The heterogeneous relationship between income and inequality: a panel co-integration approach

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Abstract

We study the relationship between per-capita income and income inequality with a heterogeneous panel co-integration approach. We extend previous studies in two respects: first, we compile a more extensive data set for 87 countries over 26-58 years and consider measures for both pre-tax and post-tax income inequality; second, we take into account country heterogeneity rather than focussing on average panel estimates alone. We find a statistically significant positive group-mean based relationship using pre-tax income inequality, but no such relationship for post-tax income inequality. However, we find estimates on the country level to be very heterogeneous in both cases. Both empirical findings invite further research: by laying out relevant empirical patterns using a robust methodology, our contribution is meant as a vantage point for further theoretical work geared towards understanding the mechanisms underlying these findings.

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1 Introduction

The relationship between national levels of income and inequality has been the subject of economic scrutiny for long. Okun (1975) formulated the famous ‘efficiency-equity trade-off’, which has later been celebrated as a ‘principle of economics’ (Mankiw, 2018, p. 5), but also criticized for theoretical (e.g. Alesina and Rodrik, 1994) and empirical (e.g. Ostry et al., 2014) reasons. Recent reviews suggest that existing evidence on the relationship has been inconclusive (see e.g. Banerjee and Duflo, 2003; Neves et al., 2016; Neves and Silva, 2014).

Empirical ambiguities might well originate from methodological advances over time, or data limitations of earlier studies. Recent advances in panel co-integration techniques may lead to more clarity, since they are more robust against common problems in panel regressions (Herzer and Vollmer, 2012). Two recent studies followed such an approach: Herzer and Vollmer (2012) used a sample of 46 countries for the period 1970-1995 and Malinen (2012) for 53 countries between 1970-1999. Both identify a negative long-run relationship between inequality and per-capita income.

We extend their work in two respects. First, we compile a more extensive data set covering 87 countries over 26 (minimum) to 58 (maximum) years during the period 1960-2017 and use measures for both pre-tax and post-tax income inequality. We find a statistically significant positive relationship with pre-tax, but not post-tax income. The distinction between pre-tax and post-tax inequality has received comparatively little attention in the empirical literature so far, with Knowles (2005) being a notable exception. His demand for using consistent data on inequality aligns well with our finding of a different relationship depending on whether pre-tax or post-tax data is used.

Our second contribution stresses heterogeneity at the country level: we find considerable heterogeneity in the estimates for both pre- and post-tax inequality measures. This indicates that looking exclusively at overall panel estimates based on group-mean comparisons shallows important differences in the experiences of single countries. Notably, our inability to detect a statistically significant relationship between post-tax inequality and income on the sample level is not due to the absence of such a relationship on the country level: rather, significantly positive and negative relationships for individual countries cancel each other out and become invisible if only the aggregate result is considered. Both empirical findings contribute to the literature on the relationship between income and income inequality and invite further research: by laying out relevant empirical patterns using a robust methodology, our contribution is meant as a vantage point for further theoretical work geared towards understanding the mechanisms underlying these findings.

The remainder of this paper is structured as follows: after describing our data and our econometric approach in section 2 we present and discuss our findings in section 3. Then we conclude the paper in section 4 and outline some promising avenues for future research suggested by our results.
2 Data and econometric approach

We measure within-country inequality with Gini indices for pre- and post-tax income, both from Solt (2016), and per-capita income using expenditure-side real GDP at chained PPPs from Feenstra et al. (2015). We use annual rather than averaged data, for the reasons summarized in, inter alia, Herzer and Vollmer (2012). The panel is unbalanced. In contrast to both Malinen (2012) and Herzer and Vollmer (2012) we consider both pre- and post-tax inequality measures since each of them highlights different aspects of the income-inequality relationship. Knowles (2005) argues that some theoretical arguments about the relationship of inequality and income level make sense only when referring to post-tax, others to pre-tax inequality. While our focus here is not on the specific channels mediating the relationship between inequality and growth, we follow his argument and consider pre-tax and post-tax inequality time series separately, which is facilitated by the separate provision of this data in Solt (2016). Descriptive information about our data can be found in table I.

<table>
<thead>
<tr>
<th>Table I: Descriptive information about the data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year range</td>
</tr>
<tr>
<td>Total number of observations</td>
</tr>
<tr>
<td>GDP per capita (mean/standard deviation)</td>
</tr>
<tr>
<td>Post-tax gini (mean/standard deviation)</td>
</tr>
<tr>
<td>Pre-tax gini (mean/standard deviation)</td>
</tr>
</tbody>
</table>

We use panel co-integration techniques to study the long-term relationship between income and income inequality. This avoids a number of technical problems that occur frequently in conventional cross-country panel studies, such as omitted variable bias or endogeneity of the regressors. In contrast to conventional panel specifications, a well-specified co-integration analysis is robust against such biases, provided the data are cointegrated of order one. As we will show show below, our time series meet this condition.

The analysis follows the typical steps of a panel co-integration approach. First, the $H_0$ that all panels contain unit roots is evaluated using the Im—Pesaran—Shin (IPS) test (Im et al., 2003). This test is suitable for unbalanced panels and does not rely on a common autoregressive parameter for all countries, which is an important feature if one wishes to take into account unobserved heterogeneity. The test estimates the equation

$$\Delta y_{i,t} = \phi_1 y_{i,t-1} + z'_{i,t} \gamma_i + \epsilon_{i,t},$$

1The data as well as the code of our analysis is available at Github: https://github.com/graebnerc/cointegration-gdp-inequality. There we also provide descriptive statistics for our data on the country level.

2Descriptive statistics on the country level are provided in the accompanying GitHub repository.

3These problems might well be an important reason for why previous research on the relationship between income and income inequality has been inconclusive.
### Table II: Unit root tests (IPS)

<table>
<thead>
<tr>
<th></th>
<th>GDP per capita (log)</th>
<th>Gini index, pre-tax (log)</th>
<th>Gini index, post-tax (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-t-bar</td>
<td>8.03</td>
<td>-2.84</td>
<td>6.58</td>
</tr>
<tr>
<td>p-value</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Time trend</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Cross-sectional</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>means removed</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td># lags (AIC)</td>
<td>1.23</td>
<td>1.67</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Number of panels: 87. Average number of periods per country: 40.48. AR parameter: panel-specific.

### Table III: Tests for a co-integration relationship between inequality and income

<table>
<thead>
<tr>
<th>GDP per capita (log) and Gini index, pre-tax income (log)</th>
<th>Pedroni</th>
<th>Westerlund</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller t/ Variance statistics*</td>
<td>2.73</td>
<td>11.00</td>
</tr>
<tr>
<td>p-value</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>Panel means included</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time trend included</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>AR-parameter</td>
<td>panel-specific</td>
<td>same</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GDP per capita (log) and Gini index, post-tax income (log)</th>
<th>Pedroni</th>
<th>Westerlund</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller t/ Variance statistics*</td>
<td>2.37</td>
<td>9.98</td>
</tr>
<tr>
<td>p-value</td>
<td>0.009</td>
<td>0.000</td>
</tr>
<tr>
<td>Panel means included</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Time trend included</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>AR-parameter</td>
<td>panel-specific</td>
<td>same</td>
</tr>
</tbody>
</table>

Number of panels: 87. Average number of periods per country: 39.48 (Pedroni) and 40.48 (Westerlund). *We report variance statistics for the Westerlund and augmented Dickey-Fuller statistics for the Pedroni test. Other statistics from the Pedroni test (variance ratio, Phillips-Perron t) yield similar results.
where $i$ indicates countries and $t$ the time-series. $z'_{i,t}$ can adopt different values and represent panel-specific means, a time trend, both or none of both. We compare different specifications below. The autoregressive parameter $\phi$ is country-specific and in case $\phi_i = 0 \ \forall \ i$, all panels contain a unit root. The IPS test confirms that all time-series contain unit roots (see table II): we cannot reject $H_0$ under a variety of specifications. All of these included panel-specific means (fixed effects). In the case of GDP per capita, we need to control for cross-sectional correlation by subtracting cross-sectional averages (Levin et al., 2002). Otherwise, the null hypothesis of a unit root should have been rejected once a linear trend had been included. This is a sensible result given that cross-sectional dependence is common in macro panels such as ours. In the case of the Gini, we have to exclude a number of countries from the analysis because their time series do not contain a unit root.\(^4\) Overall, we are confident about the presence of unit roots in all three panel time series used in our analysis.

Second, we employ two different methods to test for a potential co-integration relationship between inequality and income (see table III). We reject the $H_0$ of no co-integration relationships, with $H_1$ being that all (Pedroni test) or some (Westlund test) panels are co-integrated.

We can then estimate the bi-variate relationship between inequality and income. We employ a dynamic OLS estimator for heterogeneous co-integrated panels with homogeneous long-run covariance structure across cross-sectional units (Pedroni, 2001) and estimate the equation

$$
\log(\text{GDP per capita}_{it}) = \alpha_i + \delta_i t + \beta_i \log(\text{Gini})_{it} + \sum_{j=-q}^{q} \phi_{ij} \Delta \log(\text{Gini})_{it+j} + \epsilon_{it},
$$

where $i = 1, 2, ..., N$ indicates countries; $t = 1, 2, ..., T$ indicates years; $\phi$ is the coefficient for leads and lags (included to account for serial correlation and endogeneity); $q$ is the number of leads and lags in the regression; and $\beta$ and $\delta$ are the slope coefficients.

We follow Pedroni (2001) and compute the $\beta$ coefficients as averages over the entire panel using

$$
\hat{\beta}_{GM}^* = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \sum_{t=1}^{T} z_{i,t} z'_{i,t} \right)^{-1} \left( \sum_{t=1}^{T} z_{i,t} (y_{i,t} - \bar{y}_i) \right) \right],
$$

where $z_{i,t}$ is the vector of regressors, and the group-mean $t$ statistics as

$$
t_{\hat{\beta}_i} = (\hat{\beta}_i - \beta_0) \left\{ \hat{\sigma}_i^{-2} \sum_{t=1}^{T} (x_{i,t} - \bar{x}_i)^2 \right\}^{\frac{1}{2}}.
$$

Note that with this specification, we implicitly address the question of whether distribu-

\(^4\)More precisely, we exclude 15 countries for which we can reject the presence of a unit root in the individual time series at the 5% level. These countries are already excluded from the data set as presented in table I. The tests as such, as well as the full data set including these countries, are provided in the accompanying GitHub repository.
tion impacts economic growth, which has dominated especially the theoretical literature, by specifying per-capita income as the dependent variable and seek to describe this time series as a function of inequality. The inverse relationship might well yield different results. Further note that our specification differs from both Herzer and Vollmer (2012) – who include a control for investment – and Malinen (2012) – who includes further controls for education. We restrain from adding controls for the following reasons. First, the estimator we use requires only the assumption that all integrated variables are included into the estimation. Detecting co-integration for inequality and income implies that no further integrated variables need to be considered (see e.g. Herzer and Vollmer, 2012; Johansen, 2000). Second, controls for, say, education (as in Malinen, 2012) would absorb all effects of inequality on income operating through the channel of education. Since we are not concerned with the particular mechanisms underlying the long-run relationship between inequality and income, such an absorption runs counter to the objective of our analysis.

3 Results and discussion

We have three main results. First, the results in table IV contradict the significant negative long-term relationship between inequality and per-capita income found in Malinen (2012) and Herzer and Vollmer (2012): we find a positive relationship for pre-tax, and no statistically significant result for post-tax inequality, at least not at a significance level of at least 5%.5 The finding that results for pre-tax and post-tax inequality differ is a significant contribution to the literature since the distinction between pre- and post-tax inequality is important with regards both to theory and policy. On a theoretical level, many mechanisms linking inequality and growth implicitly refer to either pre- or post-tax inequality. For example, the argument that credit constraints in highly unequal societies limit the access of talented students to university loans implicitly refers to post-tax, not pre-tax inequality. Indeed, Knowles (2005) has argued that this is the case with most channels and mechanisms the literature has discussed. Interestingly, though, we find a statistically significant relationship with pre-tax, not post-tax inequality.6 Our results clearly invite further research on the channels through which pre- and post-tax inequality and (changes in) income levels are linked. A better understanding would be of high relevance with regards to policy, since our results raise important questions about the potential and effects of redistributive policies. In all, our findings suggest to consider both pre-tax and post-tax levels of inequality and to give questions of redistribution an important place in future research.

Third, even though we find a positive relationship between income and pre-tax inequality using Pedroni's group-mean estimator, the nature of the relationship is heterogeneous across countries (figure 1): the positive relationship between pre-tax inequality and income we find on the sample level is by no means a universal pattern on the country level: a considerable number of countries experiences exactly the opposite. Moreover, the fact

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5We are aware of one previous study that has found distinct results for different measures of inequality: Knowles (2005) finds a negative relationship of economic growth with an expenditure-based measure of inequality, but not pre-tax inequality. Since his study used a different method – a more conventional panel regression technique – we do not go deeper into the comparison of our results with this study.

6Our result for post-tax inequality would be statistically significant at the 10% level, which would, however, be very generous given our sample size of more than 3,000 observations.
Table IV: Co-integration relationships, Pedroni’s PDOLS group-mean based estimator

<table>
<thead>
<tr>
<th></th>
<th>GDP per capita (log)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini, post-tax (log)</td>
<td>0.3595</td>
</tr>
<tr>
<td></td>
<td>[1.768]</td>
</tr>
<tr>
<td>Gini, pre-tax (log)</td>
<td>1.36***</td>
</tr>
<tr>
<td></td>
<td>[9.524]</td>
</tr>
</tbody>
</table>

t-statistics in parentheses

***: p < 0.001, **: p < 0.01, *: p < 0.5

Number of panel units: 87. Average number of periods per country: 35.
Observations: 3,087. Lags and leads: 2.

that we cannot identify a statistically significant relationship between post-tax inequality and income on the sample level is not because such a relationship is absent on the country level. Rather, as a number of countries experience a (significantly) positive and others a (significantly) negative relationship, both cancel each other out if we only look at the aggregate sample result (see figure 1 and table V). In the case of pre-tax inequality, the number of countries experiencing a positive relationship is larger than those with a negative experience, leaving us with a positive relationship on average.

Overall, we can group countries into eight categories (see table V), of which only group 8 – consisting of Honduras and Hungary – corresponds to the aggregated group-mean results in the strict sense. If we allowed for a 10% significance level in our group-mean coefficient, the – considerable larger – group 1 would align with the aggregate result. Some countries (such as China or Poland) experience a consistent and statistically significant positive relationship between both measures of inequality and income (group 1). Other countries (such as Canada or Malaysia) experience a consistent and significantly negative relationship (group 2). Smaller groups obtain statistically significant relationships for one measure of inequality only, positive or negative. Israel seems to be a special case, with a positive coefficient for pre-tax and a negative for post-tax inequality. Notably, no country experiences a significantly negative relationship for pre-tax and a positive relationship for post-tax income inequality.

These results suggest that the relationship between inequality and income is context-dependent, a fact that is reflected by the ambiguous results of previous (meta) studies. In theory, a number of different mechanisms could contribute to heterogeneous experiences, only some of which we mention. Future research should study them more closely.

Processes of uneven growth, as originally envisioned by Kuznets (1955), are consistent with positive and negative relationships between inequality and growth as a country undergoes processes of economic development. Even without assuming uniform processes of economic development across countries, it is sensible to expect that economic growth could contribute to raising inequality in some phases and to reducing it in others. The time series of the countries in our sample represent these economies’ experiences at quite different levels of income and development. At the same time, considering the
### Table V: Selected country groups

<table>
<thead>
<tr>
<th>Relationship with...</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-tax inequality</td>
<td></td>
</tr>
<tr>
<td>post-tax inequality</td>
<td></td>
</tr>
</tbody>
</table>

1. Positive Positive  
Argentina, China, Colombia, Egypt, El Salvador, Fiji, Georgia, Guatemala, Hong Kong SAR, China, India, Jamaica, Japan, Kenya, Lithuania, Madagascar, Malawi, Mauritania, Morocco, Pakistan, Poland, Sierra Leone, Singapore, Sri Lanka, Sudan, Taiwan, Tunisia and Zambia

2. Negative Negative  
Botswana, Canada, Chile, Costa Rica, Croatia, Denmark, Estonia, Greece, Iran, Jordan, Kazakhstan, Malaysia, Mauritius, Panama, Rwanda, Slovenia, South Korea and Uganda

3. Positive Negative  
Israel

4. Negative Positive  
None

5. Ambiguous Negative  
Bangladesh, Bulgaria, Czechia, Germany, Ireland, Moldova and Tanzania

6. Ambiguous Positive  
Qatar and Switzerland

7. Negative Ambiguous  
Armenia, Cyprus, Netherlands and New Zealand

8. Positive Ambiguous  
Honduras and Hungary

**Note:** We speak of a positive/negative relationship only if this relationship is statistically significant at least at the 5% level, otherwise we consider the relationship to be ambiguous. Countries for which both relationships are ambiguous are: Australia, Austria, Belarus, Belgium, Brazil, Cote d’Ivoire, Dominican Republic, Finland, France, Luxembourg, Mexico, Nepal, Norway, Peru, Philippines, Portugal, Romania, Slovakia, South Africa, Spain, Sweden, Thailand, Turkey, United Kingdom, United States and Uruguay.

The heterogeneity of the country groups outlined in Table V, the current stage of development cannot account for the heterogeneous experiences alone, since most country groups contain both high- and low-income countries.

Alternatively, socio-economic institutions might well play a decisive role. For example, as the varieties-of-capitalism literature has highlighted, countries have quite different institutional architectures that impact redistribution and thus inequality as well as economic opportunities (see, e.g., Hall and Soskice, 2001). These different institutional arrangements could in theory explain the differing country experiences - at least to some extent. More broadly conceived socio-institutional contexts, e.g. gender roles and imbalances, could play a similar role. The same is true for the recent literature on growth models (see, e.g., Behringer and Treeck, 2019), which studies the various drivers of aggregate demand. Since it has been shown that countries following different growth models, at least in the European Union, react differently (in terms of inequality, but also other macroeconomic variables of interest) to an increase in economic openness (see, e.g.,
Figure 1: Heterogeneity across estimates

Note: Panel (a) refers to the estimation of equation (2) with the pre-tax Gini, (b) to the estimation of the same equation with the post-tax Gini. We speak of an ambiguous relationship when the estimates are not significant at the 5% level.

Gräbner et al. (2019), it might well be that they also show differing relationships with regards to the inequality-income relationship. Yet, to judge the ability of these approaches to make sense of the heterogeneous relationships identified in this work requires more empirical and theoretical work.

4 Summary and conclusion

We studied the long-run relationship between inequality and income using panel co-integration techniques. We find distinct results depending on whether post-tax and pre-tax inequality is used. In the case of pre-tax inequality, we find a positive relationship; in the case of post-tax inequality, we find no statistically significant relationship. These results pertain to developments within countries over time, not to cross-country comparisons. We also identified a systematic heterogeneity of estimates across countries, a finding consistent with a recent meta-study by Neves et al. (2016).

Our results and the recognition of contextual dependence of the relationship suggest several avenues for further research: on the inductive side one might use clustering techniques as in Gräbner et al. (2019) to refine the country classifications. Most importantly,
comparative investigations of the country groups summarized in table V might suggest theoretical rationalizations of the heterogeneity and lead to models proposing concrete mechanisms underlying the heterogeneity of the income-inequality relationship.

References