Economics Bulletin

Volume 39, Issue 3

Divisia Monetary Aggregates for Developing Economies: Some Theory

John Nana Francois
West Texas A&M University

Ryan S Mattson
West Texas A&M University

Abstract

Asset market development is characterized by reducing market imperfections that generate costs incurred from participating in the financial system. In developing economies where financial markets are nascent, these costs are likely to be binding. This limits the typical economic agent's ability to fully access asset market, inducing partial access. In this note, we embed financial market imperfections into the Divisia aggregate-theoretic literature and illustrate their relevance in the derivation of user cost of money and consequently, Divisia monetary aggregates. Asset market imperfections are introduced through endogenous portfolio adjustment costs that proxy for, among other things, informational, transactional, liquidity, and portfolio management costs. The presence of adjustment costs induce additional costs that alter the standard user cost of money. We show that the user cost that arises from our model can be practically implemented in the construction of Divisia aggregates as in the standard Barnett (1978) user cost.

We would like to thank the editor, associate editor, Luis De Araujo, and an anonymous referee for comments and suggestions that have significantly improved the paper. We are also grateful to William A. Barnett, Andrew Keinsley, Eric Hoffmann and seminar participants at West Texas A&M University for useful discussions and feedback. The paper has also benefited from presentation at the Monetary Policy after the Global Financial crisis conference organized by the World Economic Association. We are solely responsible for any remaining errors.
Citation: John Nana Francois and Ryan S Mattson, (2019) "Divisia Monetary Aggregates for Developing Economies: Some Theory", *Economics Bulletin*, Volume 39, Issue 3, pages 2221-2227
Contact: John Nana Francois - jfrancois@wtamu.edu, Ryan S Mattson - rmattson@the-cfs.org.
Submitted: January 07, 2019. **Published:** September 30, 2019.

1 Introduction

Financial market development is characterized by reducing or overcoming market imperfections that generate costs incurred from participating in the financial system. These costs arise from the acquisition and processing of information, making transactions, managing portfolio, and enforcing contracts. In developing economies where financial markets are still nascent, these costs are likely to be binding. This limits the typical economic agent's ability to fully access asset markets; hence, generating partial access.[1] More precisely, because asset market imperfections are ubiquitous in every transaction, they induce costs that interfere with trades that the rational agents execute, or would make, in the absence of these imperfections.[2]

In this note, we embed financial market imperfections into the Divisia aggregate-theoretic literature and illustrate their relevance in the derivation of user cost of money and consequently, Divisia monetary aggregates. We model asset market imperfection by introducing endogenous portfolio adjustment costs in the manner of Andrés *et al.* (2004), Falagiarda (2014), and Francois (2016). The adjustment costs, among other things, proxy for transactional costs, portfolio management costs, and informational costs, all of which are likely to be high in developing countries and can limit access to financial markets. Additionally, the adjustment costs are modelled in such a way that they proxy for the economic agent's behavior toward risk as argued in Tobin (1969, 1982). In particular, since non-monetary assets are illiquid relative to monetary assets, the economic agent perceives entering non-monetary asset market as riskier. Consequently, agents hold (or demand) additional monetary assets to compensate themselves for the loss of liquidity associated with holding non-monetary assets thereby self imposing a reserve requirement on their non-monetary assets (Andrés *et al.*, 2004).

The introduction of these adjustment costs in the Divisia literature in the context of developing countries is necessary and intuitive. Specifically, the pricing of monetary and non-monetary assets are central in the construction of Divisia monetary aggregrates. The user costs of money employed as weights for Divisia monetary aggregates depend on the factors that affect the pricing of assets in an economy. In this sense, incorporating these market imperfections in the Divisia aggregate-theoretic literature in developing countries should not be discounted. Furthermore, our derived user cost of money coincides with the standard Barnett (1978) user cost of money when adjustment costs are zero.

2 Model with asset market imperfections

2.1 The Economic Problem In this section we layout the basic model. We assume a representative agent who seeks to maximize the lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t U(C_t, \mathbf{m}_t),$$

[1] Demirgüç-Kunt *et al.* (2008) provide an intuitive exposition of the presence of price and nonprice barriers in the financial market in developing countries.

[2] See, DeGennaro (2005) for a detailed discussion on the definition of market imperfections.

subject to the following budget constraint,

$$p_t^* C = w_t L_t + \sum_{i=1}^{n} [r_{it-1} p_{t-1}^* m_{it-1} - (1 + \Phi(m_{it}/A_t)) p_t^* m_{it}] + [R_{t-1} p_{t-1}^* A_{t-1} - p_t^* A_t]$$

where $\beta \in (0, 1)$ is the consumer's subjective discount factor, p_t^* is the true cost-of-living index, C_t is aggregate consumption, $\mathbf{m}_t = (m_{1t}, m_{2t}, m_{3t}, ..., m_{nt})'$ is a vector of n current-period monetary assets, and L_t and w_t are per capita labor supply and expected wage rate, respectively. Additionally, and more importantly, $\Phi\left(\frac{m_{it}}{A_t}\right)$ are endogenous portfolio adjustment costs involved in holding assets A_t. As mentioned earlier, these adjustment costs proxy for transaction and informational costs, as well well as, liquidity risks. For simplicity, we assume that anytime the agent holds asset A_t, they hold additional monetary asset i, for every monetary asset $i \in \{1, 2, ..., n\}$.[3] For now we do not assign any specific functional form to the adjustment costs except that they satisfy the following properties $\Phi(0) = 0$ and $\Phi'(.) > 0$. In section 2.4, we assign a functional to $\Phi(.)$.

Setting up and solving the Lagrangian yields the following necessary conditions:

$$\frac{\partial U}{\partial C_t} = \Lambda_t p_t^* \tag{1}$$

$$\frac{\partial U}{\partial m_{it}} + \beta r_{it} p_t^* \Lambda_{t+1} - \Lambda_t p_t^* (1 + \Gamma_{it}) = 0 \tag{2}$$

$$\beta \Lambda_{t+1} R_t p_t^* + (\Pi_{it} - 1) \Lambda_t p_t^* = 0, \tag{3}$$

and the transversality condition:

$$\lim_{t \to \infty} \beta^t \Lambda_t p_t^* A_{t+1} = 0 \tag{4}$$

where $\Gamma_{it} = \Phi\left(\frac{m_{it}}{A_t}\right) + \Phi'\left(\frac{m_{it}}{A_t}\right) \cdot \left(\frac{m_{it}}{A_t}\right)$ and $\Pi_{it} = \Phi'\left(\frac{m_{it}}{A_t}\right) \cdot \left(\frac{m_{it}}{A_t}\right)^2$

For notational convenience, we follow Barnett and Su (2016) and define $r_{it}^* = \frac{p_t^*}{p_{t+1}^*} r_{it}$ and $R_t^* = \frac{p_t^*}{p_{t+1}^*} R_t$ as the real gross rates of return on monetary asset, i, and non-monetary assets, respectively. Combining Eqs.(1) and (2) and Eqs.(1) and (3), we obtain the Euler equations for monetary and non-monetary assets respectively as

$$\mathrm{E}_t \left[\frac{\partial U}{\partial m_{it}} + \beta r_{it}^* \frac{\partial U}{\partial C_{t+1}} - \frac{\partial U}{\partial C_t} (1 + \Gamma_{it}) \right] = 0 \tag{5}$$

$$\mathrm{E}_t \left[\frac{\partial U}{\partial C_t} (\Pi_{it} - 1) + \beta R_t^* \frac{\partial U}{\partial C_{t+1}} \right] = 0 \tag{6}$$

[3] It is worth mentioning that this assumption can easily be relaxed so that these costs are only present for some monetary assets. Indeed, in section 2.4, the specified functional form for $\Phi(.)$ suggests that these adjustment costs need not be present in the pricing of all monetary assets.

Eq.(5) and Eq.(6) represent the pricing equations for non-monetary and monetary assets, respectively. Notice that the presence of adjustment costs directly affect the yields of monetary and non-monetary assets non-trivially. Specifically, when these costs are non-zero, it is clear that they alter the pricing of both monetary and non-monetary assets such that the standard expectation hypothesis theory do not hold. Furthermore, the adjustment cost function depends on the quantity of monetary and non-monetary assets, which implies that relative changes of these asset holdings directly impact the money market as they generate movements in money demand (Andrés *et al.*, 2004; Zagaglia, 2009). Hence, when there is an increase in the desired stock of a bond, household's demand for money increases in order to keep the money-bond ratio constant (Zagaglia, 2009). This implies that the degree of imperfect substitutability between money and bonds affects the yields. On the other hand, if these costs are zero— i.e., there are no impediments to trading or the economy is efficient— these pricing equations collapse to the pricing equation of standard asset pricing models.[4]

2.2 User cost under portfolio adjustment costs

We now derive the user cost of monetary assets. Consider the following definition for the user-cost price of money

Definition 1. *The intratemporal real user cost price of the monetary asset i is $\wp_{it}$, defined such that*

$$\wp_{it} = \frac{\frac{\partial U}{\partial m_{it}}}{\frac{\partial U}{\partial C_t}}, \qquad i = 1, 2, ..., n$$

The definition for the intratemporal user cost states that the real user price of a monetary asset is the marginal rate of substitution between that asset and consumer goods (Barnett, 1978). Now from this definition, it is straightforward to derive the user cost of money implied by our model via the following proposition.

Proposition 1. *In the presence of portfolio adjustment costs, the real user cost of the services of monetary assets is*

$$\wp_{it} = \frac{R_t^* - r_{i,t}^*}{R_t^*} + \Omega_{it}, \quad \textit{where} \quad \Omega_{it} = \frac{\Gamma_{it} R_t^* + \Pi_{it} r_{it}^*}{R_t^*}$$

Proof. Using definition 1, the proof follows immediately from Eqs. (5) and (6) □

Notice that our derived user cost comprises two components, the first term, which is the conventional Barnett user cost and an additional term Ω_{it}, which captures the role of asset market imperfections. Recall that asset market imperfections were modelled endogenously; hence, Ω_{it} depends on quantity of assets. This suggests that equilibrium changes in the relative quantity of monetary and non-monetary assets affect the user cost of money. This is intuitive in that one would expect that in developing economies where markets are characterized by price barriers that induce partial access to asset markets, additional costs in participating in asset markets would arise. As explained earlier, the presence of these endogenous adjustment costs suggest

[4] Falagiarda and Marzo (2012) show that adjustment costs parameters generate impediments to the arbitrage activity which would equalize returns. At the same time, the presence of transaction costs determines the extent of the influence of relative assets holdings on long-term rate.

that the agent holds additional monetary assets any time they purchase non-monetary assets; hence, the opportunity cost of holding monetary assets.[5] However, as markets become more efficient, these costs disipate. An interesting implication of this result is that when markets are frictionless (or efficient), these costs approach zero, and our derived user cost coincides with the standard Barnett user cost. Hence, we have the following results from Proposition 1.

Corollary 1.1. *When adjustment costs approaches zero (i.e., $\Omega_{it} \to 0$), the user cost in Proposition 1 converges to the standard Barnett (1978).*

2.3 How applicable are these results to Divisia monetary aggregates?

In this section, we show theoretically that the derived user cost of monetary assets in Proposition 1 can be used to derive Divisia monetary aggregates. We therefore make the following assumption: assume that there exist a linearly homogenous aggregator function $M(.)$ such that U can be written in the form,

$$U(\mathbf{m}_t, C_t) = V(M(\mathbf{m}_t), C_t), \tag{7}$$

the following proposition then arises,

Proposition 2. *Let $s_{it} = \frac{\wp_{it} m_{it}}{\sum_{i=}^{n} \wp_{it} m_{it}}$ be the user-cost-evaluated expenditure share. Under the assumption of weak-separability $U(\mathbf{m}_t, C_t) = V(M(\mathbf{m}_t), C_t)$, and for any linearly homogenous monetary aggregator function, $M(.)$, the Divisia index holds under financial market imperfection:*

$$d \log M_t = \sum_{i=1}^{n} s_{it} d \log m_{it}$$

Proof. With the assumption of weak-separability, and $V_t = V(M(\mathbf{m}_{it}), c_t)$ we have that

$$\frac{\partial U_t}{\partial m_{it}} = \frac{\partial V_t}{\partial M_t} \frac{\partial M_t}{\partial m_{it}} \tag{8}$$

From definition 1 we have $\wp_t(\Lambda_t p_t) = \frac{\partial U_t}{\partial m_{it}}$, we can rewrite Eq.(8) above as

$$\frac{\partial M_t}{\partial m_{it}} = \wp_{it} \left(\frac{\Lambda_t p_t}{\frac{\partial V_t}{\partial M_t}} \right) \tag{9}$$

Additionally, given $M = M(\mathbf{m}_{it})$, we can take the total derivative to get $\mathrm{d}M_t = \sum_{i=1}^{n} \frac{\partial M}{\partial m_{it}} \mathrm{d}m_{it}$ and apply Eq. (8) to arrive at,

[5] It is important to note that this result in itself is not new as existing studies such as Tobin (1969, 1982) have discussed the role of the quantity of assets in asset pricing due to imperfect asset substitution between monetary and non-monetary assets. Indeed, recent studies have explicitly introduced the role of quantity of assets in general equilibrium models via endogenous adjustment costs that proxy for asset market imperfections (see for instance, Andrés *et al.*, 2004; Falagiarda, 2014; Francois, 2016; Zagaglia, 2009, to mention a few). The result in Proposition 1 is therefore important as it does not discount the relevance of these asset market imperfections in the Divisia literature.

$$dM_t = \left(\frac{\Lambda_t p_t}{\frac{\partial V_t}{\partial M_t}}\right)\sum_{i=1}^{n}\wp_{it}\cdot m_{it}\frac{dm_{it}}{m_{it}} = \left(\frac{\Lambda_t p_t}{\frac{\partial V_t}{\partial M_t}}\right)\sum_{i=1}^{n}\wp_{it}\cdot m_{it}d\log m_{it} \tag{10}$$

Notice also that $M_t = \sum_{i=1}^{n}\frac{\partial M}{\partial m_{it}}m_{it}$ so that from Eq. (8) we have

$$M_t = \left(\frac{\Lambda_t p_t}{\frac{\partial V_t}{\partial M_t}}\right)\sum_{i=1}^{n}\wp_{it}\cdot m_{it} \tag{11}$$

Dividing Eq.(8) and Eq.(10), we arrive at,

$$d\log M_t = \sum_{i=1}^{n} s_{it} d\log m_{it} \tag{12}$$

□

2.4 A practical example

The results in Section 2.3 suggest that these modifications can be easily implemented into the Divisia monetary aggregate literature. To further elucidate on the practicality of our results we introduce a simple functional form for the portfolio adjustment costs that practitioners can use. The functional form is as follows

$$\Phi\left(\frac{m_{it}}{A_t}\right) = \frac{\phi_i}{2}\left(\frac{m_{it}}{A_{it}}\right), \tag{13}$$

so that the overall adjustment costs with respect to monetary asset i is given by $\frac{\phi_i}{2}\left(\frac{m_{it}}{A_{it}}\right)^2\cdot A_t$. The parameter $\phi_i \in [0,1]$ governs the degree of asset market imperfections that limit access to asset markets. The smaller (larger) ϕ_i is, the less (greater) the degree of the market imperfections. Given the functional form of $\Phi\left(\frac{m_{it}}{A_t}\right)$, we can derive Ω_{it} to be

$$\Omega_{it} = \phi_i\left[R_t^* + \left(\frac{m_{it}}{A_t}\right)\cdot r_{it}^*\right]\frac{m_{it}}{R_t^* A_t} \tag{14}$$

We can see from Eq.(14) that for $\phi_i > 0$, we have Ω_{it} to be postive. The presence of adjustment costs therefore place an upward pressure on the user cost of money. More specifically, the user cost of money can be rationalized as the opportunity cost, $R_t - r_{it}$, which measures the interest forgone by holding monetary asset i when R_t is available (Barnett *et al.*, 1984). These cost barriers that require the agent to hold additional monetary asset i anytime they purchase non-monetary assets increases the opportunity cost of holding monetary asset i. We can therefore expect a higher user cost of money when informational or transactional costs are present in asset markets. It is also important to mention that all else equal, market conditions that ease or increase the supply of non-monetary assets will reduce these costs; hence, the user cost of money.[6]

[6] It is straightforward to show from Eq. (14) that $\frac{\partial \Omega_{it}}{\partial A_t} < 0$ implying that $\frac{\partial \wp_{it}}{\partial A_t} < 0$ from Proposition 1.

3 Concluding Remarks

The literature on Divisia monetary aggregates has seen a decent growth in studies that focus on developing countries (e.g., Barnett and Alkhareif, 2015; Barnett and Tang, 2016; Khainga, 2014). These studies however preclude potential cost barriers that persist in the financial markets in these economies. Consequently, they employ the conventional Barnett (1978) user cost in modelling and the construction of the user cost of money, which is ultimately employed in constructing weights for Divisia monetary aggregates. In this paper, we introduce portfolio adjustment costs, which proxy for informational, portfolio management, and transaction costs into the standard Divisia aggregate-theoretic literature. We then derive a modified user cost that explicitly accounts for impediments in the financial markets in developing economies. More importantly, we show that it is straightforward to implement the derived user cost into the theory of Divisia monetary aggregates as in the standard Barnett user cost. Additionally, we model these costs endogenously in the manner of Andrés *et al.* (2004) among others; hence, the costs also proxy for risk that augments the standard Barnett user cost. Our approach therefore provides a flexible and an alternative way of modelling risk in the Divisia literature (see for instance, Barnett and Su, 2016; Barnett and Wu, 2005). Moreover, when these costs are small or zero due to efficient financial markets, our derived user cost conveniently coincides with the Barnett user cost.

The size of these adjustment costs remain an empirical question and should not be discounted. Hence, estimating these costs and practically implementing them in the Divisia literature should constitute the object of future empirical studies.

References

Andrés, J., López-Salido, J. D. and Nelson, E. (2004) Tobin's imperfect asset substitution in optimizing general equilibrium, *Journal of Money, Credit and Banking*, pp. 665–690.

Barnett, W. and Su, L. (2016) Joint aggregation over money and credit card services under risk, *Economics Bulletin*, **36**, 2301–2310.

Barnett, W. A. (1978) The user cost of money, *Economics letters*, **1**, 145–149.

Barnett, W. A. and Alkhareif, R. M. (2015) Modern and traditional methods for measuring money supply: the case of Saudi Arabia, *International Journal of Financial Studies*, **3**, 49–55.

Barnett, W. A., Offenbacher, E. K. and Spindt, P. A. (1984) The new divisia monetary aggregates, *Journal of Political Economy*, **92**, 1049–1085.

Barnett, W. A. and Tang, B. (2016) Chinese divisia monetary index and GDP nowcasting, *Open Economies Review*, **27**, 825–849.

Barnett, W. A. and Wu, S. (2005) On user costs of risky monetary assets, *Annals of Finance*, **1**, 35–50.

DeGennaro, R. P. (2005) Market imperfections, *Journal of Financial Transformation*, **14**.

Demirgüç-Kunt, A., Honohan, P. and Beck, T. (2008) Access to finance and development: Theory and measurement, in *Finance for all?: Policies and Pitfalls in Expanding Access.*, World Bank, chap. 1, pp. 21–53.

Falagiarda, M. (2014) Evaluating quantitative easing: a dsge approach, *International Journal of Monetary Economics and Finance*, **7**, 302–327.

Falagiarda, M. and Marzo, M. (2012) A DSGE model with endogenous term structure, Tech. rep., Quaderni-Working Paper DSE.

Francois, J. N. (2016) Foreign official holdings of us treasuries, stock effect and the economy: a DSGE approach, *The BE Journal of Macroeconomics*.

Khainga, D. (2014) Divisia Monetary Aggregates and Demand for Money in Kenya, Africa Growth Initiative, working paper, Africa Growth Initiative at Brookings.

Tobin, J. (1969) A general equilibrium approach to monetary theory, *Journal of Money, Credit and Banking*, **1**, 15–29.

Tobin, J. (1982) Money and finance in the macroeconomic process, *Journal of Money, Credit and Banking*, **14**, 171–204.

Zagaglia, P. (2009) Forecasting with a DSGE model of the term structure of interest rates: The role of the feedback, *Research Papers in Economics*, **14**.