Economics Bulletin

Volume 39, Issue 4

Monotonicity in Condorcet Jury Theorem under Strategic Voting

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Abstract

The Condorcet jury theorem states that if members' voting probabilities for the better alternative are identical and independent among members, and larger than 1/2, then the probability that a committee under simple majority voting chooses the better alternative is monotonically increasing in the committee size. This implies that the committee under simple majority voting decides more efficiently than single-person decision-making. This superiority of group decision-making under strategic voting for the binary signal model has already been demonstrated. We generalize this result and prove that the monotonicity property in the Condorcet jury theorem holds in the symmetric efficient equilibrium.

The author would like to thank Luba Sapir for comments and suggestions.
Citation: Naruto Nagaoka, (2019) "Monotonicity in Condorcet Jury Theorem under Strategic Voting", *Economics Bulletin*, Volume 39, Issue 4, pages 2688-2696
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Submitted: April 09, 2019. **Published:** November 24, 2019.

1 Introduction

We consider decision-making in committees under simple majority voting in the following situation. There are two alternatives; one is the better alternative for all members, but no member knows which one is better. Each member receives a signal that conveys information about which one is better. For this collective decision-making under uncertainty, the Condorcet jury theorem (CJT), argued in Condorcet (1785), is a well-known classical result. The simplest version of the CJT is as follows. Suppose that members' voting probabilities for the better alternative are identical and independent among members. If each member's voting probability for the better alternative is larger than 1/2, then the probability that the committee chooses the better alternative satisfies the following properties.[*1] The first property is the monotonicity property in the CJT (mCJT), that is, the probability that the committee chooses the better alternative is monotonically increasing in the committee size. The second property is the superiority of the group decision-making property in the CJT (gCJT), that is, the probability that the committee chooses the better alternative is larger than the member's voting probability for the better alternative. The first property implies the second property.[*2]

In this classical version of the CJT, it is assumed that each member votes sincerely. However, Austen-Smith and Banks (1996) pointed out that sincere voting may not be an equilibrium. Since Austen-Smith and Banks (1996), many researchers have studied whether the CJT holds in the strategic voting equilibrium.

In this paper, we also study the CJT under strategic voting. Wit (1998) demonstrated that the gCJT holds under strategic voting by analyzing the symmetric efficient equilibrium in the basic model. Then, we focus on the mCJT and show that the mCJT also holds in the symmetric efficient equilibrium. Our analysis is similar to Wit (1998). The key idea of Wit (1998) is that the strategy profile that maximizes the efficiency of the decision constitutes an equilibrium, argued by McLennan (1998). In the basic model, it is assumed that each member receives a binary signal and the signal which indicates a particular state is realized with probability larger than 1/2. This implies that a committee under sincere voting decides more efficiently than single-person decision-making, by the classical CJT. Sincere voting may not be the equilibrium for the committee; however, the symmetric efficient equilibrium strategy is more efficient than the sincere voting outcome. Then, if the gCJT holds under sincere voting, there exists an equilibrium in which the gCJT holds. We generalize Wit's (1998) decomposition analysis and show that the mCJT holds in the symmetric efficient equilibrium. To show that the mCJT holds, we prove that a larger committee decides more efficiently than a smaller committee under a symmetric efficient equilibrium strategy for the smaller committee. This strategy may not be the equilibrium for the larger committee; however, the symmetric efficient equilibrium is more efficient than this strategy profile.

Another related paper on the mCJT under strategic voting is by Chakraborty and Ghosh (2003). They generalized the binary signal model to a general signal model and studied the most efficient equilibrium instead of the symmetric efficient equilibrium considered by Wit (1998). One of their results is that the mCJT holds in the most efficient equilibrium. They

[*1] The abbreviations of each property in the CJT are from McCannon (2015).

[*2] The third property is the asymptotic property in the CJT (aCJT); that is, the probability that the committee chooses the better alternative approaches 1 as the committee size goes to infinity.

also characterized the most efficient equilibrium for the binary signal model and showed that the most efficient equilibrium is typically asymmetric.[*3] In contrast to Chakraborty and Ghosh (2003), we show that the mCJT holds even when we focus on the symmetric efficient equilibrium.

The rest of this paper is organized as follows. In Section 2, we present the model. In Section 3, we examine the equilibrium strategy and the efficiency of the decision by simple majority voting. In Section 4, we establish the monotonicity property in the CJT in the symmetric efficient equilibrium. Section 5 is concluding remarks.

2 Model

We consider a committee with $2n+1$ members, where $n \geq 1$. The committee decides to choose an alternative $d \in \{A, B\}$ by simple majority voting: each member simultaneously votes for an alternative A or B without abstention, and the decision is $d = A$ if and only if at least $n+1$ members vote for A. The members have the same preference over the alternatives, depending on the state $\omega \in \{A, B\}$. The utility function of each member is as follows:

$$\begin{aligned} u(d = A|\omega = A) = u(d = B|\omega = B) = 1, \\ u(d = A|\omega = B) = u(d = B|\omega = A) = 0. \end{aligned} \tag{1}$$

State A is the state in which alternative A is better than alternative B for all members; analogously for state B. We assume that no member knows the true state and the common prior probability is $\Pr(\omega = A) = \pi_A \in (0, 1)$. Before voting, each member i receives a binary signal $s_i \in \{a, b\}$ independently across members given the state. We assume that $\Pr(s_i = a|\omega = A) = t_a > 1/2$ and $\Pr(s_i = b|\omega = B) = t_b > 1/2$. Without loss of generality, we assume that $t_a \geq t_b$.

The timing is as follows. First, a state $\omega \in \{A, B\}$ is realized. Then, each committee member receives a signal. After receiving the signal, each member votes for an alternative A or B, and the committee's decision $d \in \{A, B\}$ is made by the simple majority rule. Finally, members' utilities are determined, depending on the decision and the state.

3 Symmetric Efficient Equilibrium

In this section, we consider the symmetric efficient equilibrium of the voting game. Let (σ_a, σ_b) denote a strategy where σ_a is the probability that the i-th member votes for A when he/she receives the signal $s_i = a$ and σ_b is the probability that he/she votes for A when he/she receives the signal $s_i = b$. Let $\gamma_A(\sigma_a, \sigma_b)$ and $\gamma_B(\sigma_a, \sigma_b)$ denote the probabilities that the member votes for the better alternative under the strategy (σ_a, σ_b) at state A and state B, that is:

$$\begin{aligned} \gamma_A(\sigma_a, \sigma_b) &= t_a\sigma_a + (1 - t_a)\sigma_b, \\ \gamma_B(\sigma_a, \sigma_b) &= (1 - t_b)(1 - \sigma_a) + t_b(1 - \sigma_b), \end{aligned}$$

[*3] See also Ladha et al.(2003), Persico (2004), Dekel and Piccione (2000), and Kawamura and Vlaseros (2017).

respectively.

Given a strategy (σ_a, σ_b), the expected utility of each member is equal to the expected probability that the committee chooses the better alternative at each state, by the assumption of the utility function (1):

$$U_{2n+1}(\sigma_a, \sigma_b) = \pi_A \rho_n^A(\sigma_a, \sigma_b) + (1-\pi_A)\rho_n^B(\sigma_a, \sigma_b),$$

where $\rho_n^A(\sigma_a, \sigma_b)$ and $\rho_n^B(\sigma_a, \sigma_b)$ denote the probabilities that the committee chooses the better alternative under the strategy (σ_a, σ_b) at state A and state B, that is:

$$\begin{aligned} \rho_n^A(\sigma_a, \sigma_b) &= \sum_{m \geq n+1} \binom{2n+1}{m} (\gamma_A(\sigma_a, \sigma_b))^m (1-\gamma_A(\sigma_a, \sigma_b))^{2n+1-m}, \\ \rho_n^B(\sigma_a, \sigma_b) &= \sum_{m \geq n+1} \binom{2n+1}{m} (\gamma_B(\sigma_a, \sigma_b))^m (1-\gamma_B(\sigma_a, \sigma_b))^{2n+1-m}, \end{aligned}$$

respectively. We define the efficiency of the decision as the value of the expected utility under the symmetric efficient equilibrium:

$$V(n) \equiv \max_{\sigma_a, \sigma_b} U_{2n+1}(\sigma_a, \sigma_b). \tag{2}$$

The following Lemma shows the symmetric efficient equilibrium.

Lemma 1 (Wit 1998). *The symmetric efficient equilibrium is as follows.*

$$(\sigma_a^*, \sigma_b^*) = \begin{cases} (0,0) & for \ \frac{\pi_A}{1-\pi_A} \in \left(0, \frac{1-t_b}{t_a}\left[\frac{1-t_b}{t_a}\right]^n\right] \\ (\sigma_n^*, 0) & for \ \frac{\pi_A}{1-\pi_A} \in \left(\frac{1-t_b}{t_a}\left[\frac{1-t_b}{t_a}\right]^n, \frac{1-t_b}{t_a}\left[\frac{t_b(1-t_b)}{t_a(1-t_a)}\right]^n\right) \\ (1,0) & for \ \frac{\pi_A}{1-\pi_A} \in \left[\frac{1-t_b}{t_a}\left[\frac{t_b(1-t_b)}{t_a(1-t_a)}\right]^n, \frac{t_b}{1-t_a}\left[\frac{t_b(1-t_b)}{t_a(1-t_a)}\right]^n\right] \\ (1, \sigma_n^*) & for \ \frac{\pi_A}{1-\pi_A} \in \left(\frac{t_b}{1-t_a}\left[\frac{t_b(1-t_b)}{t_a(1-t_a)}\right]^n, \frac{t_b}{1-t_a}\left[\frac{t_b}{1-t_a}\right]^n\right) \\ (1,1) & for \ \frac{\pi_A}{1-\pi_A} \in \left[\frac{t_b}{1-t_a}\left[\frac{t_b}{1-t_a}\right]^n, +\infty\right) \end{cases}, \tag{3}$$

where σ_n^* is increasing in π_A.

Proof. First, we prove that this strategy constitutes an equilibrium. Note that $(\sigma_a, \sigma_b) = (0,0)$ and $(\sigma_a, \sigma_b) = (1,1)$ constitute an equilibrium, because it is implied that the decision is $d = A$ when all the members follow $(\sigma_a, \sigma_b) = (1,1)$ and the decision is not changed even if one member deviates from this strategy. Then, $(\sigma_a, \sigma_b) = (1,1)$ constitutes an equilibrium. Similarly, $(\sigma_a, \sigma_b) = (0,0)$ also constitutes an equilibrium. Then, we consider the strategy where $(\sigma_a, \sigma_b) \neq (0,0)$ and $(\sigma_a, \sigma_b) \neq (1,1)$. The condition by which the member with signal s_i weakly prefers voting for A to voting for B is that the expected utility from voting for A is more than or equal to the expected utility from voting for B. Note that the member's vote affects the committee's decision when he/she is pivotal; the other n-members vote for A and the other n-members vote for B. By this fact and the assumption of utility function (1), the condition is:

$$\Pr(\omega = A | s_i, piv_{(\sigma_a, \sigma_b)}) \geq \Pr(\omega = B | s_i, piv_{(\sigma_a, \sigma_b)}),$$

where $piv_{(\sigma_a,\sigma_b)}$ denotes the event that the member is pivotal when other members follow a strategy (σ_a, σ_b). By the assumption that each member's signal is realized independently given the state and Bayes' rule, the equilibrium condition is:

$$L(\sigma_a, \sigma_b) \equiv \frac{\Pr(\omega = A)\Pr(s_i|\omega = A)\Pr(piv_{(\sigma_a,\sigma_b)}|\omega = A)}{\Pr(\omega = B)\Pr(s_i|\omega = B)\Pr(piv_{(\sigma_a,\sigma_b)}|\omega = B)} = 1. \tag{4}$$

Note that:

$$\frac{\Pr(piv_{(\sigma_a,\sigma_b)}|\omega = A)}{\Pr(piv_{(\sigma_a,\sigma_b)}|\omega = B)} = \begin{cases} \left[\frac{t_a(1-t_a\sigma)}{(1-t_b)(1-(1-t_b)\sigma)}\right]^n & \text{for} \quad (\sigma, 0) \\ \left[\frac{(1-(1-t_a)(1-\sigma))(1-t_a)}{(1-t_b(1-\sigma))t_b}\right]^n & \text{for} \quad (1, \sigma) \end{cases}$$

is continuous and decreasing in σ. Then, $L(\sigma, 0)$ and $L(1, \sigma)$ are also continuous and decreasing in σ. This implies that the strategy (3) constitutes an equilibrium.

Next, we show that this strategy profile maximizes the efficiency of the decision in symmetric strategies. Note that for $f_n(p) = \sum_{m\geq n+1} \binom{2n+1}{m} p^m(1-p)^{2n+1-m}$, it holds that:

$$\begin{aligned} \frac{d}{dp}f_n(p) &= \sum_{m\geq n+1} \binom{2n+1}{m} \left(mp^{m-1}(1-p)^{2n+1-m} - (2n+1-m)p^m(1-p)^{2n-m}\right) \\ &= \binom{2n+1}{n+1}(n+1)p^n(1-p)^n, \end{aligned}$$

because $-\binom{2n+1}{m}(2n+1-m)p^m(1-p)^{2n-m} + \binom{2n+1}{m+1}(m+1)p^m(1-p)^{2n-m} = 0$. Then, the first-order condition coincides with the equilibrium condition (4), because

$$\begin{aligned} \frac{\partial}{\partial\sigma_a}U_{2n+1}(\sigma_a, \sigma_b) \geq 0 \quad &\Leftrightarrow \quad \frac{\pi_A}{1-\pi_A}\frac{\frac{\partial\gamma_A(\sigma_a,\sigma_b)}{\partial\sigma_a}}{\frac{\partial\gamma_B(\sigma_a,\sigma_b)}{\partial\sigma_a}}\left[\frac{\gamma_A(\sigma_a,\sigma_b)(1-\gamma_A(\sigma_a,\sigma_b))}{\gamma_B(\sigma_a,\sigma_b)(1-\gamma_B(\sigma_a,\sigma_b))}\right]^n \geq 1, \\ \frac{\partial}{\partial\sigma_b}U_{2n+1}(\sigma_a, \sigma_b) \geq 0 \quad &\Leftrightarrow \quad \frac{\pi_A}{1-\pi_A}\frac{\frac{\partial\gamma_A(\sigma_a,\sigma_b)}{\partial\sigma_b}}{\frac{\partial\gamma_B(\sigma_a,\sigma_b)}{\partial\sigma_b}}\left[\frac{\gamma_A(\sigma_a,\sigma_b)(1-\gamma_A(\sigma_a,\sigma_b))}{\gamma_B(\sigma_a,\sigma_b)(1-\gamma_B(\sigma_a,\sigma_b))}\right]^n \geq 1. \end{aligned}$$

Finally, it is easy to see that the monotonicity of $L(\sigma, 0)$ and $L(1, \sigma)$ guarantees the second-order condition for the cases of $(\sigma_n^*, 0)$ and $(1, \sigma_n^*)$. □

4 Monotonicity Theorem

In this section, we establish the mCJT in the symmetric efficient equilibrium. If adding two members to the committee with an arbitrary fixed size improves the efficiency of the decision, the mCJT holds. Then, we compare the efficiency of the decisions between $2n+3$ members and $2n+1$ members. We establish our main theorem, by applying the following Lemma.

Lemma 2 (The classical CJT)**.** *Let p denote the probability that each member votes for the better alternative, and $f_n(p)$ denote the probability that the committee chooses the better alternative by simple majority voting with $2n+1$ members, where $f_n(p) = \sum_{m\geq n+1} \binom{2n+1}{m} p^m(1-p)^{2n+1-m}$. It holds that:*

$$f_{n+1}(p) - f_n(p) = \binom{2n+1}{n+1} [p(1-p)]^{n+1} (2p-1).$$

Proof. Adding two members may change the decision when additional members are pivotal, that is, (i) just $n+1$ members vote for the better alternative, and, (ii) just n members vote for the better alternative. In case (i), if both of the additional members vote for the worse alternative, the committee's decision changes from the better one to the worse one (negative effect of adding members). The joint probability that $n+1$ members vote for the better alternative and that both of the additional members vote for the worse alternative is $\binom{2n+1}{n+1}p^{n+1}(1-p)^n \times (1-p)^2$. In case (ii), if both of the additional members vote for the better alternative, the committee's decision changes from the worse one to the better one (positive effect of adding members). The joint probability that n members vote for the better alternative and that both of the additional members vote for the better alternative is $\binom{2n+1}{n+1}(1-p)^{n+1}p^n \times p^2$. Then, the difference between the positive effect and the negative effect is:

$$
\begin{aligned}
& \binom{2n+1}{n+1}(1-p)^{n+1}p^n \times p^2 - \binom{2n+1}{n+1}p^{n+1}(1-p)^n \times (1-p)^2 \\
= & \binom{2n+1}{n+1}\left[p(1-p)\right]^{n+1}(2p-1).
\end{aligned}
$$

Therefore, $f_{n+1}(p) - f_n(p) = \binom{2n+1}{n+1}\left[p(1-p)\right]^{n+1}(2p-1)$. □

Theorem 1 (The mCJT under strategic voting)**.** *It holds that $V(n+1) \geq V(n)$ for $\pi_A \in (0,1)$, where $V(\cdot)$ is defined in (2).*

Proof. Let $(\sigma_a^{**}, \sigma_b^{**})$ and (σ_a^*, σ_b^*) denote the symmetric efficient equilibrium in a $2n+3$ member committee and a $2n+1$ member committee, respectively. As $(\sigma_a^{**}, \sigma_b^{**})$ is the symmetric efficient equilibrium for $2n+3$ members, it holds that $U_{2n+3}(\sigma_a^{**}, \sigma_b^{**}) \geq U_{2n+3}(\sigma_a^*, \sigma_b^*)$. Hence, if it holds that $U_{2n+3}(\sigma_a^*, \sigma_b^*) \geq U_{2n+1}(\sigma_a^*, \sigma_b^*)$, we can conclude that $V(n+1) = U_{2n+3}(\sigma_a^{**}, \sigma_b^{**}) \geq U_{2n+3}(\sigma_a^*, \sigma_b^*) \geq U_{2n+1}(\sigma_a^*, \sigma_b^*) = V(n)$.

First, we consider the cases of $(\sigma_a^*, \sigma_b^*) = (0,0)$ and $(\sigma_a^*, \sigma_b^*) = (1,1)$. We focus on the latter case. For the case of $(\sigma_a^*, \sigma_b^*) = (1,1)$, $\rho_n^A(1,1) = \rho_{n+1}^A(1,1) = 1$ and $\rho_n^B(1,1) = \rho_{n+1}^B(1,1) = 0$. Then, $U_{2n+3}(\sigma_a^*, \sigma_b^*) = U_{2n+1}(\sigma_a^*, \sigma_b^*)$ for the case of $(\sigma_a^*, \sigma_b^*) = (1,1)$. An analogous argument holds for the case of $(\sigma_a^*, \sigma_b^*) = (0,0)$. Hence, we conclude that $V(n+1) = U_{2n+3}(\sigma_a^{**}, \sigma_b^{**}) \geq U_{2n+3}(\sigma_a^*, \sigma_b^*) = U_{2n+1}(\sigma_a^*, \sigma_b^*) = V(n)$ for the cases of $(\sigma_a^*, \sigma_b^*) = (0,0)$ and $(\sigma_a^*, \sigma_b^*) = (1,1)$.

Second, we consider the case of $(\sigma_a^*, \sigma_b^*) = (1,0)$. Note that $\gamma_A(1,0) = t_a > 1/2$ and $\gamma_B(1,0) = t_b > 1/2$. Then, it holds that $\rho_{n+1}^A(1,0) > \rho_n^A(1,0)$ and $\rho_{n+1}^B(1,0) > \rho_n^B(1,0)$, by applying the classical CJT. This implies that $U_{2n+3}(\sigma_a^*, \sigma_b^*) > U_{2n+1}(\sigma_a^*, \sigma_b^*)$ for the case of $(\sigma_a^*, \sigma_b^*) = (1,0)$. Hence, we conclude that $V(n+1) = U_{2n+3}(\sigma_a^{**}, \sigma_b^{**}) \geq U_{2n+3}(\sigma_a^*, \sigma_b^*) > U_{2n+1}(\sigma_a^*, \sigma_b^*) = V(n)$ for the cases of $(\sigma_a^*, \sigma_b^*) = (1,0)$.

Third, we consider the cases of $(\sigma_a^*, \sigma_b^*) = (\sigma_n^*, 0)$ and $(\sigma_a^*, \sigma_b^*) = (1, \sigma_n^*)$. Let $\gamma_A = \gamma_A(\sigma_a^*, \sigma_b^*)$ and $\gamma_B = \gamma_B(\sigma_a^*, \sigma_b^*)$. By Lemma 2,

$$
\begin{aligned}
& U_{2n+3}(\sigma_a^*, \sigma_b^*) - U_{2n+1}(\sigma_a^*, \sigma_b^*) \\
= & \binom{2n+1}{n+1}\left\{\pi_A[\gamma_A(1-\gamma_A)]^{n+1}(2\gamma_A - 1) + (1-\pi_A)[\gamma_B(1-\gamma_B)]^{n+1}(2\gamma_B - 1)\right\}.
\end{aligned}
$$

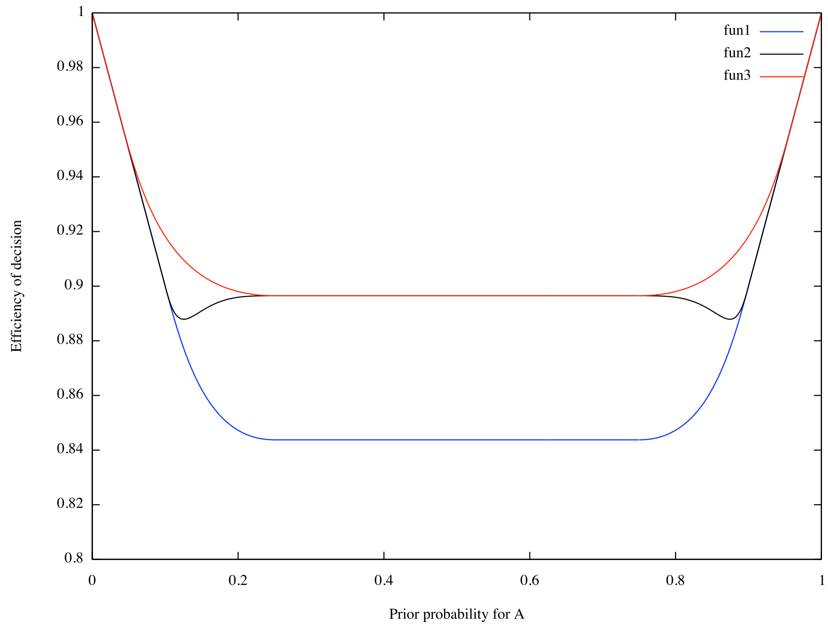

Figure 1: Efficiency of decision for $\pi_A \in (0,1)$

We focus on the case of $(\sigma_a^*, \sigma_b^*) = (\sigma_n^*, 0)$. For this case, it holds that $\gamma_B = (1-t_b)(1-\sigma_n^*) + t_b > 1/2$. When $\gamma_A = t_a\sigma_n^* > 1/2$, an analogous argument holds for the case of $(\sigma_a^*, \sigma_b^*) = (1, 0)$. Thus, we assume that $\gamma_A \leq 1/2$. Then,

$$\begin{aligned}
U_{2n+3}(\sigma_a^*, \sigma_b^*) \geq U_{2n+1}(\sigma_a^*, \sigma_b^*) \quad &\Leftrightarrow \quad \frac{\pi_A}{1-\pi_A}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^{n+1}\frac{2\gamma_A-1}{2\gamma_B-1} \geq -1 \\
&\Leftrightarrow \quad \frac{1-t_b}{t_a}\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\frac{2\gamma_A-1}{2\gamma_B-1} \geq -1 \\
&\Leftrightarrow \quad \frac{1-\gamma_A}{\gamma_B}\frac{2\gamma_A-1}{2\gamma_B-1} \geq -1
\end{aligned}$$

by the equilibrium condition $\frac{\pi_A}{1-\pi_A}\frac{t_a}{1-t_b}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^n = 1$, and $\frac{\gamma_A}{1-\gamma_B} = \frac{t_a}{1-t_b}$. Moreover,

$$\begin{aligned}
\frac{1-\gamma_A}{\gamma_B}\frac{2\gamma_A-1}{2\gamma_B-1} \geq -1 \quad &\Leftrightarrow \quad (1-\gamma_A)(1-2\gamma_A) \leq \gamma_B(2\gamma_B-1) \\
&\Leftrightarrow \quad (1-\gamma_A)((1-\gamma_A)-\gamma_A) \leq \gamma_B(\gamma_B-(1-\gamma_B)) \\
&\Leftrightarrow \quad \gamma_B^2 + \gamma_A(1-\gamma_A) \geq (1-\gamma_A)^2 + \gamma_B(1-\gamma_B).
\end{aligned}$$

This inequality holds by the fact that $\gamma_B > 1-\gamma_A$ and $\gamma_A(1-\gamma_A) > \gamma_B(1-\gamma_B)$ for $(\sigma_n^*, 0)$, since $\gamma_B > 1-\gamma_A \geq 1/2 \geq \gamma_A > 1-\gamma_B$. An analogous argument holds for the case of $(\sigma_a^*, \sigma_b^*) = (1, \sigma_n^*)$. Hence, we conclude that $V(n+1) = U_{2n+3}(\sigma_a^{**}, \sigma_b^{**}) \geq U_{2n+3}(\sigma_a^*, \sigma_b^*) \geq U_{2n+1}(\sigma_a^*, \sigma_b^*) = V(n)$ for the cases of $(\sigma_a^*, \sigma_b^*) = (\sigma_n^*, 0)$ and $(\sigma_a^*, \sigma_b^*) = (1, \sigma_n^*)$.

Therefore, it holds that $V(n+1) \geq V(n)$ for all cases. □

Figure 1 illustrates Theorem 1 for the case of three member committee ($n = 1$), where we assume $t_a = t_b = 3/4$. The graph of "fun 1" is the efficiency of decision $V(1) =$

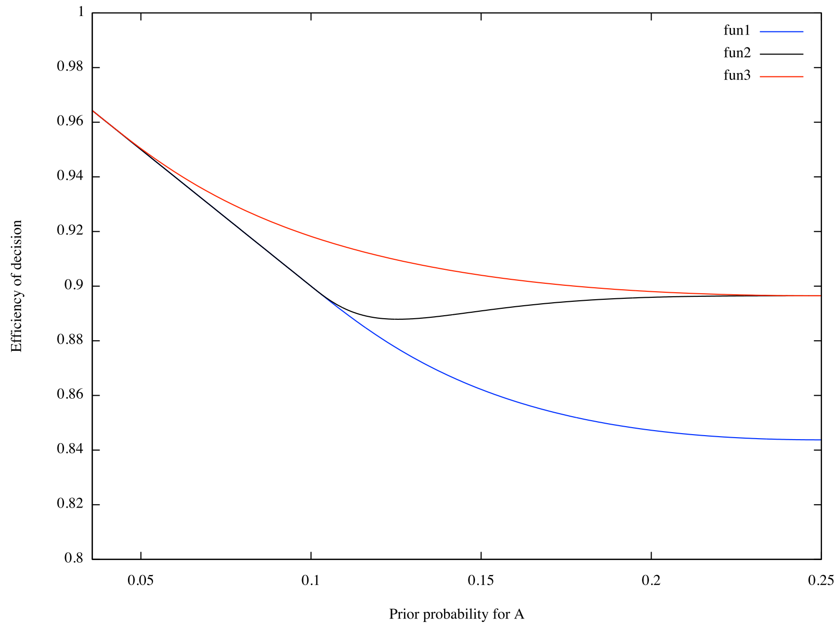

Figure 2: Efficiency of decision for $\pi_A \in \left(\frac{1}{28}, \frac{1}{4}\right)$

$U_3(\sigma_a^*, \sigma_b^*)$ for the prior probability $\pi_A \in (0,1)$. In this example, $(\sigma_a^*, \sigma_b^*) = (\sigma_1^*, 0)$ for $\pi_A \in (1/10, 1/4)$, and $(\sigma_a^*, \sigma_b^*) = (1, \sigma_1^*)$ for $\pi_A \in (3/4, 9/10)$. The graph of "fun 2" is the efficiency of decision of five member committee under the same strategy, $U_5(\sigma_a^*, \sigma_b^*)$. It holds that $U_5(\sigma_a^*, \sigma_b^*) > V(1)$ for $\pi_A \in (1/10, 9/10)$ and $U_5(\sigma_a^*, \sigma_b^*) = V(1)$ for $\pi_A \notin (1/10, 9/10)$. The graph of "fun 3" is the efficiency of decision $V(2) = U_5(\sigma_a^{**}, \sigma_b^{**})$. In this example, $(\sigma_a^{**}, \sigma_b^{**}) = (\sigma_2^*, 0)$ for $\pi_A \in (1/28, 1/4)$, and $(\sigma_a^{**}, \sigma_b^{**}) = (1, \sigma_2^*)$ for $\pi_A \in (3/4, 27/28)$. Figure 2 shows the same graphs for $\pi_A \in (1/28, 1/4)$. It holds that $V(2) > U_5(\sigma_a^*, \sigma_b^*)$ for $\pi_A \in (1/28, 1/4) \cup (3/4, 27/28)$ and $V(2) = U_5(\sigma_a^*, \sigma_b^*)$ for $\pi_A \notin (1/28, 1/4) \cup (3/4, 27/28)$, because $(\sigma_a^{**}, \sigma_b^{**})$ is the symmetric efficient equilibrium of five member committee. Then, $V(2) \geq U_5(\sigma_a^*, \sigma_b^*) \geq V(1)$ for $\pi_A \in (0,1)$.

5 Concluding Remarks

In this paper, we established the monotonicity in CJT under strategic voting. We assumed that each member receives a binary signal with the same probability and the signals are realized independently among members conditional on the state.

Under the sincere voting, these assumptions correspond to the assumptions on the voting probability made in the simplest version of the CJT. It is known that the issues of mCJT without these assumptions are not so simple, even when the sincere voting is assumed. For example, if the members' voting probabilities are not identical, adding two members does not necessarily improve the efficiency of decision (Karotkin and Paroush 2003; Sapir 2005), but the committee decides more efficiently than any random subgroups of the committee (Ben-Yashar and Paroush 2000; Berend and Sapir 2005). On the other hand, it depends on the details of model specification whether the mCJT holds without the independence assumption (Boland 1989).

Then, studying the robustness of the monotonicity in CJT under strategic voting with respect to the assumptions on the information structure of signals is left for the future research.

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