for each i.

Plugging Eq. (1.7) into Eq. (1.5) and using $1 + \theta := \alpha_i + \alpha_j$ yield

$$\sigma_i = \frac{\alpha_i R(1-\delta)}{(1+\theta)(1-\delta) + \beta\delta} (=: \sigma_i^*)$$

for each i. Therefore, we have the equilibrium strategy of minister i as $g_i = \sigma_i^* w$, or Eq (5), which leads to Proposition 1.

With regard to Proposition 1, we note the following:

Corollary to Proposition 1: For a given governmental net wealth, w, each minister is urged to spend more when the present bias of the ministers becomes stronger (i.e., $\beta \downarrow$).

This claim is intuitively plausible, and follows immediately from the fact that σ_i^* is strictly decreasing in the degree of present bias, β.

Appendix 2

To consider Woo's (2005) case where $\beta = 1$ and $\rho = R - 1 + p$ $(=: \varrho)$, we define a pair of feedback strategies, $\{\phi_A(w), \phi_B(w)\}$, as a feedback Nash equilibrium if and only if, given $\phi_j(\cdot)$, minister i cannot become better off by deviating from $\phi_i(\cdot)$ for each i and every possible w, where $i, j \in \{A, B\}$ and $i \neq j$.

Below, we derive the equilibrium strategy of minister i given in Eq. (9) using the method of dynamic programming.

For minister i, consider a function $J_i(w)$ that satisfies the Bellman equation:

$$J_i(w) = \max_{g_i} \left[\alpha_i \ln g_i + (1 - \alpha_i) \ln \phi_j(w) + \frac{1}{1+\varrho} J_i(w') \right], \tag{2.1}$$

where

$$w' = Rw - g_i - \phi_j(w).$$

The first-order condition for the maximization appearing in Eq. (2.1) is

$$\frac{\alpha_i}{g_i} + \frac{1}{1+\varrho} \cdot \frac{\mathrm{d}}{\mathrm{d}g_i} J_i(w') = 0. \tag{2.2}$$

As in Appendix 1, for each i, we guess that the strategy is linear in w, and that

$$J_i(w) = r_i + s_i \ln w, \tag{2.3}$$

where r_i and s_i are undetermined constants. From Eq. (2.2), together with Eq. (2.3), we have

$$\frac{\alpha_A}{g_A} = \frac{1}{1+\varrho} \cdot \frac{s_A}{Rw - g_A - g_B} \tag{2.4}$$

for minister A, while

$$\frac{\alpha_B}{g_B} = \frac{1}{1+\varrho} \cdot \frac{s_B}{Rw - g_A - g_B} \tag{2.5}$$

for minister B. Solving Eqs. (2.4) and (2.5) simultaneously for g_A and g_B yields

$$g_i = \frac{\alpha_i(1+\varrho)Rs_j}{(1+\varrho)\alpha_i s_j + (1+\varrho)\alpha_j s_i + s_i s_j} w,$$

where $j \in \{A, B\}$, $i \neq j$. Namely, the strategies take the form of $\phi_i(w) = \xi_i w$ where ξ_i is a constant to be determined;

$$\xi_i := \frac{\alpha_i(1+\varrho)Rs_j}{(1+\varrho)\alpha_i s_j + (1+\varrho)\alpha_j s_i + s_i s_j}. \tag{2.6}$$

For each i, by substituting Eq. (2.3) and $\phi_i(w) = \xi_i w$ into Eq. (2.1), we obtain

$$r_i + s_i \ln w = \mathcal{C}_i + \left(1 + \frac{s_i}{1+\varrho}\right) \ln w, \tag{2.7}$$

where

$$\mathcal{C}_i := \alpha_i \ln \xi_i + (1-\alpha_i) \ln \xi_j + \frac{r_i}{1+\varrho} + \frac{s_i}{1+\varrho} \ln(R - \xi_i - \xi_j).$$

Because Eq. (2.7) should hold for every w, by comparing the coefficients of $\ln w$ on both sides of Eq. (2.7), we have

$$s_i = \frac{1+\varrho}{\varrho} \tag{2.8}$$

for each i.

Plugging Eq. (2.8) into Eq. (2.6) and using $1+\theta := \alpha_i + \alpha_j$ yields

$$\xi_i = \frac{\alpha_i R \varrho}{(1+\theta)\varrho + 1} (=: \xi_i^*).$$

Therefore, in Woo's (2005) case, we have the equilibrium strategy of minister i as $g_i = \xi_i^* w$, or Eq. (9).

References

Adda, J. and R. Cooper, (2003) *Dynamic Economics*, The MIT Press.

Ainslie, G. (2001) *Breakdown of Will*, Cambridge University Press.

Alesina, A. and A. Passalacqua, (2016) "The political economy of government debt," in J. Taylor and H. Uhlig (ed.), *Handbook of Macroeconomics 2*, 2599-2651, North Holland.

Barro, R. J. (1999) "Ramsey meets Laibson in a neo-classical growth model," *Quarterly Journal of Economics*, 114, 1125-1152.

Cabo, F., G. Matín-Herrán, and M-P. Martínez-García, (2015) "Non-constant discounting and *Ak*-type growth models," *Economics Letters*, 131, 54-58.

Futagami, K. and Y. Nakabo, (2018) "Capital accumulation game with quasi-geometric discounting and consumption externalities," No. 1811, Discussion Papers in Economics and Business, Osaka University.

Ikeda, S. (2016) *The Economics of Self-Destructive Choices*, Springer.

Laibson, D. (1997) "Golden eggs and hyperbolic discounting," *Quarterly Journal of Economics*, 112, 443-478.

Loewenstein, G. and R. Thaler, (1989) "Anomalies: Intertemporal choice," *Journal of Economic Perspectives*, 3, 181-193.

O'donoghue, T. and M. Rabin, (1999) "Doing it now or later," *American Economic Review*, 89, 103-124

Strulik, H. (2015) "Hyperbolic discounting and endogenous growth," *Economics Letters*, 126, 131-134.

Woo, J. (2003a) "Economic, political, and institutional determinants of public deficits," *Journal of Public Economics*, 87, 387-426.

Woo, J. (2003b) "Social polarization, industrialization, and fiscal instability: theory and evidence," *Journal of Development Economics*, 72, 223-252.

Woo, J. (2005) "Social polarization, fiscal instability and growth," *European Economic Review*, 49, 1451-1477.

Velasco, A. (1999) "A model of endogenous fiscal deficits and delayed fiscal reforms," in J. M. Portaba and J. Von Hagen (ed.), *Fiscal Institutions and Fiscal Performance*, 37-58, University of Chicago Press.