Risk Aversion and Optimal Hedge Ratio in Commodities Futures Markets

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Abstract

In this paper, our main objective is to show that the determination of the optimal hedge ratio for a raw material producer, who is submitted to income risk, depends on the type of its utility function. More precisely, we maximize the expected utility of wealth for the following four utility functions: quadratic, exponential, power and expo-power. We then derive an explicit formula of the optimal hedge ratio when using the first two functions and an implicit function when the agent's preferences are modeled by power or expo-power utility functions. The results obtained are then applied to data on quantity and prices collected from the NCCB and ICCO from 1980 to 2013. The implementation with some Matlab programs provides the estimated value of the optimal hedge ratio for a Cameroonian cocoa producer around 80% for quadratic and exponential utility functions, and 87.9% for power utility and between 50% and 65% for the expo-power utility function.

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1 Introduction

Agricultural commodities are of great interest in most Sub-Saharan Africa countries (SSA) because their production provides jobs, they are used as inputs in major production processes, and they generate most of the export revenue and in some cases, most state’s revenue in such countries. Those commodities are often characterized by a variety of risk that affect stakeholders at different stages in their value chain. Among those risks, we will focus on quantity and production risk.

Most agricultural commodities markets have experienced substantial short-term fluctuations in their prices which have an undesirable effect on producers’ total income and on commercial balance sheet of major producing countries. The most usual definition of price risk exposure for a producer is the difference between the expected sale price, on the basis of which a producer makes production and marketing decisions, and the actual sale price (Gemetch et al., 2009). These unpredictable differences in prices are commonly explained by the fluctuations in input prices, the quality of the infrastructure, the quality and quantity of the harvest, the state of the economy in importer’s and exporters’ countries. The major component of the production risk, quantity risk, can be explained by factors like climate changes, labor quality, biological factors and the quality of the production amongst others.

For instance, according to NIS report (2011), the farm’s gate cocoa’s prices have increased to $500CFAf/kg in 1994 compared to $172CFAf/kg observed during the previous season. Those cocoa prices have more than double (from $831.7CFAf/kg to $1853CFAf/kg) from 2007/2008 to 2009/2010. In contrast, the same report revealed a drop in production of cocoa beans from 163701t in 2005 to 131127t two years later even if it has been followed by an increase in 2009 to reach 194000t approximatively. Such movements have also been observed in rice’s prices where they were 10% higher at the third quarter of 2014 compared to its level one quarter before and 9% less than its level at the same time one year before. After dropping by over 7% from 2007 to 2008, the world production of soybean went to a decent growth of nearly 23% from 2008 to 2009 (UNCTAD, 2014).

Due to their major consequences on each stakeholder’s decision, the international community together with governments, have put an emphasis on the findings of a set of efficient risk management tools for agricultural co-

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1. the difference between the expected and the realized production
2. National Institute of Statistics, Cameroon
3. CFAf is the Africa Financial Community Franc, a local currency of sub-saharan countries : 1CFAf≈655.96Euros
modities.
A number of management tools, with mitigated effect, have so far been pro-
pposed in the literature or in many SSA countries. For the producer, we have
the crops and commercial partners diversification, the respect of good prac-
tises during the production process, the building of much more warehouses
and some public measures such as regulatory funds and buffer stocks. Those
tools, which were not always based on markets’ fundamentals, have failed or
are insufficient to hedge producers against high income fluctuations.

Therefore, scholars (McKinnon, 1967; Johnson, 1960) come up with more
market based instruments which can efficiently reduce the effect of revenue
risk. The main idea is that a part of the anticipated production level must be
held in the financial market by using instruments such as forwards, options,
futures or swap. This becomes an important research domain in SSA coun-
tries when activities of their Stock exchanges are increasing. Following this
research direction, a first branch of literature (McKinnon, 1967; Beninga et
al., 1983; Lence, 2009 and Bond et al. 1985) suggested that the fraction of the
anticipated production to be hedged must be independent of a given utility
function. In this article we do not follow this branch of the litterature. Some
scholars (Johnson, 1960, Ederington, 1970; Rolfo, 1980; Heaney, 1991; Ohe-
ngeng, 2013 and Armah, 2013) have found evidence of the impact of the risk
aversion, which can vary with wealth (Pope cited by Saha, 1993), on the
determination of the optimal hedge ratio (OHR). In other words, this ratio
depends on the way producer’s preferences have been represented. Conse-
quently, the purpose of this paper is to show that different utility functions
might lead to different OHRs and that a more flexible utility function like
the expo-power utility function, may yield to more realistic results for SSA
countries.
The paper is organized as follow. Section II reviews the literature on optimal
hedging and present some major properties of the four utility functions that
we have used. In Section III, we study the sensitivity of the OHR for each of
those utility functions and we end with a case study on cameroonian cocoa
producer. Section V presents some concluding remarks.

2 The Relation between the OHR and the utility
function

2.1 A Literature Review on Hedging Revenue Risk On Com-
modity Futures Markets

Rolfo (1980) found the OHR using the utility maximization problem
(UMP) for a cocoa producer from Ghana, Ivory Coast, Nigeria and Brazil.
He used data from London financial market and quadratic and logarithm util-

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lity functions to obtain the following results: given production risks, limited or no use of the futures markets is more beneficial than a full hedge; taking into account revenue risk yields to an OHR near the unity for Brazilian producer. Despite this major contribution, there are at least two shortcomings: since his data run from 1956 to 1976, period characterized by a lower and less volatile prices compared to the current structural breaks that is observed on commodities’ prices, his results are dated and cannot take into account such breaks (Armah, 2013). He also assumed not suitable hypothesis which stipulates a gaussian distribution of producer income.

Using a consumption-Investment model in continuous time, Ho (1984) has determined the OHR for an American wheat producer. Based on monthly data collected from 1977 to 1980 and working in a revenue risk context, he found a less than one OHR. But he assumed CARA agents and his study is dated.

Sy (1990) compared the relative costs and benefits that an Ivorian producer of coffee, cocoa or cotton may have from hedging the revenue risk. By applying Ordinary Least Square on NYSE data from 1973 to 1984, he computed OHR in a Mean-Variance framework. He found a positive and less than one OHR but only when cocoa price risk is assumed. When including quantity risk, he came up with some negative OHR. He found that risk management through financial market yields to more benefits than those obtained from stabilization’s programs. His data are few and dated so that his major results have to be update.

Torkamani and Rahimi (2001) used four utility functions to examine whether an Azerbaijan farmer’s attitude towards risk depends on his preference’s representation. They found that different utility functions can yield to different information about a particular farmer’s attitude towards risk but they have not determined the OHR. Moosa’s (2003) used four models to access the sensitivity of the OHR to the model specification. He concluded that, when he includes exchange rate markets, the model specification has not a significant effect on the optimal hedge strategy, the more important being the level of the correlation between cash price and the price of the chosen financial instrument. But revenue risk was not included in this study.

According to Lence (2009), hedge aims to reduce risk exposure, increase productivity and allow decision making based on less volatile price series. He assumed a null basic risk and a CARA utility function. His major findings

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4. CARA = Constant Absolute Risk Aversion, IARA = Increasing Absolute Risk Aversion, DARA = Decreasing Absolute Risk Aversion
5. Basic risk is the difference between futures price and cash price. (Benninga and al., 1983)
is that hedging for a SSA country’s is sub-optimal.

By considering the necessary condition of efficiency \(^6\), cocoa futures markets are not efficient in short run according to Armah (2012) even if he also found that this market is efficient in short and long run when sufficient condition of efficiency is used. This result suggests that futures’ hedging of price risk is a feasible market based risk management strategy for SSA’s cocoa exporter. Armah (2013) assessed the usefulness of hedging risk on futures markets for the ghanian cocoa board who is subject to revenue risk. He used a CARA utility function in a Utility Maximization Problem framework to see that quadratic and Nelson & Escalante utility functions yield to different OHRs and that only limited use of cocoa futures markets is beneficial for a ghanian producer. Moreover taking into account transactions costs reduce substantially the value of the optimal hedge ratio.

Microeconomists use utility functions to describe the satisfaction that a consumer may have through the consumption of a bundle of goods. As far as the determination of OHR is concerned, we are interested on the capability of the utility function to describe the commodity producer’s attitude towards risk and his risk preference structure. One of the fundamental property of any utility function is its uniqueness by any increasing transformation. In the following, we describe the main properties of the four utility functions used in this paper.

2.2 Utility Functions : Expression and Properties

2.2.1 Quadratic Utility

The general form of the quadratic utility function is : \( U(w) = w - bw^2 \) with \( w \) being the agent’s income and \( b > 0 \) its risk parameter representing the price measured in units of expected income and paid in order to maintain the same utility as the variance of income change. It is an IARA and IRRA \(^7\) utility function and its expected utility is defined as \( EU(w) = E(w) - b[Var(w)] \). Despite the fact that quadratic utility function is one of the most used utility function in the relevant literature - due to the simplicity that it offers while doing empirical studies- it posses some limits:

1. As from a given value of income, its marginal utility decreases, a quite undesirable property in utility theory. In fact quadratic utility function increases only if \( U'(w) = 1 - 2bw > 0 \), that is, if \( w < \frac{1}{2b} \). So for any income level \( w \) such that \( w \geq \frac{1}{2b} \), the agent will be less satisfied when his income increases.

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\(^6\) Price at each moment contains all available information up to this moment (FAMA, 1970).
\(^7\) IRRA=Increasing Relative Risk Aversion
2. Quadratic utility does not reflect the agent’s actual behaviour. It assumes a normal distribution of income that means any agent can only accept a given deviation from the expected level, no matter if it is above or below that level. But the International Task Force (ITF) suggests, in its 2005 report, that the primary concern of commodity producers is to avoid unfavorable outcomes, period of price slump, in which they cannot meet their essential cash expenditures.

3. A substantial body of research has found that risk measures based on third (Skewness) and fourth (kurtosis) moments give more realistic results. That is not the case with quadratic utility which uses only the first two moments. Skewness helps to check any asymmetry in the distribution meanwhile kurtosis detects fat tails in the distribution.

4. Its IARA property is also an undesirable one (Pratt, 1964).

2.2.2 Exponential utility

Exponential utility function has widely been used in Economics (Holt, 2002) and is defined by: $U(w) = -e^{-\lambda w}$ where $\lambda$ is its positive risk parameter. CARA and IRRA are its major properties as far as risk is concerned. While assuming a gaussian distributed income and recalling mathematical notions on expectation in continuous time, the expected value of this utility function is: $EU(w) = E(w) - \frac{1}{2}Var(w)$.

We can therefore point out that when wealth follows a normal distribution, the expected value of the exponential utility function with $\lambda$ as risk parameter is the same as that of the quadratic utility function with $b = 0.5\lambda$ as its parameter. As regard to one of the weaknesses of exponential utility, Alpanda and Wogloma (2009) said "...Exponential utility was abandoned largely because it implies increasing relative risk aversion in a cross-section of individuals and a non-stationary aggregate consumption to wealth ratio, contradicting macroeconomic data....". In addition, as in the quadratic utility, it assumes a gaussian distributed income.

2.2.3 Power utility

The power utility function can be expressed as follow: $U(w) = \begin{cases} w^\alpha & \text{if } \alpha > 0 \\ \ln(w) & \text{if } \alpha = 0 \\ -w^\alpha & \text{for } \alpha < 0 \end{cases}$.

It has the following properties: (i) Power utility approaches logarithm utility when $\alpha \to 0$, (ii) Power utility functions are DARA (when $\alpha < 1$) and CRRA. Its associated Arrow-Pratt coefficient is: $A(w) = \frac{1-\alpha}{w}$ and (iii) Power utility is a suitable utility function for risk averse agent only if $\alpha < 1$. 

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Its major limit is formulated by Alpanda et Wogloma (2009): "...Utility modeled as a power function ...is unbounded and generates asset pricing puzzles. The unboundedness property leads to St. Petersburg paradoxes and indifference to compound gambles."

Further, Saha (1993) states that all these three functions are not flexible in the sense that they do not assume that there can exist a change in agent’s preference, and such utility function may lead to bias and inconsistent estimators.

2.2.4 Expo-power utility

Saha (1993) introduced the expo-power utility function defined by: $U(w) = \theta - e^{-\beta w^\alpha}$ where $\theta > 1$ and $\alpha \beta > 0$. Contrary to the three previous ones, expo-power utility function exhibits, depending on its parameters values, decreasing, constant or increasing absolute risk aversion and decreasing or increasing relative risk aversion. Thus he established the following result.

Lemma 1. (See Saha (1993), page 906)

1. Expo-power exhibits DARA when $\alpha < 1$; CARA for $\alpha = 1$ and IARA if $\alpha > 1$.
2. It exhibits DRRA for $\alpha < 0$ and IRRA if $\alpha > 0$.
3. Expo-power utility function is suitable for risk averse agent if $\alpha > 1$ & $w^\alpha < \frac{\alpha - 1}{\alpha \beta}$.

In the next Section, We derive the optimal hedge ratio expression’s (which can be on explicit or implicit form) for each utility function obtained in an utility maximization framework.

3 Utility function and the derivation of the Optimal Hedge Ratio

Let us consider a commodity producer who wants to buy $n$ contracts on a commodity futures market where anticipated and realized prices are $f$ and $p_f$ are respectively. Its wealth is: $w = \tilde{p} \tilde{q} + n(f - \tilde{p})$ where $\tilde{q}$ and $\tilde{p}$ are the random quantities produce and cash prices respectively. Introducing the price, quantity and revenue risk expressions as defined by Rolfo\textsuperscript{8} and the anticipated level of its production $q^*_e$ at a given season, its wealth becomes

$$w = f q^*_e (1 + e^p_t)(1 + e^q_t) - n f_t e^p_f. \tag{1}$$

\textsuperscript{8} Spot price risk $e^p_t = \frac{p - f_t}{f_t}$; Futures price risk : $e^p_f = \frac{p_t - f}{f_t}$; Quantity risk : $e^q_t = \frac{q - q^*_e}{q^*_e}$. (Rolfo [18])
3.1 Optimal Hedge Ratio for Quadratic and Exponential utility functions

Rolfo (1980) established that the expression of the optimal hedge ratio ($x^*$) derived from the maximization of the expected quadratic utility function is:

$$x^* = \frac{Cov((1 + e^p_t)(1 + e^q_t), e^{p'}_t)}{Var(e^{p'}_t)} - \frac{E(e^{p'}_t)}{2b f_t q_t Var(e^{p'}_t)}. \quad (2)$$

From (2), the optimal number of contracts to be hold in futures markets is the sum of two terms: the first term is the slope of the regression of spot income on futures prices and the second term is the stochastic term, which capture the hedge effect on returns. The Optimal hedge ratio, $x^*$, is an explicit function of the quantity and price distributions and of output forecasted\(^9\).

As we raised it earlier in Subsection 2.2.2, by replacing $2b$ by $\lambda$, we deduce the following expression of the OHR ($x^*$) obtained with the Exponential utility function:

$$x^* = \frac{Cov((1 + e^p_t)(1 + e^q_t), e^{p'}_t)}{Var(e^{p'}_t)} - \frac{E(e^{p'}_t)}{\lambda f_t q_t Var(e^{p'}_t)}. \quad (3)$$

3.2 Optimal Hedge Ratio for Power and Expo-power utility functions

Let us start with the expressions of expected values of these two utility functions which are useful for the determination of their OHRs.

With the Power utility function $U(w) = w^\alpha (w > 0)$ for $\alpha > 0$, the expected utility becomes $EU(w) = E(w^\alpha)$. By replacing the expression of $w$ defined by (1) in $EU(w)$, we have

$$EU(w) = \frac{1}{k} \sum_{t=1}^{k} [f_t q_t^c (1 + e^p_t)(1 + e^q_t) - x f_t q_t^s e^{p'}_t]^\alpha, \text{ where } x = \frac{n}{q_t^c}. \quad (4)$$

With the Expo-power utility function and the expression of the wealth defined by (1), the expected expo-power utility is:

$$EU(w) = \theta - \frac{1}{k} \sum_{t=1}^{k} e^{-\beta[f_t q_t^c (1 + e^p_t)(1 + e^q_t) - x f_t q_t^s e^{p'}_t]^\alpha}. \quad (5)$$

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\(^9\) See Rolfo for more details.
Let us deduce the OHRs. The OHR of a producer which preferences are modeled by a Power and Expo-power utility functions are obtained by looking for the $x^*$ which maximizes (4) and (5) respectively.

Contrary to the first two cases where we have explicit expressions of the OHR, the power and the expo-power utility functions give an implicit relation of the OHR. These OHRs shall be approximated by using the Generalized Least Squares method in the Matlab software and on data collected from cameroonian cocoa producers.

### 3.3 Application to cameroonian cocoa producers

#### 3.3.1 On Cocoa production in Cameroon

The cocoa economy represents about 2% of the cameroonian’s GNP, 6% of the GNP of the primary sector and almost 30% of the GNP of agricultural commodities. Cameroon is the 5th world cocoa producer (ICCO, 2014) and cocoa is its most exported agricultural commodity. Cocoa bean in Cameroon is produced mostly by small farmers and approximately 60% of them are working on farm of about one to five hectares, the income from cocoa being an important component of their total income. Due to high short-term volatility in its price and quantity produced and its importance for the cocoa producer, appropriated financial risk management strategy must be settle. The ITF price risk management mechanism suggests that small producers access the instrument through local intermediaries (commodity traders, exporters, importers based in producing countries, local banks or future merchants, producer co-operatives or even large producers). As some scholars (Armah (2013), Rolfo (1980), Armah (2012)), we suggest the use of futures contracts instead options even though according to ITF studies, some farmers are interested on put options because quantity risks make them unwilling to commit themselves to deliver a fixed amount on maturity of the futures contracts and purchased of put options has the merit of maintaining upside price potential while limiting downside risk but it requires additional payment of an up-front premium thing that producers have some difficulty to paid. Some scholars agree that producers are reluctant on long term contracts, our study focusses on three months issued which is not the case in Armah (2013) and Rolfo (1980) where 6 months contracts were considered. We also assume that while going on financing markets, producers only want to hedge themselves so they do not have any speculative idea.

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10. cited by Gemech et al. (2009).
3.3.2 Data analysis and discussions

We use yearly observations from the Quarterly Bulletin of Cocoa statistics published on the 31st March 2014 by ICCO on cash and futures cocoa’s prices. In the National Board of Cocoa and Coffee (NBCC) database, we take the quantity of cocoa beans produced from 1980 to 2013. We use a R.2.15.0 program to compute the anticipated level of production at the beginning of each cocoa campaign. We consider three months contracts negotiated on September with the maturity on December. So the cash price \( p \) is the average one observed in the month of December at the farm gate level, the value of the futures prices at maturity when the contract is negotiated \( f \) is the average ICCO prices in September and the futures prices at the delivery date \( p_f \) is the average ICCO price in December.

Analysis of the mean and the standard deviation of price and quantity series reported in the two tables of Figure 1 reveals the presence of revenue risk faced by cameroonian cocoa producer. Therefore risk management strategies need to be proposed. In the table on the right of Figure 1, we see that \( E(f) \) is not statistically different from \( E(p_f) \). This suggests an OHR independent of the risk parameter with the first two utility functions. Armah (2013) and Rolfo (1980) found different results due to some difference in contract duration. In fact, most cocoa experts agree on the fact that cocoa producers are unwilling to do forward selling because of their low level of income and it is their main source of income. So it is difficult for small cocoa farmers to buy forward contract with a maturity date being greater than three months. The test of the ratio of spot price over futures price as shown in the table on the left of Figure 1 indicates that futures prices are less volatile than the spot one in cocoa market. This also justifies the used of market based tools for managing revenue risk.

Tables of Figure 2 show that estimated futures price’s error and quantity’s error are unbiased which is not the case with cash price’s error which is biased. The correlation matrix between prices as illustrated in the table on the left of Figure 3, indicates that cash and futures prices are highly correlated. That is why financial market is been used as indicator for cash price determination. The table on the right of Figure 3 presents the correlation amongst errors, that is, a little correlation between risk on spot price and the quantity risk. In fact only 6% of world cocoa beans come from Cameroon and

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11. Cocoa’s futures prices are those observed in LIFFE.
12. Due to the lack of updated data on locally used of cocoa and the fact that only a small proportion of the total quantity produced is transformed locally, we assume that the quantity exported as the total quantity produced.
13. The anticipated level of production is the sum of a brownian motion with a null mean and a standard deviation highly correlated with the observed quantity produced and the quantity effectively produced.
the methods used to find the desired level of production at the beginning of each campaign can explained this little correlation. Rolfo (1980) found a similar result with Brazilian cocoa farmers. We have also realized that revenue risk is more correlated with price risk than quantity risk.

3.3.3 Implementation for the determination of OHR

**Quadratic and Exponential utility functions**: By using appropriated data and expressions (2) and (3), a Matlab program display an optimal hedge ratio of 87.9% whatever the exposure level of the cocoa producer to revenue risk. Both utility functions, even though they have different characteristics, give the same value of the OHR. This can be explained by the fact that analysis of the data reveals that $E(f)$ was not statistically different from $E(p_f)$. This value of the OHR is different from the one obtained by Ohemeng (2013) even though his sample space and time period is similar to our. This can be explain by the difference in the estimation methods and in the contract duration.

**Power utility function**: For values of its risk parameter $\lambda$ in $[0; 1]$, we use a Matlab program to find values of the OHR which maximizes equation (4). The obtained average level of OHR is 80%. It is different from the one found earlier, thus confirming the sensitivity of the OHR to the way the agent’s preferences are represented.

**Expo - Power utility function**: A first simulation on parameter’s $\alpha$ and $\beta$ with Matlab shows that cameroonian cocoa producer have a decreasing absolute risk aversion and an increasing relative risk aversion. The second simulation shows that the values of the optimal hedge ratio depend on initial conditions and on $\alpha$’s values. With appropriated initial values, as suggested by Saha (1993), and for different values of $\alpha$ belonging in $[0.6; 1]$, OHR is ranging from 50% to 65%. It is important to notice the following features: (i) we know the farmer’s attitude towards risk based only on values taken by parameters $\alpha$ and $\beta$ and (ii) the obtained range of values for the OHR takes into account the possible difference in perception of risk between cocoa producers.

4 Conclusion

Our main concern in this paper was to show that different utility functions may lead to different optimal hedge ratios (OHR) and that a more flexible utility function, namely the expo-power utility function, may lead to more realistic results. In order to achieve this, we have selected four utility functions amongst them three widely used in the related literature. After outlining some of their major properties, we found the explicit or implicit expression of the part of the anticipated production level that a commodity
producer have to optimally hedge in futures markets.

As a case study, we have chosen cocoa since it a very complex commodity as far as revenue risk is concerned, and we choose Cameroon, a developing country which is the 5th producer in the world, because of its level of cocoa production which come mostly from small farmers. Using ICCO and NBCC databases for quantities and prices and after computing the level of desired production at the beginning of each cocoa campaign, we reach to some important results. The first three utility functions assume that the farmer has a particular attitude toward risk whereas their attitude by the expo-power utility is obtained from analysis of data. We come up with different values of OHR when changing the utility function. The OHR has the same value (87.9%) for quadratic utility function and exponential utility function and that value of OHR decreases of 7% for Power utility function and belongs between 50% and 60% for Expo-power utility function.

In the developing country context, is it possible and efficient to take into account transactions costs and will that change significantly our results? Is it relevant to introduce the price determination in this problem? That are some concerns that need to be focused on.

Références


Appendix

Figure 1 – Equality Test

Zero mean in quantity error

Zero mean in spot price error

Figure 2 – Zero mean error test.
**Figure 3 – Correlation Matrix.**

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Price correlations.

Error correlations.