Economics Bulletin

Volume 39, Issue 3

There is no such thing as the zero lower bound

Joshua R Hendrickson
University of Mississippi

Abstract

Conventional discussion of the zero lower bound on nominal interest rates relies on static reasoning. According to the conventional argument, people who are holding interest-bearing assets should switch to currency the instant that the nominal interest rate falls below zero since currency has a fixed nominal rate of interest equal to zero. In this paper, I argue that the presence of uncertainty about the expected future path of the nominal interest rate and the non-negative fixed costs associated with the storage of cash require a dynamic rather than a static analysis. People who are holding an interest-bearing asset have the option, but not the obligation to switch to currency at any point in time. The economic decision is to determine at what point to exercise this option. I show that the lower bound on the nominal interest rate in this context is below zero. This is true even if storage costs are approximately zero. Since my calculation does not depend on storage costs, it implies that the effective lower bound on the nominal interest rate might be considerably lower than previously thought.

I would like to thank the anonymous reviewer for useful comments.
Citation: Joshua R Hendrickson, (2019) "There is no such thing as the zero lower bound", *Economics Bulletin*, Volume 39, Issue 3, pages 1870-1875
Contact: Joshua R Hendrickson - jrhendr1@olemiss.edu.
Submitted: July 10, 2019. **Published:** August 20, 2019.

1 Introduction

Modern central banks primarily communicate changes in the stance of monetary policy through changes in the short term nominal interest rate. Over the past couple of decades monetary economists have given particular attention to zero lower bound on nominal interest rates.[1] If monetary policy is primarily transmitted through interest rates, the inability of central bankers to lower interest rates below zero can limit the effectiveness of policy.[2]

In recent years, with central banks around the world approaching the zero lower bound, a number of economists have argued in favor of using negative nominal interest rates as a policy tool. In addition, others have advocated eliminating cash altogether. The motivation for eliminating cash is as follows. A person can always sell an interest-bearing asset for currency. Since currency earns a constant nominal interest rate of 0%, the introduction of negative rates might induce people to sell their assets with negative rates for currency. Thus, even if negative nominal interest rates are deemed to be desirable for policy purposes, these negative rates might not be feasible. Eliminating cash would take away this option and therefore eliminate the source of the zero lower bound. While central banks have not sought to eliminate cash, some have experimented with negative nominal interest rates (Jackson, 2015).

The experiments with negative nominal interest rates as well as observations of market rates below zero for assets with long maturities have called into question whether zero is the effective lower bound on nominal interest rates. For example, Witmer and Yang (2015) argue that the actual return on currency is negative due to costs associated with storing currency and, as a result, the effective lower bound for the Bank of Canada is approximately -0.50%.

In this paper, I provide an argument as to why there is no such thing as a zero lower bound. In making this claim, I do not deny that there is floor for nominal interest rates, but rather take the position that the floor is significantly below zero. I show this using a simple framework that solves for the threshold at which a person should switch from a typically interest-bearing deposit to cash. The logic of my argument can be summarized as follows. Conventional discussions of the zero lower bound treat the decision as a static, now-or-never decision. Whenever the nominal interest rate on a deposit is less than zero, people will immediately switch to cash. The zero lower bound is simply a no-arbitrage condition. However, if there is uncertainty associated with the expected future path of the nominal interest rate and non-negative fixed costs of storing currency, then the decision should not be examined as a static, now-or-never decision. Any person holding a deposit has the option, but not the obligation to switch to cash. The important question is when to exercise this option. The threshold for exercising this option is dependent on the expected future path of the nominal interest rate. As I demonstrate below, when viewed from this perspective, the

[1] The literature is too exhaustive to summarize. However, for a representative sample of this work, see McCallum (2000), Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe and Uribe (2001), Svensson (2001), Eggertsson and Woodford (2003), Bernanke, Reinhart, and Sack (2004), and Hamilton and Wu (2011).

[2] There is, of course, some question about whether or not monetary policy is actually transmitted primarily through the nominal interest rate. I put those issues aside for the sake of simplicity and take it as given that the interest rate channel is of primary importance.

lower bound on the nominal interest rate can be significantly lower than zero.

What is novel about my result is that there is an effective lower bound on the nominal interest rate that is below zero *even when storage costs are approximately zero*. In other words, I show that uncertainty about the expected future path of interest rates is sufficient to generate an effective lower bound on the nominal interest rate that is below zero. Nonetheless, this does not imply that storage costs are irrelevant. The magnitude of storage costs plays a role in determining how low the effective lower bound can go. Finally, it is important to note that my back-of-the-envelope calculation of the effective lower bound is -1.17%. This calculation is substantially lower than the estimate of Witmer and Yang (2015). Unlike these previous authors, my calculation relies solely on the uncertainty surrounding the future path of the short term nominal interest rate and not on storage costs. As a result, the effective lower bound on the nominal interest rate might be significantly lower than previously thought.

2 The Model

Time is continuous and infinite. There are two assets: an interest-bearing deposit and currency. Assume that each person holds a \$1 deposit that earns a nominal interest rate, i. Currency earns a nominal interest rate of 0% with absolute certainty. By holding the deposit, a person owns an option to convert the deposit into currency at any time. The objective is to determine the threshold for the nominal interest rate at which to switch from the deposit to currency.

In determining the threshold for switching to currency, one must take into account two factors. First, when switching from holding the deposit to holding cash, one must have a place to store the currency. Suppose that the cost of storing currency is $F \geq 0$. Second, each person takes the deposit interest rate as given and there is some degree of uncertainty about the nominal interest rate on deposits. In particular, assume that the nominal interest rate, i, follows a Brownian motion:

$$di = \mu dt + \sigma dz \tag{1}$$

where μ is the expected change in the nominal interest rate, σ is the conditional standard deviation, and dz is an increment of a Wiener process (i.e. $dz = \epsilon\sqrt{dt}$, where ϵ is drawn from a standard normal distribution).

Let $V(i)$ denote the value of the option to convert the deposit to currency. It follows that the Bellman equation for a person holding the deposit can be written as

$$\rho V(i) = \frac{1}{dt} EdV \tag{2}$$

where $\rho \in (0, 1)$ is the rate of time preference and E is the expectations operator. Using Ito's Lemma and equation (1), it follows that the equation above can be re-written as

$$\frac{1}{2}\sigma^2 V''(i) + \mu V'(i) - \rho V(i) = 0 \tag{3}$$

This is a second-order differential equation that has a solution of the form:

$$V(i) = Ae^{\beta_1 i} + Be^{\beta_2 i} \tag{4}$$

where A and B are non-negative constants and β_1 and β_2 are solutions to the following quadratic equation:

$$\frac{1}{2}\sigma^2\beta^2 + \mu\beta - \rho = 0 \tag{5}$$

Equation (4) captures the value of the option to switch from the deposit to holding currency. Given this interpretation, it is possible to impose boundary conditions on equation (4) using economic reasoning. First, note that as the nominal interest rate on the deposit becomes sufficiently high, the value of the option to convert the deposit into currency becomes worthless. Formally, this implies that

$$\lim_{i\to\infty} V(i) = 0 \tag{6}$$

Recall that equation (5) has a positive and negative solution. Let β_1 denote the negative solution and β_2 be the positive solution. It follows that for this boundary condition to be satisfied, it must be true that $B = 0$.

Second, at the point at which a person wants to convert the deposit into currency, she will be indifferent between holding currency and the deposit. Let i^* denote the interest rate at which she is indifferent between holding the deposit and holding currency. Given that the interest rate on cash is 0%, the cost of switching is the foregone interest and the cost of storage, F. It follows that

$$V(i^*) = -i^* - F \tag{7}$$

Or, using the solution for $V(i)$, evaluated at i^*, implies that

$$A = e^{-\beta_1 i^*}\Bigg(- i^* - F\Bigg) \tag{8}$$

Plugging this into (4) yields

$$V(i) = e^{\beta_1(i-i^*)}\Bigg(- i^* - F\Bigg) \tag{9}$$

This solution for $V(i)$ now has an economic interpretation. The value of the option to convert the deposit into currency is equal to the payoff from doing so at the optimal exercise point multiplied by a stochastic discount factor. The actual threshold at which a person should switch is yet to be defined, but the incentives that she faces are captured by the valuation equation. By exercising the option, she loses the foregone interest, i^*, and pays a storage cost F. A positive payoff at the exercise point requires that the nominal interest rate is negative. Suppose that $i > 0 > i^*$. In this case, the lower the threshold chosen for the nominal interest rate, the higher the payoff at the exercise point, but the lower the present value of the option because it takes longer to reach the threshold. In choosing the threshold, she must optimally balance this tradeoff. It follows that her objective is to choose the threshold for nominal rate that maximizes this option value. Maximizing (9) with respect to i^* yields

$$i^* = \frac{1}{\beta_1} - F \tag{10}$$

Note that since β_1 is negative and the fixed cost associated with storing cash is non-negative, the threshold for the nominal interest rate is negative.

I can now use this expression to solve for the lower bound on nominal interest rates. Since I've considered the choice between holding a \$1 deposit and cash, it is reasonable to assume that $F \approx 0$. It follows that $i^* = 1/\beta_1$. To solve for this lower bound, I need to calibrate the rate of time preference and the time series properties of i. I begin by setting $\rho = 0.04$, a standard calibration for the rate of time preference. To calibrate the time series properties of the nominal interest rate, I use the daily effective federal funds rate for the period from July 1, 1954 to July 20, 2016 from the St. Louis Federal Reserve's FRED database. I calculate the daily change in the federal funds rate. I then calculate the mean of this change to calibrate $\mu \approx 0$ and the standard deviation of this change to calibrate $\sigma = 0.33$. This implies that equation (5) can be written as

$$0.5(.33^2)\beta^2 = 0.04 \tag{11}$$

This implies that $\beta \approx \pm 0.857$. We know from the boundary conditions that β must be less than zero. It follows that

$$i^* \approx -\frac{1}{0.857} \approx -1.17 \tag{12}$$

The lower bound on the nominal interest rate is below zero.

3 Discussion

In the previous section, I showed that zero is not the lower bound on the nominal interest rate. In fact, my back-of-the-envelope calculation suggests that the lower bound on the nominal interest rate is approximately -1.17%. In this section, I discuss the intuition of this result and the implications of this really simple model. In addition, I also discuss the role that storage costs can play in influencing the lower bound. I argue that a number of proposals to enact negative nominal interest rates can be understood as attempts to increase the storage cost associated with holding cash. I also suggest that within the framework I have presented here, these proposals could be substantially more modest than they have been presented in the past. Finally, I note that with modest increases in storage costs, central banks could likely target any nominal interest rate that they desire. This means that society need not eliminate cash in order to circumvent the zero lower bound.

It is important to begin the discussion of the model with something of a novel result. While it is generally understood that storage costs can lead to negative nominal interest rates, the back-of-the-envelope calculation that I presented above ignores storage costs. As a result, it is important to consider why zero is not the lower bound even in the absence of storage costs. As discussed above, traditional analysis looks at the decision to hold cash or some interest-bearing asset as a static decision. If the nominal interest rate on deposits is greater than the nominal interest rate on cash, then one should hold deposits. On the other hand, if the nominal interest rate on deposits is less than the nominal interest rate on cash, then one should hold cash. Since the nominal interest rate on cash is zero, this becomes the lower bound on nominal interest rates in the context of a static decision. In the present model, however, the decision is dynamic. Since there is uncertainty about the future path of the nominal interest rate, a person might be willing to accept a negative nominal interest rate if she expects the nominal interest rate to rise in the future. It is only after the nominal interest rate falls to the threshold i^* that she is no longer willing to wait to see if the interest rate will rise.

The back-of-the-envelope calculation is useful because it captures the importance of the role of uncertainty surrounding the nominal interest rate. Nonetheless, more realistic calculations would likely be a function of the size of one's deposit. In addition, it is also plausible that the cost of storage is proportional to the size of the deposit that needs to be converted to cash. As shown in equation (10), the higher the storage cost, the more negative the lower bound becomes on the nominal interest rate.

One can understand a number of proposals to eliminate the zero lower bound within this context. For example, a simple and commonly recognized solution to the zero lower bound is simply to eliminate cash altogether. Within the context of the model, this is akin to making it infinitely costly to hold cash, such that the lower bound on the nominal interest rate goes to $-\infty$. Other proposals to relax the zero lower bound have been more modest. Goodfriend (2000) suggests using a magnetic strip inside of currency so that it can be taxed. Mankiw (2009) suggests having a lottery that would select serial numbers on currency. Following the lottery, currency with the selected serial numbers would be removed from circulation (or simply not accepted as payment). Others, such as Agarwal and Kimball (2015), have argued for a variable exchange rate between cash and deposits. Each of these proposals represents an attempt to increase the storage costs associated with cash.

The importance of the model presented here is that it is capable of showing that even modest storage costs can reduce the lower bound on nominal interest rates significantly. For example, if the objective of monetary policy is to set the market interest rate equal to the natural interest rate, it is important to consider how low the natural interest rate could possibly go. During the most recent financial crisis, the Taylor Rule (Taylor, 1993) suggested that the federal funds rate should be -2.5 to -5% at its trough, depending on the Federal Reserve's preferred inflation target. For the Federal Reserve's preferred inflation target of 2%, the trough for the federal funds rate under the Taylor Rule was -3.56%.[3] Assuming that the Taylor Rule (or something similar) should be used to conduct policy, it is important to consider whether creative solutions that allow for the implementation of negative interest rate policy are even necessary. Assessing the need for such creative solutions requires careful analysis of the storage costs associated with currency. It is possible that the lower bound is sufficiently negative that no creative solution is required. Nonetheless, the back-of-the-envelope calculations of my model as well as the recent experience of negative interest rates, including the negative rates on Danish debt across the yield curve (Worrachate 2019), suggest that central banks have room to maneuver below the so-called zero lower bound.

[3] These calculations come from the St. Louis Federal Reserve: https://research.stlouisfed.org/datatrends/mt/page10.php.

References

[1] Agarwal Ruchir and Miles Kimball (2015) "Breaking Through the Zero Lower Bound." IMF Working Paper.

[2] Bernanke, Ben S., Vincent R. Reinhart, and Brian P. Sack (2004) "Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment." *Brookings Papers on Economic Activity*, 1 - 78.

[3] Eggertsson, Gauti B., and Michael Woodford (2003) "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 139 - 211.

[4] Goodfriend, Marvin (2000) "Overcoming the Zero Lower Bound on Interest Rate Policy." *Journal of Money, Credit and Banking* 32(4), 1007 - 1035.

[5] Hamilton, James, and Jing Cynthia Wu (2011) "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment." *Journal of Money, Credit, and Banking* 44, 3 - 46.

[6] Jackson, Harriet (2015) "The International Experience with Negative Policy Rates." Bank of Canada Staff Discussion Paper No. 2015-13.

[7] Mankiw, N. Gregory (2009) "It May Be Time for the Fed to Go Negative." New York Times, April 2009.

[8] McCallum, Bennett T. (2000) "Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates." *Journal of Money, Credit and Banking* 32(4), 870 - 904.

[9] Reifschneider, David, and John C. Williams (2000) "Three Lessons for Monetary Policy in a Low Inflation Era." *Journal of Money, Credit and Banking* 32(4), 936 - 66.

[10] Svensson, Lars E. O. (2001) "The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap." *Monetary and Economic Studies* 19(S-1), 277 - 312.

[11] Taylor, John B. (1993) "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Series on Public Policy* 39, 195 - 214.

[12] Witmer, Jonathan and Jing Yang (2015) "Estimating Canada's Effective Lower Bound." *Bank of Canada Staff Analytical Note 2015-2*.

[13] Worrachate, Anchalee (2019) "In Land of Sub-Zero Debt, a Whole Yield Curve May Turn Negative." Bloomberg, June 19. https://www.bloomberg.com/news/articles/2019-06-19/in-land-of-sub-zero-debt-a-whole-yield-curve-may-turn-negative