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Two-sided platforms, heterogeneous tastes, and coordination

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Abstract

This paper studies duopolistic price competition in a two-sided market with positive and negative indirect network externalities on both sides. I develop a model in which the externality is positive for some agents and negative for the others on each side. The paper shows that (i) a platform in equilibrium attracts a larger number of agents on both sides if the proportion of agents who incur a negative externality is small and (ii) each platform in equilibrium obtains a larger market share on one side and a lower market share on the other side if the proportion is large. Social welfare is not maximized in these equilibria because the platform with the lower market share on each side attracts too many agents in the former case while each platform attracts too many agents on the side with a lower market share in the latter case.

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1. Introduction

Advertising-supported media are two-sided platforms in that advertising slots help firms reach consumers, but the consequence of advertising is not simple. Consumers tend to incur disutility each time they see an advertisement. Some empirical studies, however, claim that consumers may obtain utility that increases in the number of advertisements (e.g., Kaiser and Song 2009).[1] It is thus relevant that some consumers obtain higher utility while others incur greater disutility from a medium with a larger number of advertisements. Firms in general can earn higher expected revenues by showing their advertisements to a larger number of consumers. Regarding profits, nevertheless, firms with limited capacities (e.g., individual professionals and family-owned restaurants) might face excess demand and incur additional costs. This fact implies that some firms might exhibit higher willingness to pay for media with smaller audiences (e.g., local newspapers) because those media yield higher expected profits to the firms. The consideration above suggests that an indirect network externality (simply called an "externality" hereafter) is positive for some economic agents and negative for others on both sides of a media market. To the best of my knowledge, the literature seldom investigates this situation.[2] An exception is Sokullu (2016a, 2016b), who empirically shows that the market demand functions are not monotone in opposite-side demand on both sides of the U.S. newspaper and German magazine industries. Sokullu (2016a, 2016b), however, constructs a model for a monopolistic medium.[3] The present paper studies price competition when positive and negative externalities coexist on each side of a duopolistic two-sided market.

I find that the pattern of the equilibrium configuration varies according to the proportion of potential users who incur a negative externality. If the proportion is smaller than half, one platform exceeds the rival platform in market share and price on both sides. This equilibrium configuration replicates the pattern in Gabszewicz and Wauthy (2004, 2014), who analyze the case of a positive externality exerted on each potential user and interpret the configuration as vertical differentiation in terms of market share. The configuration in the present paper is notable in that the lower market share is an advantage for the rival to attract agents who incur a negative externality, in which sense each platform engages in horizontal differentiation. If the externality is negative for the majority of potential users, then each platform attracts a larger number of agents on one side than the rival but fewer agents on the other side. Ambrus and Argenziano (2009) analyze the case of a positive externality exerted on each potential user and apply the concept of coalitional rationalizability proposed by Ambrus (2006),[4] and they show that a similar user allocation may arise in equilibrium. The difference is that each platform charges higher fees on its side with a larger number of users (incurring a weaker negative externality) in the present paper, whereas each platform chooses a lower price on such a side (where a weaker positive externality is exerted) in Ambrus and Argenziano (2009).

[1] Kaiser and Song (2009) conduct an empirical analysis of the German magazine industry and obtain the following results. First, consumer utility tends to increase in the number of advertising pages divided by that of content pages. Second, simulated models with consumer heterogeneity suggest the existence of both consumers who enjoy advertisements and who do not in some magazine segments.

[2] Rochet and Tirole (2003, 2006), Weyl (2010), and White and Weyl (2016) develop general models that allow for this situation but do not explicitly discuss the competitive outcome in this situation.

[3] Sokullu (2016b) calculates the relative prices of the magazines to account for competition but does not explicitly model price competition among magazines.

[4] Coalitional rationalizability rules out any bundle of strategies that are never optimal for an arbitrary group of players given other players' strategies.

Platforms avoid fierce competition in my model because each platform mildly competes on its side with fewer users. In sum, two different patterns of user allocations arise under a standard equilibrium concept in a single model with the coexistence of positive and negative externalities.

This paper shows that social welfare is maximized only if one platform attracts all agents on one side (say, side A). If the externality is positive for a sufficiently large number of agents, all agents on the other side (side B) should join the platform. As the proportion of agents who incur a negative externality grows, the number of side-B agents who should choose the platform decreases because the welfare impacts of the negative externality cannot be ignored. In particular, the other platform should attract all agents on side B if the proportion is sufficiently high. This result arises only in the case in which positive and negative externalities coexist.

2. Model

This section develops a duopoly model for a two-sided market á la Gabszewicz and Wauthy (2004, 2014) but with two differences. First, the externality is positive for some potential users and negative for others on each of the two sides. Second, both sides of the market are assumed to be fully covered.

There are two different groups of unit-mass agents on sides A and B of a platform market. Platforms 1 and 2 are symmetric firms that provide agents on both sides with their services, charging participation fees. Each platform consists of stand-alone and intermediation services. A stand-alone service has an agent-, platform-, and side-common intrinsic value, which is denoted by $v \in \mathbb{R}_{++}$ and is high enough for any potential user to enjoy a strictly positive payoff from either platform. An intermediation service connects agents on side A with those on side B, which causes the side-A users of a platform to exert an externality on the side-B users of the platform and *vice versa*. The agents on each side are assumed to have different valuations of intermediation services, in that the externality is exerted positively on some of them and negatively on others, and that the impact of the externality depends on each agent's type.[5] The types are uniformly distributed on a unit interval $[-\alpha, 1-\alpha]$, where $\alpha \in (0,1)$ is an exogenous side-common parameter that indicates the proportion of agents who incur a negative externality.[6] Potential users on each side simultaneously choose one platform after each platform determines its prices. Provided that platform 1 attracts $n_1^A \in [0,1]$ agents on side A and charges $p_1^B \in \mathbb{R}$ on

[5]This formulation can apply to the advertising side of a media market. Firms generally obtain higher benefits from a medium with a larger audience, which is the case of a positive externality. Some firms, however, possibly run their businesses with too small staffs to accept a large number of consumers (e.g., individual professionals and family-owned firms). These firms might incur higher costs if showing their advertisements to a larger audience because they need to address demand that exceeds their capacities. In this sense, a negative externality may be exerted on some firms.

The formulation can also apply to the subscription side. Although some consumers might enjoy advertisements *per se*, it is a natural assumption that consumers are likely to incur disutility by seeing advertisements. Nevertheless, the latter consumers can also obtain benefits if they see matched advertisements and purchase the advertised products. The sign of such a consumer's payoff from advertisements is determined by the relation between the total disutility of seeing them and the total utility from his/her purchase(s). In sum, positive and negative externalities plausibly coexist on the side.

[6]Appendix B shows that the main results are robust if the parameter α is side-specific as long as the side-A and side-B parameters are not substantially different.

side B, an agent of type $\theta \in [-\alpha, 1-\alpha]$ on side B receives a payoff of

$$u_1^B\left(p_1^B, n_1^A; \theta\right) \equiv v + \theta n_1^A - p_1^B$$

from that platform. Let $u_2^B(p_2^B, n_2^A; \theta)$ denote the payoff obtained by the agent from platform 2, where the notations of p_2^B and n_2^A are analogous. Define $u_1^A(p_1^A, n_1^B; \theta)$, $u_2^A(p_2^A, n_2^B; \theta)$, p_1^A, p_2^A, n_1^B, and n_2^B similarly. Notice here that θ is the coefficient on n_1^A or n_2^A and that each market share is a consequence of platform choices on side A. Thus, the relation between each side-B agent's expectations of n_1^A and n_2^A plays a crucial role in determining the configuration of the allocation on side B. The rest of the present paper assumes potential users to (rationally) expect that

$$n_1^{Ae} > n_2^{Ae} \quad \text{and} \quad n_1^{Be} \neq n_2^{Be}, \tag{1}$$

where $n_1^{Ae} \in [0,1]$, $n_2^{Ae} \in [0,1]$, $n_1^{Be} \in [0,1]$, and $n_2^{Be} \in [0,1]$ denote their expectations of the respective market shares. Note that this expression represents all cases in which $n_1^{Ae} \neq n_2^{Ae}$ and $n_1^{Be} \neq n_2^{Be}$ because the market is symmetrically formulated.[7] Define $\widetilde{\theta}^B(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae})$ as a type such that

$$\begin{aligned} & u_1^B\left[p_1^B, n_1^{Ae}; \widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right)\right] = u_2^B\left[p_2^B, n_2^{Ae}; \widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right)\right] \\ \Longleftrightarrow\ & \widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right) = \frac{p_1^B - p_2^B}{n_1^{Ae} - n_2^{Ae}}. \end{aligned} \tag{2}$$

If $n_1^{Ae} > n_2^{Ae}$ and under the full-coverage assumption,[8] platform 1 (which attracts a larger number of side-A agents) is chosen by the side-B potential users of type θ such that

$$\begin{aligned} & u_1^B\left(p_1^B, n_1^{Ae}; \theta\right) \geq u_2^B\left(p_2^B, n_2^{Ae}; \theta\right) \\ \Longleftrightarrow\ & \theta \geq \widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right) \quad \text{and} \quad -\alpha \leq \theta \leq 1-\alpha, \end{aligned}$$

and platform 2 is chosen by those of type θ such that

$$\begin{aligned} & u_1^B\left(p_1^B, n_1^{Ae}; \theta\right) < u_2^B\left(p_2^B, n_2^{Ae}; \theta\right) \\ \Longleftrightarrow\ & \theta < \widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right) \quad \text{and} \quad -\alpha \leq \theta \leq 1-\alpha. \end{aligned}$$

Let $D_1^B(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae})$ and $D_2^B(p_2^B, p_1^B; n_2^{Ae}, n_1^{Ae})$ denote the market demand functions for platforms 1 and 2 on side B, respectively:

$$\begin{aligned} D_1^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right) &= \begin{cases} 1-\alpha-\widetilde{\theta}^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right) & \text{if } \widetilde{\theta}^B(\cdot) \in [-\alpha, 1-\alpha] \\ 1 & \text{if } \widetilde{\theta}^B(\cdot) < -\alpha \\ 0 & \text{if } \widetilde{\theta}^B(\cdot) > 1-\alpha \end{cases} \\ D_2^B\left(p_2^B, p_1^B; n_2^{Ae}, n_1^{Ae}\right) &= 1 - D_1^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right). \end{aligned}$$

The market demand for platforms 1 and 2 on side A, denoted by $D_1^A(p_1^A, p_2^A; n_1^{Be}, n_2^{Be})$ and $D_2^A(p_2^A, p_1^A; n_2^{Be}, n_1^{Be})$, respectively, is analogously obtained.

[7] See footnote 14 for the derivation of an equilibrium when $n_1^{Ae} = n_2^{Ae}$ and/or $n_1^{Be} = n_2^{Be}$, which is unstable in that it may arise only if both platforms attract exactly the same number of agents on one or both side(s).

[8] Side B is fully covered if agents of the lowest type ($\theta = -\alpha$) eventually obtain weakly positive payoffs, where $v \geq \alpha n_2^A + p_2^B$.

Before proceeding to profit maximization, I formulate the process to form market-share expectations. This paper assumes that potential users expect the opposite-side market shares independently of the opposite-side prices and that their expectations are fulfilled in equilibrium.[9] Under this formulation, the market demand functions are defined as those of the own-side prices and the expected opposite-side market shares only.

Platforms 1 and 2 maximize their own profits with respect to their participation fees, given one another's price strategy and the potential users' expectations of the market shares. Specifically, platform 1 chooses (p_1^A, p_1^B) that maximizes

$$\pi_1\left(p_1^A, p_1^B; p_2^A, p_2^B, n_1^{Ae}, n_1^{Be}, n_2^{Ae}, n_2^{Be}\right) \equiv p_1^A D_1^A\left(p_1^A, p_2^A; n_1^{Be}, n_2^{Be}\right) + p_1^B D_1^B\left(p_1^B, p_2^B; n_1^{Ae}, n_2^{Ae}\right)$$

given (p_2^A, p_2^B) and $(n_1^{Ae}, n_1^{Be}; n_2^{Ae}, n_2^{Be})$. Platform 2's profit maximization is symmetrically formulated, where $\pi_2(p_2^A, p_2^B; p_1^A, p_1^B, n_2^{Ae}, n_2^{Be}, n_1^{Ae}, n_1^{Be})$ denotes the platform's profit. Note that the marginal and fixed costs of production are normalized to zero for both platforms. Platform 1's optimal side-B price, denoted by $p_1^B(p_2^B; n_1^{Ae}, n_2^{Ae})$, follows from the first-order condition:

$$\frac{\partial \pi_1\left(p_1^A, p_1^B; \cdot\right)}{\partial p_1^B} = 0 \iff p_1^B\left(p_2^B; n_1^{Ae}, n_2^{Ae}\right) = \frac{p_2^B + (1-\alpha)\left(n_1^{Ae} - n_2^{Ae}\right)}{2}$$

because the second-order condition that $\partial^2 \pi_1(p_1^A, p_1^B; \cdot)/\partial(p_1^B)^2 < 0$ holds for any p_1^B.[10] Platform 2's optimal side-B price is analogous:

$$\frac{\partial \pi_2\left(p_2^A, p_2^B; \cdot\right)}{\partial p_2^B} = 0 \iff p_2^B\left(p_1^B; n_2^{Ae}, n_1^{Ae}\right) = \frac{p_1^B + \left(n_1^{Ae} - n_2^{Ae}\right)\alpha}{2},$$

and $\partial^2 \pi_2(p_2^A, p_2^B; \cdot)/\partial(p_2^B)^2 < 0$ for any p_2^B. One can similarly derive each platform's price strategy on side A.

An equilibrium consists of the $(p_1^{A*}, p_1^{B*}; p_2^{A*}, p_2^{B*})$ and $(n_1^{A*}, n_2^{A*}; n_1^{B*}, n_2^{B*})$ that solve the following equation system:

$$p_1^{A*} = p_1^A\left(p_2^{A*}; n_1^{B*}, n_2^{B*}\right) \qquad p_1^{B*} = p_1^B\left(p_2^{B*}; n_1^{A*}, n_2^{A*}\right) \tag{3}$$
$$p_2^{A*} = p_2^A\left(p_1^{A*}; n_2^{B*}, n_1^{B*}\right) \qquad p_2^{B*} = p_2^B\left(p_1^{B*}; n_2^{A*}, n_1^{A*}\right) \tag{4}$$
$$n_1^{A*} = D_1^A\left(p_1^{A*}, p_2^{A*}; n_1^{B*}, n_2^{B*}\right) \qquad n_2^{A*} = D_2^A\left(p_2^{A*}, p_1^{A*}; n_2^{B*}, n_1^{B*}\right) \tag{5}$$
$$n_1^{B*} = D_1^B\left(p_1^{B*}, p_2^{B*}; n_1^{A*}, n_2^{A*}\right) \qquad n_2^{B*} = D_2^B\left(p_2^{B*}, p_1^{B*}; n_2^{A*}, n_1^{A*}\right). \tag{6}$$

In equations (3) and (4), each platform's profit-maximizing prices are consistent with the competitor's expectation of them. In equations (5) and (6), the potential users' expectations of each platform's market share are fulfilled.

[9] This formulation follows Gabszewicz and Wauthy (2004, 2014), who adapt Katz and Shapiro's (1985) fulfilled-expectation concept to the context of a two-sided market. There is another expectation concept (see Hagiu and Hałaburda 2014 for a discussion), employed for instance by Armstrong (2006), that allows for expectation dependent on the opposite-side prices. Appendix B discusses the robustness of the main results if the latter concept applies.

[10] The derivatives of $\pi_1(p_1^A, p_1^B; \cdot)$ contain no term derived from the respective other sides because (i) platform choices are made independently of the opposite-side prices and (ii) the platform incurs zero production cost. The platform therefore maximizes its side-A and side-B profits separately. In particular, this separation causes the platform to charge a positive price on each side. This discussion would also apply if the market were partially covered.

The policymaker is interested in a welfare-maximizing user allocation.[11] In this paper, social welfare is the sum of the total benefits on sides A and B because the costs are zero. Consider first the total benefit on side B, denoted by $W^B(n_1^B, n_2^B; n_1^A, n_2^A)$. The agents of higher types should join the platform with a larger number of opposite-side users. Recall that the types are uniformly distributed on a unit interval. Similar to the equilibrium analysis, hereafter, assume that

$$n_1^A > n_2^A \quad \text{and} \quad n_1^B \neq n_2^B \tag{7}$$

and focus on the full-coverage case.[12] Then,

$$W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right) = v + n_1^A \int_{1-\alpha-n_1^B}^{1-\alpha} \theta \mathrm{d}\theta + n_2^A \int_{-\alpha}^{n_2^B-\alpha} \theta \mathrm{d}\theta.$$

The total benefit on side A, denoted by $W^A(n_1^A, n_2^A; n_1^B, n_2^B)$, is analogous, but one should note that its functional form depends on the relation between n_1^B and n_2^B:[13]

$$W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) = v + \begin{cases} n_1^B \displaystyle\int_{1-\alpha-n_1^A}^{1-\alpha} \theta \mathrm{d}\theta + n_2^B \int_{-\alpha}^{n_2^A-\alpha} \theta \mathrm{d}\theta & \text{if } n_1^B > n_2^B \\ n_1^B \displaystyle\int_{-\alpha}^{n_1^A-\alpha} \theta \mathrm{d}\theta + n_2^B \int_{1-\alpha-n_2^A}^{1-\alpha} \theta \mathrm{d}\theta & \text{if } n_1^B < n_2^B. \end{cases}$$

Social welfare is therefore

$$W\left(n_1^A, n_2^A, n_1^B, n_2^B\right) \equiv W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right).$$

Let $(n_1^{A**}, n_2^{A**}, n_1^{B**}, n_2^{B**})$ denote $(n_1^A, n_2^A, n_1^B, n_2^B)$ that maximizes $W(n_1^A, n_2^A, n_1^B, n_2^B)$.

3. Equilibrium and Its Welfare Consequence

This section discusses the equilibrium and welfare maximization. I show that each platform in equilibrium chooses a different price strategy according to the proportion of agents who incur a negative externality. The section establishes that the efficient allocation pattern also depends on the proportion and differs from the equilibrium configuration (except when $\alpha = 1/2$). See Appendix A for proofs of the propositions.

[11] This paper formulates welfare maximization as the problem to obtain an efficient outcome that would arise if the policymaker could directly allocate all agents. One can instead consider an alternative formulation such that (i) the policymaker announces an efficient user allocation and (ii) agents simultaneously make platform choices as announced. The former and latter formulations are equivalent if agents follow the announcement while expecting the opposite-side market shares. By contrast, both formulations result in different outcomes if agents somehow do not incorporate the announcement in their market-share expectations. The latter case can be addressed by reformulating welfare maximization as the problem to maximize social welfare given each agent's expectation, which is parallel to competition described in this paper. Appendix B discusses this welfare-maximization problem and establishes that the welfare implications of the competitive outcome are qualitatively unchanged.

[12] Appendix B shows that full coverage is efficient because social welfare decreases as any agent on each side exits from the market.

[13] If $n_1^B < n_2^B$, for instance, the side-A agents of higher types should use platform 2.

Table 1: Equilibrium Configuration If $0 < \alpha < 1/2$ in Proposition 1

	Side A	Side B
Platform 1	n^*: high / p^*: high types: $+$	n^*: high / p^*: high types: $+$
Platform 2	n^*: low / p^*: low types: $+$ and $-$	n^*: low / p^*: low types: $+$ and $-$

Note: The sub- and superscripts of n^* and p^* are omitted, which applies to Table 2.

3.1. Competitive Outcome

The following proposition states the equilibrium configurations.[14]

Proposition 1. The equilibrium under condition (1) is characterized as follows.

1. If $\alpha \in (0, 1/2)$, $p_1^{A*} = p_1^{B*} = (1-2\alpha)(2-\alpha)/9$, $p_2^{A*} = p_2^{B*} = (1-2\alpha)(1+\alpha)/9$, $n_1^{A*} = n_1^{B*} = (2-\alpha)/3$, and $n_2^{A*} = n_2^{B*} = (1+\alpha)/3$.
2. If $\alpha \in (1/2, 1)$, $p_1^{A*} = p_2^{B*} = (2\alpha-1)(1+\alpha)/9$, $p_2^{A*} = p_1^{B*} = (2\alpha-1)(2-\alpha)/9$, $n_1^{A*} = n_2^{B*} = (1+\alpha)/3$, and $n_2^{A*} = n_1^{B*} = (2-\alpha)/3$.

Table 1 summarizes the properties of the equilibrium configuration when the externality is positive for the majority of potential users. Platform 1 obtains larger market shares on both sides and attracts agents of higher types under higher participation fees. Platform 2 forms smaller networks on both sides, where agents of lower types participate and pay lower fees. Gabszewicz and Wauthy (2004, 2014) demonstrate a similar configuration in the absence of a negative externality and regard the configuration as the occurrence of vertical differentiation in opposite-side market share. The configuration when $0 < \alpha < 1/2$ in the present paper, on the other hand, exhibits horizontal differentiation due to the coexistence of positive and negative externalities. Platform 1 attracts only agents of positive (and higher) types, who choose the platform because it yields higher benefits to them. Platform 2 attracts all of the agents incurring a negative externality, who can mitigate their disutilities by choosing the platform. One can clarify this property by altering the type distributions. Suppose, in addition to the agents of types $\theta \in [-\alpha, 1-\alpha]$, that there is a small mass of potential users whose type is $\delta < -\alpha$ on each side. Once δ decreases enough, platform 2 can improve its profit by attracting only the type-δ agents ($n_1^{A*} > n_2^{A*}$ and $n_1^{B*} > n_2^{B*}$) under participation fees higher than those of platform 1 ($p_2^{A*} > p_1^{A*}$ and $p_2^{B*} > p_1^{B*}$),[15] some of whose users incur a weak negative externality. This example supports the possibility of the platform with the lower market shares charging higher fees as a consequence of horizontal differentiation.

Table 2 shows the equilibrium configuration when the externality is negative for the majority of potential users. Each platform has a side with a larger market share (called its

[14] There also exist equilibria in which both platforms are expected on, say, side B to attract the same number of side-A agents. The platforms face Bertrand competition on side A, and the allocation on that side is determined by the expectation formed on side B. If the side-B expectation is that $n_1^{Ae} > n_2^{Ae}$, $p_1^{A*} = p_1^{B*} = p_2^{A*} = p_2^{B*} = 0$, $0 \leq n_2^{A*} < n_1^{A*} \leq 1$, and $n_1^{B*} = n_2^{B*} = 1/2$, which arises only when $\alpha = 1/2$. If the expectation is that $n_1^{Ae} = n_2^{Ae}$, $n_1^{A*} = n_2^{A*} = n_1^{B*} = n_2^{B*} = 1/2$ and $p_1^{A*} = p_2^{A*} = p_1^{B*} = p_2^{B*} = 0$ for all α, as in Gabszewicz and Wauthy (2004, 2014).

[15] Weyl (2010) and White and Weyl (2016) propose pricing that enables the platform to attract a desired number of agents only.

Table 2: Equilibrium Configuration If $1/2 < \alpha < 1$ in Proposition 1

	Side A	Side B
Platform 1	n^*: high / p^*: high types: $-$	n^*: low / p^*: low types: $+$ and $-$
Platform 2	n^*: low / p^*: low types: $+$ and $-$	n^*: high / p^*: high types: $-$

Table 3: Welfare-Maximizing Allocation Configuration If $0 < \alpha < 1/2$ in Proposition 2

	Market Share on Side A	Market Share on Side B
Platform 1	1	1 ($0 < \alpha \leq 1/4$) high ($1/4 < \alpha < 1/2$)
Platform 2	0	0 ($0 < \alpha \leq 1/4$) low ($1/4 < \alpha < 1/2$)

larger side) occupied totally by negative types of agents and a side with a smaller market share, which enables each platform to mitigate the negative externality incurred by the platform's users on its larger side and to charge higher participation fees there. Each platform also makes price competition less severe because a platform obtains a larger market share on a side if the other platform attracts fewer agents on that side. Note that this configuration displays a similar user allocation to Ambrus and Argenziano's (2009). However, the characteristics of Ambrus and Argenziano's (2009) configuration are that each user on a larger side enjoys a small benefit and each platform cannot charge high fees on its larger side. This difference occurs because the present paper allows for the coexistence of positive and negative externalities.

3.2. Welfare Maximization and Efficiency

The following proposition shows the efficient allocation pattern for each α.

Proposition 2. Under condition (7), social welfare is maximized if and only if the agents are allocated as follows.[16]

1. If $\alpha \in (0, 1/4]$, $n_1^{A**} = n_1^{B**} = 1$ and $n_2^{A**} = n_2^{B**} = 0$.
2. If $\alpha \in (1/4, 3/4)$, $n_1^{A**} = 1$, $n_1^{B**} = (3 - 4\alpha)/2$, $n_2^{A**} = 0$, and $n_2^{B**} = (4\alpha - 1)/2$.
3. If $\alpha \in [3/4, 1)$, $n_1^{A**} = 1$, $n_1^{B**} = 0$, $n_2^{A**} = 0$, and $n_2^{B**} = 1$.

Tables 3 and 4 display the efficient allocation configurations according to the proportion of agents who incur a negative externality. When the externality is positive for a sufficiently large number of agents, social welfare is maximized if platform 1 gathers all agents on both sides because most agents have high enough types to enjoy the highest benefits from the platform. When the externality is negative for a sufficiently large number of agents, social welfare is maximized if platform 1 attracts all agents on side A but

[16] This statement holds whenever $n_1^A \not< n_2^A$ because $W(1/2, 1/2, 1/2, 1/2) = \lim_{(n_1^A, n_1^B) \to (1/2, 1/2)} W(\cdot)$.

Table 4: Welfare-Maximizing Allocation Configuration If $1/2 < \alpha < 1$ in Proposition 2

	Market Share on Side A	Market Share on Side B
Platform 1	1	0 ($3/4 \leq \alpha < 1$) low ($1/2 < \alpha < 3/4$)
Platform 2	0	1 ($3/4 \leq \alpha < 1$) high ($0 < \alpha \leq 1/4$)

none on side B because most agents have sufficiently low types and that allocation minimizes their disutilities. When the proportion of agents who incur a negative externality is moderate, social welfare is maximized if platform 1 attracts all agents on side A and some agents on side B. The platform's efficient side-B market share decreases in the proportion of agents who incur a negative externality. The configurations discussed above significantly differ from those in Gabszewicz and Wauthy (2004, 2014) and Ambrus and Argenziano (2009), where social welfare is maximized if one platform attracts all agents on both sides. This difference arises because agents who incur a negative externality play important roles in reducing social welfare in the present paper.

Proposition 2 implies that none of the equilibrium configurations established in Proposition 1 is efficient. On side A, platform 1 in equilibrium always attracts a smaller number of potential users than the efficient level ($n_1^{A*} < n_1^{A**}$). On side B, the equilibrium and efficient market shares differ. Platform 1 in equilibrium obtains a smaller market share than the efficient level ($n_1^{B*} < n_1^{B**}$) if the externality is positive for more than half of agents ($0 < \alpha < 1/2$), which means that the positive externality on each side is less enhanced. The platform in equilibrium attracts a larger number of potential users than the efficient level ($n_1^{B*} > n_1^{B**}$) if the externality is negative for more than half of agents ($1/2 < \alpha < 1$), where the negative externality on each side is less mitigated.

4. Conclusion

This paper studies the equilibrium and efficient outcomes in a two-sided market where positive and negative externalities coexist on both sides. Each potential user's expectation of opposite-side market demand differentiates the platforms such that a positive externality is enhanced and a negative externality is mitigated. Social welfare is maximized only if a platform attracts all agents on one side, in which the platform's efficient market share on the other side weakly decreases as the proportion of agents who incur a negative externality grows. The equilibrium and efficient outcomes almost always differ.

Appendix

A. Proofs

This section contains proofs of the propositions established in the main text.

A.1. Proof of Proposition 1

Consider the equilibrium allocation and prices on side B. Equations (3) and (4) yield each platform's side-B price:

$$p_1^{B*} = \frac{\frac{p_1^{B*}+\left(n_1^{A*}-n_2^{A*}\right)\alpha}{2} + (1-\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{2} \iff p_1^{B*} = \frac{(2-\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right) \tag{8}$$

$$p_2^{B*} = \frac{\frac{(2-\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{3} + \left(n_1^{A*}-n_2^{A*}\right)\alpha}{2} = \frac{(1+\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right). \tag{9}$$

The price difference on side B is

$$p_1^{B*} - p_2^{B*} = \frac{(1-2\alpha)\left(n_1^{A*}-n_2^{A*}\right)}{3}.$$

Equation (6) yields the equilibrium side-B demand for each platform:

$$n_1^{B*} = 1-\alpha-\frac{\frac{(1-2\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right)}{n_1^{A*}-n_2^{A*}} = \frac{2-\alpha}{3} \tag{10}$$

$$n_2^{B*} = \frac{\frac{(1-2\alpha)}{3}\left(n_1^{A*}-n_2^{A*}\right)}{n_2^{A*}-n_2^{A*}} + \alpha = \frac{1+\alpha}{3}. \tag{11}$$

Therefore, $n_1^{B*} > n_2^{B*}$ if $\alpha \in (0, 1/2)$, and $n_1^{B*} < n_2^{B*}$ if $\alpha \in (1/2, 1)$. The difference in market share on side B is

$$n_1^{B*} - n_2^{B*} = \frac{1-2\alpha}{3}.$$

The equilibrium prices and side-A allocation are obtained as follows. If $\alpha \in (0, 1/2)$, the derivations of p_1^{A*}, p_2^{A*}, n_1^{A*}, and n_2^{A*} are analogous to those of expressions (8) to (11), respectively. The equilibrium prices are

$$p_1^{B*} = \frac{2-\alpha}{3}\cdot\frac{1-2\alpha}{3} = p_1^{A*} \qquad p_2^{B*} = \frac{1+\alpha}{3}\cdot\frac{1-2\alpha}{3} = p_2^{A*}.$$

If $\alpha \in (1/2, 1)$, the same discussion applies except that (p_1^{A*}, n_1^{A*}) and (p_2^{A*}, n_2^{A*}) replace one another.

A.2. Proof of Proposition 2

This proof consists of two parts. The first part shows that the first-order conditions for the welfare-maximization problem violate one of the second-order conditions, which implies that the welfare-maximizing outcomes are corner solutions. The second part obtains the welfare-maximizing allocation under condition (7).

A.2.1. Second-Order Conditions for Welfare Maximization

Suppose that $n_1^B > n_2^B$. Note that $n_2^A = 1 - n_1^A$ and $n_2^B = 1 - n_1^B$, which implies that $\mathrm{d}n_2^A/\mathrm{d}n_1^A = \mathrm{d}n_2^B/\mathrm{d}n_1^B = -1$. I have the following:

$$\begin{aligned}
\frac{\partial W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_1^A} &= n_1^B \cdot \left(1-\alpha-n_1^A\right) - \left(n_2^A - \alpha\right) n_2^B \\
&= n_1^B \cdot \left(1-\alpha-n_1^A\right) - \left(1 - n_1^A - \alpha\right)\left(1 - n_1^B\right) \\
&= n_1^A - 2n_1^A n_1^B + 2\left(1-\alpha\right) n_1^B - \left(1-\alpha\right) \\
\frac{\partial W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)}{\partial n_1^A} &= \left(1-\alpha\right) n_1^B - \frac{\left(n_1^B\right)^2}{2} - \frac{\left(n_2^B\right)^2}{2} + \alpha n_2^B \\
&= \left(1-\alpha\right) n_1^B - \frac{\left(n_1^B\right)^2}{2} - \frac{\left(1-n_1^B\right)^2}{2} + \left(1 - n_1^B\right)\alpha \\
&= -\left(n_1^B\right)^2 + 2\left(1-\alpha\right) n_1^B - \frac{1}{2} + \alpha.
\end{aligned}$$

The first-order condition with respect to n_1^A is that

$$\begin{aligned}
\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A} &= \frac{\partial}{\partial n_1^A}\left[W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)\right] \\
&= n_1^A - 2n_1^A n_1^B - \left(n_1^B\right)^2 + 4\left(1-\alpha\right) n_1^B + 2\alpha - \frac{3}{2} = 0.
\end{aligned}$$

Analogously, the first-order condition with respect to n_1^B is that

$$\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^B} = -\left(n_1^A\right)^2 + 4\left(1-\alpha\right) n_1^A - 2n_1^A n_1^B + n_1^B + 2\alpha - \frac{3}{2} = 0.$$

Extracting the condition on side B from that on side A yields the following:

$$\begin{aligned}
& n_1^A - n_1^B - \left(n_1^B\right)^2 + \left(n_1^A\right)^2 + 4\left(1-\alpha\right)\left(n_1^B - n_1^A\right) = 0 \\
\iff & \left(n_1^A - n_1^B\right)\left(n_1^A + n_1^B + 4\alpha - 3\right) = 0 \\
\iff & n_1^A = n_1^B \quad \text{or} \quad n_1^A + n_1^B = 3 - 4\alpha.
\end{aligned}$$

If the latter equality is the case (which holds only if $1/4 < \alpha < 1/2$), the first-order condition with respect to n_1^A is rewritten as

$$\begin{aligned}
\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A} &= n_1^A \underbrace{-\left(n_1^A + n_1^B\right)^2 + \left(n_1^A\right)^2}_{=-2n_1^A n_1^B - \left(n_1^B\right)^2} + 4\left(1-\alpha\right) n_1^B + 2\alpha - \frac{3}{2} \\
&= n_1^A + \left[-\left(3-4\alpha\right)^2 + \left(n_1^A\right)^2\right] + 4\left(1-\alpha\right)\underbrace{\left(3 - 4\alpha - n_1^A\right)}_{=n_1^B} + 2\alpha - \frac{3}{2} \\
&= \underbrace{\left(n_1^A\right)^2 - \left(3-4\alpha\right) n_1^A + \frac{1}{2}\left(3-4\alpha\right)}_{\equiv f\left(n_1^A\right)} = 0.
\end{aligned}$$

The function $f(n_1^A)$ is minimized if

$$\frac{\mathrm{d}f\left(n_1^A\right)}{\mathrm{d}n_1^A} = 0 \iff n_1^A = \frac{\left(3-4\alpha\right)}{2}.$$

The first-order condition with respect to n_1^A does not hold in this case because

$$\begin{aligned} f\left(\frac{3-4\alpha}{2}\right) &= \frac{(3-4\alpha)^2}{4} - \frac{(3-4\alpha)^2}{2} + \frac{1}{2}(3-4\alpha) \\ &= \frac{2(3-4\alpha)-(3-4\alpha)^2}{4} \\ &= \frac{-16\alpha^2 - 3 + 16\alpha}{4} \\ &= \frac{-16\left(\alpha-\frac{1}{2}\right)^2+1}{4} > 0 \end{aligned}$$

as long as $1/4 < \alpha < 1/2$ (the value is close to 0 as $\alpha \to 1/4$). Therefore, $n_1^A = n_1^B \equiv n_1 \in (1/2, 1)$ if n_1^A and n_1^B solve the first-order conditions. One can derive the following single condition from the original first-order condition with respect to n_1^A multiplied by 2:

$$-6n_1^2 + 2(5-4\alpha)n_1 + 4\alpha - 3 = 0 \tag{12}$$

for all α. The second-order partial derivatives of $W(n_1^A, n_2^A, n_1^B, n_2^B)$ regarding platform 1 are

$$\begin{aligned} \frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^A\right)^2} &= -2n_1^B + 1 < 0 \\ \frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^B\right)^2} &= -2n_1^A + 1 < 0 \\ \frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A \partial n_1^B} &= -2\left(n_1^A + n_1^B\right) + 4(1-\alpha), \end{aligned}$$

which is simplified as follows:

$$\begin{aligned} \left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^A\right)^2}\right|_{n_1^A=n_1^B=n_1} &= \left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial\left(n_1^B\right)^2}\right|_{n_1^A=n_1^B=n_1} = -2n_1 + 1 \\ \left.\frac{\partial^2 W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^A \partial n_1^B}\right|_{n_1^A=n_1^B=n_1} &= -4\left[n_1 - (1-\alpha)\right]. \end{aligned}$$

The determinant of the Hessian matrix is a function of n_1:

$$\begin{aligned} H(n_1) &\equiv \left(4n_1^2 - 4n_1 + 1\right) - \left[16n_1^2 - 32(1-\alpha)n_1 + 16(1-\alpha)^2\right] \\ &= -12n_1^2 + 4(7-8\alpha)n_1 - 16\alpha^2 + 32\alpha - 15. \end{aligned}$$

Multiplying equation (12) by 2 and solving the equation for $H(n_1)$ yields

$$\begin{aligned} & -12n_1^2 + 4(5-4\alpha)n_1 + 8\alpha - 6 = 0 \\ \iff & -12n_1^2 + 4(7-8\alpha)n_1 - 4(2-4\alpha)n_1 + \left(-16\alpha^2 + 32\alpha - 15\right) \\ & \quad + 16\alpha^2 - 24\alpha + 9 = 0 \\ \iff & H(n_1) = 8(1-2\alpha)n_1 - 16\alpha^2 + 24\alpha - 9, \end{aligned}$$

which is linear in n_1. One can obtain the supremum of $H(n_1)$ as follows:

$$\begin{cases} \lim_{n_1 \to 1} H(n_1) = -16\alpha^2 + 8\alpha - 1 = -16\left(\alpha - \frac{1}{4}\right)^2 \leq 0 & \text{if } 0 < \alpha < \frac{1}{2} \\ H(n_1) = 0 \cdot n_1 - 16 \cdot \left(\frac{1}{2}\right)^2 + 24 \cdot \frac{1}{2} - 9 = -1 < 0 & \text{if } \alpha = \frac{1}{2} \\ \lim_{n_1 \to \frac{1}{2}} H(n_1) = -16\alpha^2 + 16\alpha - 5 = -16\left(\alpha - \frac{1}{2}\right)^2 - 1 < 0 & \text{if } \frac{1}{2} < \alpha < 1. \end{cases}$$

The determinant of the Hessian matrix cannot be strictly positive if the first-order conditions hold. There is no interior welfare-maximizing allocation such that $n_1^B > n_2^B$ for any α.

Suppose now that $n_1^B < n_2^B$. The first-order derivatives of social welfare with respect to n_2^A and n_1^B are mirror images of one another. Similarly to the case in which $n_1^B > n_2^B$,

$$\begin{aligned} \frac{\partial W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_2^A} &= n_2^A - 2n_2^A n_2^B + 2(1-\alpha) n_2^B - (1-\alpha) \\ &= 2n_2^A n_1^B - n_2^A - 2(1-\alpha) n_1^B + (1-\alpha) \\ \frac{\partial W^B\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_2^A} &= -\frac{\partial W^B\left(n_1^A, n_2^A; n_1^B, n_2^B\right)}{\partial n_1^A} \\ &= \left(n_1^B\right)^2 - 2(1-\alpha) n_1^B - \alpha + \frac{1}{2}. \end{aligned}$$

The derivative with respect to n_2^A thus equals

$$\begin{aligned} \frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_2^A} &= \frac{\partial}{\partial n_2^A}\left[W^A\left(n_1^A, n_2^A; n_1^B, n_2^B\right) + W^B\left(n_1^B, n_2^B; n_1^A, n_2^A\right)\right] \\ &= 2n_2^A n_1^B - n_2^A + \left(n_1^B\right)^2 - 4(1-\alpha) n_1^B - 2\alpha + \frac{3}{2}. \end{aligned}$$

Regarding n_1^B, an analogous calculation is presented:

$$\frac{\partial W\left(n_1^A, n_2^A, n_1^B, n_2^B\right)}{\partial n_1^B} = \left(n_2^A\right)^2 - 4(1-\alpha) n_2^A + 2n_2^A n_1^B - n_1^B - 2\alpha + \frac{3}{2}.$$

Both derivatives correspond to the values of $-\partial W(n_1^A, n_2^A, n_1^B, n_2^B)/\partial n_1^A$ and $-\partial W(n_1^A, n_2^A, n_1^B, n_2^B)/\partial n_1^B$ obtained when $n_1^B > n_2^B$, respectively, but replace n_1^A with n_2^A. The first-order conditions can thus be rewritten as analogous equalities to those in the preceding paragraph. The own-variable second-order derivatives equal those in that paragraph that are multiplied by -1 and replace n_1^A with n_2^A; thus, the own-variable second-order conditions hold (because $0 < n_2^A < 1/2$ and $0 < n_1^B < 1/2$). Moreover, the cross-variable second-order derivatives are also those in that paragraph that are multiplied by -1 and replace n_1^A with n_2^A, which implies that the determinant of the Hessian matrix is analogous. One can therefore establish the absence of an interior welfare-maximizing outcome such that $n_1^B < n_2^B$ for any α in the same way as above. I summarize below how to prove this statement. First, the first-order conditions imply that

$$n_2^A = n_1^B \quad \text{or} \quad n_2^A + n_1^B = 3 - 4\alpha,$$

but the latter equality is incompatible with the condition of n_2^A. Second, if $n_2^A = n_1^B$, the second-order condition with regard to the Hessian matrix does not hold for any α.

A.2.2. Welfare-Maximizing Allocation

The preceding discussion establishes that the welfare-maximization problem has a corner solution only: $n_1^{A**} = 1$ and $n_2^{A**} = 0$. Social welfare can thus be expressed as $\overline{W}(n_1^B) \equiv W(1, 0, n_1^B, 1 - n_1^B)$. If $1/4 < \alpha < 3/4$, n_1^{B**} is n_1^B derived from the first-order condition to maximize $\overline{W}(n_1^B)$:

$$\frac{\mathrm{d}\overline{W}\left(n_1^B\right)}{\mathrm{d}n_1^B} = \frac{1-2\alpha}{2} + \left(1 - \alpha - n_1^B\right) = 0 \iff n_1^B = \frac{3-4\alpha}{2},$$

which satisfies the second-order condition ($\mathrm{d}^2\overline{W}(n_1^B)/\mathrm{d}(n_1^B) = -1 < 0$ for any n_1^B). Otherwise, $n_1^{B**} = 1$ if $0 < \alpha \leq 1/4$, and $n_1^{B**} = 0$ if $3/4 \leq \alpha < 1$.

B. Discussions on the Major Assumptions

This section reviews a few major assumptions made in the main text. I relax these assumptions and examine their impacts on this paper's main statements. For simplicity, this section focuses on the case in which conditions (1) and (7) in the main text hold.

B.1. Efficiency of Full Coverage

To examine the relevance of focusing on the full-coverage case in the welfare analysis, consider the welfare impacts of a deviation from the outcome in Proposition 2 such that an agent who has the lowest type ($\theta = -\alpha$) exits. First, I remark that the lowest-type agents on side A are allocated platform 1 for any α in the case of welfare maximization. Suppose that $0 < \alpha \leq 1/4$, where the lowest-type agents on side B are also allocated platform 1. Exit by an agent of the lowest type on side A reduces social welfare in

$$\underbrace{(v-\alpha)}_{\text{side } A} + \underbrace{\frac{1-2\alpha}{2}}_{\text{side } B},$$

which is positive for any strictly positive v because this decrement equals

$$v + \underbrace{\left(-2\alpha + \frac{1}{2}\right)}_{\in\left[0, \frac{1}{2}\right)} > 0.$$

The decrement of social welfare as an agent of the lowest type on side B exits is analogous. Suppose that $1/4 < \alpha < 1/2$, where the lowest-type agents on side B are allocated platform 2. Exit by an agent of the lowest type on side A reduces social welfare in

$$\begin{aligned}\underbrace{\left(v - \alpha n_1^{B**}\right)}_{\text{side } A} + \underbrace{\frac{2\left(1-\alpha\right) - \left(n_1^{B**}\right)^2}{2}}_{\text{side } B} &= v - \left[\frac{\left(n_1^{B**}\right)^2}{2} + \alpha n_1^{B**}\right] + (1-\alpha) \\ &= v + \frac{4\alpha - 1}{8},\end{aligned}$$

which is positive for any strictly positive v because

$$0 < \frac{4\alpha - 1}{8} < \frac{1}{8}.$$

The decrement of social welfare as an agent of the lowest type on side B exits is just $v > 0$ because platform 2 obtains no market share on side A. Suppose that $1/2 < \alpha < 3/4$.[17] Notice that the lowest-type agents on each side are allocated the same as when $1/4 < \alpha < 1/2$ (i.e., platform 1 for those on side A and platform 2 for those on side B) and that the mathematical form of n_1^{B**} is also the same, which implies that social welfare decreases as an agent of the lowest type on each side exits for any strictly positive v. If $3/4 \leq \alpha < 1$, for any strictly positive v, exit by an agent of the lowest type on each side reduces social welfare in v because all agents incur no negative externality (i.e., $n_1^{A**} = 1$ but $n_1^{B**} = 0$ and $n_2^{A**} = 0$ but $n_2^{B**} = 1$).

I now discuss the welfare implications of the above result. Social welfare decreases for any strictly positive v as an agent of the lowest type exits from the market. Exit by any other agent on each side also reduces social welfare for any strictly positive v because, under the assumption that both platforms have the same intrinsic value, the agent obtains a higher benefit than that of the lowest type. Therefore, the policymaker does not have an incentive to make the market partially covered for any strictly positive v.

B.2. Side-Asymmetric Type Distributions

This subsection discusses how the main results are changed if the types of agents on each side are asymmetrically distributed. I maintain the type distribution on side A but modify the type distribution on side B in that $(\alpha + \epsilon)$ replaces α, where $\epsilon \in \mathbb{R}$ is an exogenous parameter such that $0 < (\alpha + \epsilon) < 1$ (i.e., $-\alpha < \epsilon < 1 - \alpha$), which means that the externality is negative for $(\alpha + \epsilon)$ potential users on side B. In sum, the main results are robust unless the type distributions on both sides substantially differ.

I first investigate the impacts of this extension on the equilibrium configurations. Equations (8) to (11) apply to (p_1^{A*}, p_2^{A*}) and (n_1^{A*}, n_2^{A*}) with α being replaced by $(\alpha + \epsilon)$ and to (p_1^{A*}, p_2^{A*}) and (n_1^{A*}, n_2^{A*}) with no change. The equilibrium allocation on side B is

$$n_1^{B*} = \frac{2 - (\alpha + \epsilon)}{3} \qquad n_2^{B*} = \frac{1 + (\alpha + \epsilon)}{3},$$

which implies that

$$\begin{aligned} n_1^{B*} > n_2^{B*} &\iff \frac{1}{2} < n_1^{B*} < 1 \\ &\iff \underbrace{-1 - \alpha}_{<-\alpha} < \epsilon < \frac{1}{2} - \alpha \\ n_1^{B*} < n_2^{B*} &\iff 0 < n_1^{B*} < \frac{1}{2} \\ &\iff \frac{1}{2} - \alpha < \epsilon < \underbrace{2 - \alpha}_{>1-\alpha}. \end{aligned}$$

The equilibrium allocation on side A is unchanged; therefore, its property is the same as in the original case. The equilibrium prices on side B are

$$p_1^{B*} = \frac{[2 - (\alpha + \epsilon)]\,(1 - 2\alpha)}{9} \qquad p_2^{B*} = \frac{[1 + (\alpha + \epsilon)]\,(1 - 2\alpha)}{9},$$

[17] The statement below holds if $\alpha = 1/2$ by relaxing condition (7) regarding the allocation on side B.

which means that $p_1^{B*} > p_2^{B*}$ if $0 < \alpha < 1/2$ and $n_1^{B*} > n_2^{B*}$ and that $p_1^{B*} < p_2^{B*}$ if $1/2 < \alpha < 1$ and $n_1^{B*} < n_2^{B*}$. The properties of p_1^{A*} and p_2^{A*} are qualitatively the same as in the original case because p_1^{A*} and p_2^{A*} do not depend on ϵ if n_1^{B*} and n_2^{B*} are given. Proposition 1 is therefore robust to the extent that

$$\begin{cases} -\alpha < \epsilon < \dfrac{1}{2} - \alpha & \text{if } 0 < \alpha < \dfrac{1}{2} \\ \dfrac{1}{2} - \alpha < \epsilon < 1 - \alpha & \text{if } \dfrac{1}{2} < \alpha < 1. \end{cases}$$

Consider how the side-asymmetric type distributions affect welfare maximization. Appendix A shows that no interior solution exists under the symmetric type distributions because the second-order conditions do not totally hold. The statements of that section are robust if α is not close to $1/4$ and $|\epsilon|$ is moderate. The proof of Proposition 2 applies with the following modification:

$$\begin{aligned} \frac{\mathrm{d}W\left(1, 0, n_1^B, 1 - n_1^B\right)}{\mathrm{d}n_1^B} &= \frac{1 - 2\alpha}{2} + \left[1 - (\alpha + \epsilon) - n_1^B\right] = 0 \\ \iff n_1^{B**} &= \frac{3 - 4\left(\alpha + \frac{\epsilon}{2}\right)}{2}, \end{aligned}$$

where the marginal benefit yielded on side A is unchanged. The efficient allocation configuration has the same properties for each α except that $(\alpha + \epsilon/2)$ replaces α.

B.3. Active Beliefs

In this subsection, the concept called “active beliefs” (Gabszewicz and Wauthy 2004) or “responsive expectations” (Hagiu and Hałaburda 2014) applies to the formation of each potential user’s demand expectation. This concept allows for agents who form the rational expectations of the opposite-side market demand functions, which depend on the opposite-side prices. If the price of platform 1 increases on side A, for instance, potential users on side B expect the platform to lose some users on side A. Each platform considers this additional price effect when the platform determines its price strategy. To briefly see the impacts of this expectation formation, I investigate how the platforms deviate from the respective equilibrium price strategies in the main text, how their deviations change the equilibrium allocation on each side, and how robust the comparison between the equilibrium and efficient outcomes in the main text is.[18]

Suppose that $0 < \alpha < 1/2$, where the original equilibrium is characterized by strictly positive threshold types and side-symmetric expectations. Assume that these properties remain to hold. The partial derivative of a platform’s profit with respect to each of its prices evaluated at the original equilibrium is strictly negative because the platform can attract additional agents on a side by reducing the participation fees on the other side. The partial derivatives with regard to sides A and B are symmetric. The partial derivative

[18]This subsection examines none of the second-order conditions for profit maximization. However, I below focus on the situation in which each platform chooses a price strategy that enhances its profit (i.e., the price strategy does not minimize the platform’s profit), and show that both platforms have no incentive to change their prices intensely and that the sign of the resulting threshold type on each side is unchanged (i.e., the solution candidates seem to be interior and unique). This discussion supports the uniqueness and optimality of the price strategy derived as a solution to each platform’s profit maximization.

of platform 1's profit is strictly lower than that of platform 2's regarding each side because the former platform charges higher participation fees in the original equilibrium. Thus, the participation fees of a platform decrease on both sides in the same value, and the decrements of platform 1's prices are higher than those of platform 2's. These imply that the threshold types on both sides decrease and are symmetric. The new threshold types are strictly negative because the cross-side price effects shrink as the threshold types approach zero while there exist own-side price countereffects. Both platforms in this case obtain market shares such that $(1/2 <)(2-\alpha)/3 < n_1^A = n_1^B < 1-\alpha (< 1)$ and $(0 <)\alpha < n_2^A = n_2^B < (1+\alpha)/3 (< 1/2)$, where $n_1^A < n_1^{A**}$ and $n_1^B < n_1^{B**}$ for any $\alpha \in (0, 1/2)$. Platform 1 keeps its prices and market shares strictly higher than platform 2 ($p_1^A = p_1^B > p_2^A = p_2^B$ and $n_1^A = n_1^B > n_2^A = n_2^B$).[19] This outcome is consistent with the above two properties. Therefore, the equilibrium configuration and its welfare consequence are qualitatively unchanged from those in the main text in that $p_1^{A*} > p_2^{A*}$, $p_1^{B*} > p_2^{B*}$, $n_2^{A*} < n_1^{A*} < n_1^{A**}$, and $n_2^{B*} < n_1^{B*} < n_1^{B**}$.

Suppose next that $1/2 < \alpha < 1$, where the original equilibrium is characterized by strictly negative threshold types and side-asymmetric expectations. Assume that these properties remain to be satisfied. The partial derivative of a platform's profit with respect to each of its prices evaluated at the original equilibrium is strictly positive because the platform can attract additional agents on a side by raising the participation fees on the other side. The partial derivative regarding the side with fewer users is higher than that regarding the other side because the platform in the original equilibrium charges higher participation fees on the latter side. Each platform thus chooses a higher price on its side with fewer users and reduces the number of users on the side, which also enables the platform to raise its price on the other side with the number of users kept larger than that in the original equilibrium. The threshold types on both sides increase (from a strictly negative value) and are symmetric except for the allocations. One can find that the new threshold types are strictly negative from, again, the balance between the cross-side price effects and the own-side price countereffects. The two platforms here obtain market shares such that $(1/2 <)(1+\alpha)/3 < n_1^A = n_2^B < \alpha (< 1)$ and $(0 <)1-\alpha < n_2^A = n_1^B < (2-\alpha)/3 (< 1/2)$, where $n_1^A < n_1^{A**}$ and $n_1^B > n_1^{B**}$ for any $\alpha \in (1/2, 1)$. The property of the allocation on each side is not substantially changed from that in the original equilibrium in that $p_1^A = p_2^B > p_1^B = p_2^A$ and $n_1^A = n_2^B > n_1^B = n_2^A$. The outcome derived in this paragraph is consistent with the above two properties. Therefore, the equilibrium configuration and its welfare consequence have qualitatively the same properties as those in the main text in that $p_1^{A*} > p_2^{A*}$, $p_1^{B*} < p_2^{B*}$, $n_2^{A*} < n_1^{A*} < n_1^{A**}$, and $n_1^{B**} < n_1^{B*} < n_2^{B*}$.

B.4. Welfare Maximization Given Agents' Expectations

This subsection investigates social-welfare maximization when the policymaker also treats each agent's market-share expectation as given. The main text follows the work of Gabszewicz and Wauthy (2004, 2014), and assumes in the equilibrium analysis that agents form market-share expectations independently of the opposite-side prices and that their expectations are fulfilled with the realizations. The first assumption is adapted to welfare maximization by redefining social welfare as

$$\widetilde{W}\left(n_1^A, n_2^A, n_1^B, n_2^B; n_1^{Ae}, n_2^{Ae}, n_1^{Be}, n_2^{Be}\right)$$

[19]The process to derive expression (2) implies that the denominator of the threshold type on each side is strictly positive, which also applies to the next paragraph.

$$\equiv W^A\left(n_1^A,n_2^A;n_1^{Be},n_2^{Be}\right)+W^B\left(n_1^B,n_2^B;n_1^{Ae},n_2^{Ae}\right),$$

where the total benefit equals $W^A(n_1^A,n_2^A;n_1^{Be},n_2^{Be})$ on side A and $W^B(n_1^B,n_2^B;n_1^{Ae},n_2^{Ae})$ on side B. The second assumption is adapted by reformalizing welfare maximization as the problem to maximize $\widetilde{W}(n_1^A,n_2^A,n_1^B,n_2^B;\cdot)$ with respect to n_1^A, n_2^A, n_1^B, and n_2^B (given n_1^{Ae}, n_2^{Ae}, n_1^{Be}, and n_2^{Be}) subject to $n_1^{Ae}=n_1^{A**}$, $n_2^{Ae}=n_2^{A**}$, $n_1^{Be}=n_1^{B**}$, and $n_2^{Be}=n_2^{B**}$. Notice that the first-order derivatives of the function contain no term of the realized market shares on the respective other sides under this framework, which implies that the second-order condition with regard to the Hessian matrix does not need to be examined.

I solve the problem above. Recall that $\mathrm{d}n_2^A/\mathrm{d}n_1^A=\mathrm{d}n_2^B/\mathrm{d}n_1^B=-1$ because both sides are fully covered. Consider first the case in which $n_1^{Be}>n_2^{Be}$. The first-order derivatives of social welfare are

$$\begin{aligned}
\frac{\partial\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial n_1^A}&=\left(1-\alpha-n_1^A\right)n_1^{Be}-\left(n_2^A-\alpha\right)n_2^{Be}\\
&\qquad\left(\text{The calculation process for }\frac{\partial W^A(\cdot)}{\partial n_1^A}\text{ in Appendix A applies.}\right)\\
&=n_1^A-2n_1^An_1^{Be}+2(1-\alpha)n_1^{Be}-(1-\alpha)\\
&=\left(1-2n_1^{Be}\right)n_1^A-(1-\alpha)\left(1-2n_1^{Be}\right)\\
\frac{\partial\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial n_1^B}&=\left(1-\alpha-n_1^B\right)n_1^{Ae}-\left(n_2^B-\alpha\right)n_2^{Ae}\\
&\qquad\left(\text{A similar calculation process to that of }\frac{\partial\widetilde{W}(\cdot)}{\partial n_1^A}\text{ applies.}\right)\\
&=\left(1-2n_1^{Ae}\right)n_1^B-(1-\alpha)\left(1-2n_1^{Ae}\right),
\end{aligned}$$

and the second-order derivatives are

$$\frac{\partial^2\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial\left(n_1^A\right)^2}=1-2n_1^{Be}\qquad\frac{\partial^2\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial\left(n_1^B\right)^2}=1-2n_1^{Ae}.$$

The first-order conditions are that

$$\begin{aligned}
\frac{\partial\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial n_1^A}=0&\iff n_1^A=1-\alpha\\
\frac{\partial\widetilde{W}\left(n_1^A,n_2^A,n_1^B,n_2^B;\cdot\right)}{\partial n_1^B}=0&\iff n_1^B=1-\alpha,
\end{aligned}$$

and the second-order conditions hold. Notice that the allocations derived above can arise if and only if $0<\alpha<1/2$. Welfare maximization is characterized in this case by the following outcome: $n_1^{A**}=n_1^{B**}=1-\alpha$ and $n_2^{A**}=n_2^{B**}=\alpha$. Suppose next that $n_1^{Be}<n_2^{Be}$. The first-order and second-order conditions with respect to n_1^B are unchanged from those in the preceding case. The conditions with respect to n_2^A (not n_1^A) are parallel to those with respect to n_1^A in the preceding case. Notice that agents form consistent expectations with the allocations to be derived here if and only if $1/2<\alpha<1$. Social welfare is thus maximized in the current case by the following outcome: $n_1^{A**}=n_2^{B**}=\alpha(>1/2)$ and $n_2^{A**}=n_1^{B**}=1-\alpha(<1/2)$.

An interpretation of this result is that all agents of positive types should join the platform with the larger market share on the other side and the other agents should participate in the other platform. This outcome is equivalent to what would arise if both platforms chose identical prices and each potential user joined the platform to maximize his/her payoff. The outcome differs (except when $\alpha \to 0$ or $\alpha \to 1$) from that in Proposition 2 because the former one abstracts the cross-side welfare impacts of each agent's participation in a particular platform; therefore, social welfare is enhanced the most only in the case of Proposition 2. Nevertheless, one can straightforward find that the comparison between the equilibrium and efficient outcomes in the main text is robust if welfare maximization is formulated as in this subsection: $n_1^{A*} < n_1^{A**}$ for any $\alpha \in (0, 1)$, and $n_1^{B*} < n_1^{B**}$ if $0 < \alpha < 1/2$ and $n_1^{B*} > n_1^{B**}$ if $1/2 < \alpha < 1$.

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