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The gap between equilibrium expected payoffs in contests with linear externalities

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Abstract

Two-player Tullock-type contests with linear externalities are classified into two, those with cost externality and those with rent externality. We consider the players' symmetric externalities and their asymmetric abilities and examine the gap between their equilibrium expected payoffs. We find that (1) in the two types of contests, the expected payoff is larger for the favorite than for the underdog, (2) in the cost-externality contest, the gap between the players' expected payoffs is constant regardless of changes in extent of externalities, and (3) in the contest with rent externality, the gap increases when it becomes more enhanced with externalities.

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1. Introduction

The literature on contest theory assumes that the size of rent is exogenously given. However, the rents of several contests are affected by the aggregate efforts of players. Thus, several authors have examined the impact of externalities on Tullock-type contest outcomes (Chung, 1996; Lee and Kang, 1998; Lee, 2007; Eggert and Kolmar, 2006; Shaffer, 2006; Chowdhury and Sheremeta, 2011a, 2011b, 2015; and Keskin and Sağlam, 2018). The contests examined in these papers can be classified into two types, those with cost externality and those with rent externality. In contests with cost externality, the external effects generated by player efforts influence the costs as regards rent seeking. In contrast, the contests with rent externality represent the case of such external effects altering the rent size.

Many contests are associated with cost externalities. An example is an R&D competition wherein the R&D external effects of one firm alter the extent of positive externalities through the cost of firms (D'Aspremont and Jacquemin, 1988). Another example is a territorial fight. Each player's efforts may increase the rival's real cost as well as his real cost (Baye *et al.*, 2012). Contests with cost externalities are investigated by Lee and Kang (1998) and Lee (2007).

Contests with rent externality are indeed ubiquitous. For instance, the prize money for a lottery winner depends on the total expenditure on the lottery tickets. The more tickets the people buy, the larger is the winner's prize money (Chung, 1996). Another example is a labor tournament. A typical labor tournament rewards behavior that increases the total surplus available to the organization members. Thus, there would be more incentives for labor with high competitiveness (Shaffer, 2006). Contests with rent externality have been examined by Chung (1996), Eggert and Kolmar (2006), Shaffer (2006), and Keskin and Sağlam (2018).[1]

Most of the studies on two-player Tullock-type contests with externalities focus on one of these two types. An exception is the work of Chowdhury and Sheremeta (2011a, 2011b, 2015), who, like this study, treat the two types simultaneously.[2] While Chowdhury and Sheremeta (2011a) show that multiple equilibria may exist in contests with *asymmetric* externalities, Chowdhury and Sheremeta (2011b) find that when the players' reaction functions are positive, the first-order conditions for maximizing their expected payoffs are necessary and sufficient for a unique equilibrium. Furthermore, Chowdhury and Sheremeta (2015) introduce equivalence among Tullock-type contests and define the contests that generate the same reaction functions strategically equivalent. They then show that the various types of contests may be strategically equivalent, and the contest designer may be able to achieve different goals using strategically equivalent contests yielding different equilibrium expected payoffs.

Like Chowdhury and Sheremeta (2011a, 2011b, 2015), this article investigates two types of contests, those with cost externality and those with rent externality. The main differences between those studies and this article pertain to the relative externality as well as ability of the players. Unlike those studies, we consider the players' *symmetric* externalities and *asymmetric* abilities in the two types of contests.

This article explains the unique equilibrium due to symmetric externalities in each contest. We scrutinize the equilibrium outcomes of each contest by exploring how the aggregate effort externalities are related to the gap between the equilibrium payoffs of two asymmetric players, to obtain three main results. First, in both types of contests, the equilibrium

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[1] Chowdhury and Sheremeta (2011b) provide various field examples of two types of contests with externalities.

[2] Other contest settings treat cost externality as well as prize externality. For example, Baye *et al.* (2012) study the symmetric equilibria of the two-player contests class wherein the winner is determined by the all-pay auctions selection rule. In Baye *et al.* (2012), each player's strategy affects the rival's payoff and/or her own, and the nature of this effect relies on the rank order of her decision variable.

payoff of the favorite is larger than that of the underdog.[3] Second, in the contest with cost externality, the gap between the equilibrium expected payoffs of the two players does not change even if the extent of externality changes. Third, in the contest with rent externality, the gap increases as the contest enhances with externalities. Our main results are due to the differences in abilities of the two players. These differences lead to two situations: (*i*) The reaction function of the favorite is different from that of the underdog in each contest, and (*ii*) the reaction function of the player participating in the cost-externality contest changes when he participates in the rent-externality contest. Situation (*ii*) implies that the two types of contests are not strategically equivalent.

This article is organized as follows. Section 2 examines the two-player Tullock-type contest with cost externality. Section 3 examines the contest with rent externality. We interpret our results in Section 4.

2. Contest with cost externality

Consider a two-player Tullock-type contest with cost externality. Players 1 and 2 compete for the basic (or initial) rent v. Let x_1 and x_2 be the players' respective effort levels in rent seeking, p_1 be the probability of player 1 winning the rent, and $p_2 = 1 - p_1$ be the probability of player 2 winning the rent. Now, p_1 is defined by the Tullock-form contest success function represented by even productivity of effort as[4]

$$\begin{aligned} p_1(x_1, x_2) &= hx_1/(hx_1 + x_2) \qquad && \text{for } x_1 + x_2 > 0 \\ &= 1/2 && \text{for } x_1 = x_2 = 0, \end{aligned}$$

where the relative ability parameter $h > 1$ reflects the contest bias term, which indicates that player 1 has more ability than player 2 (Tullock, 1980; Lee and Baik, 2017). Let π_i denote the expected payoff of player i (= 1, 2). Now, the expected payoffs of players 1 and 2 are

$$\begin{aligned} \pi_1 &= p_1\{v + \delta(x_1 + x_2) - x_1\} + p_2\{\delta(x_1 + x_2) - x_1\} \\ &= p_1 v + \delta x_2 - (1 - \delta)x_1 \end{aligned} \tag{1}$$

and

$$\begin{aligned} \pi_2 &= p_2\{v + \delta(x_1 + x_2) - x_2\} + p_1\{\delta(x_1 + x_2) - x_2\} \\ &= p_2 v + \delta x_1 - (1 - \delta)x_2, \end{aligned} \tag{2}$$

respectively, where δ is an externality parameter, and $\delta x_j - (1 - \delta)x_i$ can be rewritten as $\delta(x_i + x_j) - x_i$; here, $\delta(x_i + x_j)$ represents the external effects of changes in costs ($i \neq j$). Thus, the real cost of player i is $C_i = -\delta(x_i + x_j) + x_i$. In view of the even productivity efforts, we consider a linear symmetric externality for the two players. When $0 < \delta < 1$, the contest is associated with positive externality, and when $-1 < \delta < 0$, the contest is associated with negative externality.

Player i expends effort x_i to maximize π_i, taking player j's effort x_j as given. The first-

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[3] The favorite is defined as the player winning more than 50%, and the underdog, as the player winning less than 50% (Dixit, 1987).

[4] Katz (1988) considers a contest success function for player 1 given as $p_1(x_1, x_2) = hx_1^{r1}/(hx_1^{r1} + x_2^{r2})$, where $r1$ and $r2$ reflect the productivities of efforts. If $r1 \neq r2$, the players have access to different productivities of efforts. If $r1 = r2$, they have access to identical productivities of efforts (see Clark and Riis, 1998; Choi *et al.*, 2016). In this article, like Gradstein (1995), we assume $r1 = r2 = 1$ to focus on the difference between players' abilities. Gradstein (1995), considering n-player Tullock-type contests, analyzes the participation in contests of players with different abilities.

order condition for maximizing π_i reduces to

$$(\partial p_i/\partial x_i)v - (1 - \delta) = 0. \tag{3}$$

We derive the two players' reaction functions, $b_1(x_2)$ and $b_2(x_1)$, respectively, from the first-order conditions

$$b_1(x_2) = -x_2/h + [x_2 v/\{(1 - \delta)h\}]^{1/2} \tag{4}$$

and

$$b_2(x_1) = -hx_1 + [hx_1 v/(1 - \delta)]^{1/2}. \tag{5}$$

The reaction function of player 1 in (4) is different from that of player 2 in (5), since the abilities of the two players are asymmetric. Using the two reaction functions, we obtain a unique equilibrium for the contest with cost externality,[5] denoted by (x_1^*, x_2^*).

Lemma 1. *In the equilibrium of the contest with cost externality, players* 1 *and* 2 *expend* $x_1^* = x_2^* = hv/\{(1 - \delta)(1 + h)^2\}$. *The probability of winning for player* 1 *is* $p_1^* = h/(1 + h)$. *The expected payoffs of players are* $\pi_1^* = \{\delta + (1 - \delta)h\}hv/\{(1 - \delta)(1 + h)^2\}$ *and* $\pi_2^* = \{1 + \delta(h - 1)\}v/\{(1 - \delta)(1 + h)^2\}$.

Although the effort levels of both players are equal, we can easily find that $p_1^* > 1/2$ and $\partial p_1^*/\partial\delta = 0$. This implies that the symmetric expenditure in the rent-seeking contest with "bias" discussed in Tullock (1980) holds in the contest with cost externality as well. Using Lemma 1, we compare the equilibrium expected payoffs of the favorite and the underdog, and examine the effects of increasing parameter δ. Proposition 1 summarizes the comparison and comparative-statics results.

Proposition 1: (*i*) *The equilibrium expected payoff of the favorite is higher than that of the underdog.* (*ii*) *The equilibrium total expected payoff increases as the parameter* δ *increases.* (*iii*) *The players' equilibrium expected payoffs increase at the same size in* δ. (*iv*) *The gap between the players' equilibrium expected payoffs is constant regardless of change in* δ.

We can easily find that $\pi_1^* > \pi_2^*$ and $\partial(\pi_1^* + \pi_2^*)/\partial\delta > 0$. However, the reason for the equilibrium expected payoffs of the players to increase at the same size in δ, $\partial\pi_1^*/\partial\delta = \partial\pi_2^*/\partial\delta = hv/\{(1 - \delta)(1 + h)\}^2 > 0$ is not clear. Furthermore, we need to explain why the gap between the equilibrium expected payoffs of the players does not change even when the externality parameter changes. From (1), (2), and Lemma 1, we obtain the equilibrium expected payoffs of the favorite and the underdog as $\pi_1^* = p_1^* v - (1 - 2\delta)x^*$ and $\pi_2^* = p_2^* v - (1 - 2\delta)x^*$, respectively, where $x_1^* = x_2^* = x^*$. The gap between the players' equilibrium expected payoffs depends on the gap between $p_1^* v$ and $p_2^* v$, since the players' equilibrium real costs are the same: $C_1 = C_2 = (1 - 2\delta)x^*$. We clearly have $p_1^* v > p_2^* v$ and $\partial\pi_1^*/\partial\delta = \partial\pi_2^*/\partial\delta = -\partial C_1^*/\partial\delta = -\partial C_2^*/\partial\delta > 0$. Here, the two players' equilibrium expected payoffs are increasing at the same size in δ, and the gap between the payoffs is constant, regardless of the change in δ.

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[5] Chowdhury and Sheremeta (2011b) show that when the reaction function of player i is positive, the first-order condition for maximizing π_i is necessary and sufficient for unique equilibrium. In (4) and (5), we obtain $b_1(x_2) > 0$ for $0 < x_2 < hv/(1 - \delta)$; and $b_2(x_1) > 0$ for $0 < x_1 < v/h(1 - \delta)$. Using Lemma 1, we show that the reaction functions of players are positive: $x_2^* < hv/(1 - \delta)$ implying $b_1(x_2^*) > 0$; and $x_1^* < v/h(1 - \delta)$, as a result, $b_2(x_1^*) > 0$, which is due to the symmetric externalities for the players.

3. Contest with rent externality

Consider a two-player Tullock-type contest with rent externality, similar to the one in Section 2, except that the external effects generated by the players' efforts influence the size of rent, and not costs. The expected payoffs of the players are, respectively,

$$\begin{aligned}\pi_1 &= p_1\{v + \delta(x_1 + x_2) - x_1\} + p_2\{- x_1\} \\ &= p_1\{v + \delta(x_1 + x_2)\} - x_1\end{aligned} \tag{6}$$

and

$$\begin{aligned}\pi_2 &= p_2\{v + \delta(x_1 + x_2) - x_2\} + p_1\{- x_2\} \\ &= p_2\{v + \delta(x_1 + x_2)\} - x_2,\end{aligned} \tag{7}$$

where the term $\delta(x_i + x_j)$ represents the external effects of changes in size of rent. Thus, the endogenous rent for player i is $v + \delta(x_i + x_j)$. Player i selects x_i to maximize π_i, taking player j's effort x_j as given. The first-order condition for maximizing π_i reduces to

$$(\partial p_i/\partial x_i)\{v + \delta(x_1 + x_2)\} + p_i\delta - 1 = 0. \tag{8}$$

We derive the two players' reaction functions, $b_1(x_2)$ and $b_2(x_1)$, from respectively the following first-order conditions:

$$r_1(x_2) = -x_2/h + [\{hv + (h-1)\delta x_2\}x_2/(1-\delta)]^{1/2}/h \tag{9}$$

and

$$r_2(x_1) = -hx_1 + [\{v - (h-1)\delta x_1\}hx_1/(1-\delta)]^{1/2}. \tag{10}$$

(9) and (10) show different reaction functions for the players since their abilities are asymmetric. Let (x_1^{**}, x_2^{**}) denote the unique equilibrium of the rent-externality contest.[6] Also, assume that $x_1^{**} = qx_2^{**}$, where $q > 0$ is a function of δ and h to be solved for Lemma 2. Next, using the two reaction functions and $x_1^{**} = qx_2^{**}$, we obtain the equilibrium of the contest.

Lemma 2. *In the equilibrium of the contest with rent externality, players* 1 *and* 2 *expend* $x_1^{**} = hq^2v/\{(h-\delta)hq^2 + (1-\delta)(1+2hq)\}$ *and* $x_2^{**} = hqv/\{(h-\delta)hq^2 + (1-\delta)(1+2hq)\}$, *where* $q = [(h-1) + \{4(1-\delta)^2h + (h-1)^2\}^{1/2}]/\{2(1-\delta)h\}$. *The probability of winning for player* 1 *is* $p_1^{**} = hq/(1+hq)$. *The expected payoffs of the players are* $\pi_1^{**} = \{1-\delta+(h-1)q\}hqv/\{(h-\delta)hq^2 + (1-\delta)(1+2hq)\}$ *and* $\pi_2^{**} = (1-\delta)v/\{(h-\delta)hq^2 + (1-\delta)(1+2hq)\}$.

We can easily show that $x_1^{**} < x_2^{**}$ if $\delta < 0$, and $x_1^{**} > x_2^{**}$ if $\delta > 0$. From Lemma 2, $p_1^{**} > 1/2$, regardless of who expends greater effort level. Using Lemma 2, we compare the equilibrium expected payoffs of the players and examine the external effects. Proposition 2 illustrates the comparison and comparative-statics results.

Proposition 2: (*i*) *The equilibrium expected payoff of the favorite is higher than that of the underdog.* (*ii*) *The equilibrium total expected payoff increases as the parameter δ increases.* (*iii*) *The favorite's equilibrium expected payoff increases in δ, while the underdog's decreases.* (*iv*) *The gap between the equilibrium expected payoffs of the players widens as δ increases.*

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[6] From (9) and (10), we find that $r_1(x_2) > 0$ for $\delta h \geq 1$ and $x_2 > 0$ or for $\delta h < 1$ and $0 < x_2 < hv/(1-\delta h)$; and $r_2(x_1) > 0$ for $0 < x_1 < v/(h-\delta)$. Using Lemma 2, we show that the reaction functions of the players are positive: $x_2^{**} < hv/(1-\delta h)$, implying that $r_1(x_2^{**}) > 0$; and $x_1^{**} < v/(h-\delta)$, as a result, $r_2(x_1^{**}) > 0$.

We find that $\pi_1^{**} > \pi_2^{**}$ and $\partial(\pi_1^{**} + \pi_2^{**})/\partial\delta > 0$.[7] Because $\partial\pi_1^{**}/\partial\delta > 0$ and $\partial\pi_2^{**}/\partial\delta < 0$, the gap between the equilibrium expected payoffs of the players widens in δ; this needs some explanations.[8] An increase in δ enhances the favorite's equilibrium probability of winning p_1^{**}. At the same time, an increase in δ increases $p_1^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$, and also raises x_1^{**}. However, the former positive effect on $p_1^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ dominates the latter negative effect on x_1^{**}. The comparative-statics result, $\partial\pi_2^{**}/\partial\delta < 0$, is more complicated. We know that $\partial p_2^{**}/\partial\delta < 0$. However, $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ and x_2^{**} increase and then decrease in δ. Intuitively, the result of $\partial\pi_2^{**}/\partial\delta < 0$ is as follows. (*a*) When $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ and x_2^{**} increase in δ, the former positive effect on $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ is dominated by the latter negative effect on x_2^{**}. (*b*) When $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ decreases and x_2^{**} increases in δ, the result is self-evident. (*c*) When $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ and x_2^{**} decreases in δ, the former negative effect on $p_2^{**}\{v + \delta(x_1^{**} + x_2^{**})\}$ dominates the latter positive effect on x_2^{**}.

4. Discussion

This article shows that the reaction functions of the two asymmetric players in *each* two-player Tullock-type contest are different and therefore generate asymmetric equilibrium efforts and asymmetric expected payoffs. In addition, we show that if the players' abilities are different, the two Tullock-type contests would not be strategically equivalent. The condition that the contests are strategically equivalent is $b_i(x_j) = r_i(x_j)$ (see Chowdhury and Sheremeta, 2015). However, using the reaction functions of the players in (4) and (9), we can obtain $b_1(x_2) < r_1(x_2)$ for $0 < (h - 1)\delta x_2$, and using (5) and (10), we can obtain $b_2(x_1) > r_2(x_1)$ for $0 > -(h - 1)\delta x_1$.[9] In conclusion, our main results, especially those presented in Propositions 1 and 2, are due to the players' asymmetric abilities.

The implications of this article are as follows. If a contest designer wants to sustain the gap between the players' expected payoffs, he would have to choose a contest with cost externality. This can be observed in the Olympics Games (see Lee and Kang, 1998). At the Olympics, the athletes' records continue to increase every four years, but the difference between them may not change. The contest with rent externality is observed in a labor tournament (see Shaffer, 2006). From this article, a performance-based pay system where only worker(s) with high achievements earn a bonus is not advisable if the designer does not want to deepen the wage gap between workers.

[7] From Lemma 2, we obtain $\pi_1^{**} + \pi_2^{**} = \{1 - \delta + (h - 1)q\}hqv/\{(h - \delta)hq^2 + (1 - \delta)(1 + 2hq)\}$. Then, the comparative statics with respect to δ are as follows: $\partial(\pi_1^{**} + \pi_2^{**})/\partial\delta = h[h(h - 1)(1 + t)q^3 + (1 - \delta)\{h(h + \delta - 2)q^2 - 2(1 - \delta)q - (1 - \delta)\}\partial q/\partial\delta]/\{(h - \delta)hq^2 + (1 - \delta)(1 + 2hq)\}^2$, where $\partial q/\partial\delta = (h - 1)[h - 1 + \{1 + 2h(2\delta^2 - 4\delta + 1) + h^2\}^{1/2}]/[2h(1 - \delta)^2\{1 + 2h(2\delta^2 - 4\delta + 1) + h^2\}^{1/2}] > 0$. For both $h \to 1$ and $h \to \infty$, we obtain $\partial(\pi_1^{**} + \pi_2^{**})/\partial\delta = 0$. Also, from $\partial^2(\pi_1^{**} + \pi_2^{**})/\partial\delta\partial h = 0$, we obtain a unique value for h: $h = \{-2\delta^2 + 10\delta^2 - 16\delta + 9 + 2(\delta^6 - 10\delta^5 + 41\delta^4 - 89\delta^3 + 108\delta^2 - 69\delta + 18)^{1/2}\}/(3 - 2\delta)$. By inserting the value of h into $\partial(\pi_1^{**} + \pi_2^{**})/\partial\delta$, we obtain $\partial(\pi_1^{**} + \pi_2^{**})/\partial\delta > 0$. We use the computer program Maple to solve the comparative statics with respect to δ.

[8] we use Maple to obtain: $\partial\pi_1^{**}/\partial\delta = [(h - 1)(hq^2 + hq + 1)q^2 + (1 - \delta)\{1 - \delta + 2(h - 1) + h(h + \delta - 2)q^2\}\partial q/\partial\delta]v/\{(h - \delta)hq^2 + (1 - \delta)(1 + 2hq)\}^2 > 0$; and $\partial\pi_2^{**}/\partial\delta = -[(h - 1)hq^2 + 2h(1 - \delta)\{(h - \delta)q + (1 - \delta)\partial q/\partial\delta]v/\{(h - \delta)hq^2 + (1 - \delta)(1 + 2hq)\}^2 < 0$.

[9] If $h = 1$, as in Chowdhury and Sheremeta (2015), the two types of contests are strategically equivalent, to generate the same reaction functions, and, as a result, the same equilibrium effort levels, with different equilibrium expected payoffs: $b_i(x_j) = r_i(x_j) = -x_j + \{vx_j(1 - \delta)\}^{1/2}$, $b_j(x_i) = r_j(x_i) = -x_i + \{vx_i/(1 - \delta)\}^{1/2}$, $x_i^* = x_j^{**} = v/\{4(1 - \delta)\}$, and $\pi_i^* = \pi_j^* = v/\{4(1 - \delta)\} > \pi_i^{**} = \pi_j^{**} = v/4$ (see Lemmas 1 and 2).

References

Baye, M., D. Kovenock and C. Vries (2012) “Contests with Rank-order Spillovers” *Economic Theory* **51**, 315-350.

Choi, J. P., S. M. Chowdhury and J. Kim (2016) “Group Contests with Internal Conflict and Power Asymmetry” *Scandinavian Journal of Economics* **118**, 816-840.

Chowdhury, S. M and R. Sheremeta (2011a) “Multiple Equilibria in Tullock Contests” *Economics Letters* **112**, 216-219.

Chowdhury, S. M and R. Sheremeta (2011b) “A Generalized Tullock Contest” *Public Choice* **147**, 413-420.

Chowdhury, S. M and R. Sheremeta (2015) “Strategically Equivalent Contests” *Theory and Decision* **78**, 587-601.

Chung, T.-Y (1996) “Rent-seeking Contest When the Prize Increases with Aggregate Efforts” *Public Choice* **87**, 55-66.

Clark, D. J. and C. Riis (1998) “Contest Success Functions: An Extension” *Economic Theory* **11**, 201-204.

D’Aspremont, C. and A. Jacquemin (1988) “Cooperative and Non-cooperative R&D in Duopoly with Spillovers” *American Economic Review*, **78**, 1133-1137.

Dixit, A (1987) “Strategic Behavior in Contests” *American Economic Review* **77**, 891-898.

Eggert, W. and M. Kolmar (2006) “Contests with Size Effects” *European Journal of Political Economy* **22**, 989-1008.

Gradstein, M. (1995) “Intensity of Competition, Entry and Entry Deterrence in Rent Seeking Contests” *Economics and Politics* **7**, 79-91.

Katz, A (1988) “Judicial Decisionmaking and Litigation Expenditure” *International Review of Law and Economics* **8**, 127-143.

Keskin, K. and Ç. Sağlam (2018) “A Territorial Conflict: Trade-offs and Strategies” *Defence and Peace Economics* **29**, 658-665.

Lee, D. and K. H. Baik (2017) “Concealment and Verification over Environmental Regulations: A Game-theoretic Analysis” *Journal of Regulatory Economics* **51**, 235-268.

Lee, S. (2007) “Contests with Size Effect through Costs” *European Journal of Political Economy* **23**, 1190-1193.

Lee, S. and J. Kang (1998) “Collective Contests with Externalities” *European Journal of Political Economy* **14**, 727-738.

Shaffer, S. (2006) “War, Labor Tournaments, and Contest Payoffs” *Economics Letters* **92**, 250-255.

Tullock, G. (1980) “Efficient Rent Seeking” in *Toward a Theory of the Rent-Seeking Society* by J. Buchanan, R. Tollison, and G. Tullock, Eds., College Station: Texas A&M University Press, 97-112.