Abstract

In this note, we integrate the long-run concept of risk into the stock valuation process, using the conditional capital asset pricing model. Our main result indicates that the intrinsic value of a stock is positively related to its long-run dividend growth rate, and negatively related to its long-run covariance between dividends and aggregate dividends. This result suggests that the theoretical framework of the conditional capital asset pricing model can be used to examine the effect of long-run risk on firm values. This result also suggests that the long-run covariance between dividends and aggregate dividends represents a relevant measure of risk, without assuming anything about aggregate consumption.
1. Introduction

In this note, we develop a stock valuation model that integrates the long-run covariance between a firm’s dividends and aggregate dividends. Our development is based on the capital asset pricing model (CAPM) in a conditional form (the conditional-CAPM).

According to Sharafoddin and Emsia (2016, p. 128), one of the most significant issues in investment management is stock valuation. Following the classic valuation models of Gordon (1962), Basu (1977), and Ohlson (1995), many models have been proposed to estimate the intrinsic value of a stock (see, for example, Hurley and Johnson, 1998; Feltham and Ohlson, 1999; Pastor and Veronesi, 2003; Bakshi and Chen, 2005; Dong and Hishleifer, 2005; and Yee, 2008a, 2010b). More recently, Bergeron (2013a and 2013b) introduced the long-run concept of risk into the stock valuation process, and Bergeron (2019) extended the approach with recursive preferences.

As mentioned by Bansal et al. (2016, p. 52), the long-run risk model developed by Bansal and Yaron (2004) has motivated a significant amount of research in financial economics. Along this line, Bansal et al. (2005) reveal that cash flow betas, measured by the long-run covariance between dividends and aggregate consumption, account for more than 60% of the cross-sectional variation in risk premia. The subsequent works of Hansen et al. (2008), Bekaert et al. (2009), Bansal and Kiku (2011), Bansal and Shaliastovich (2013), Calvet and Czellar (2015), and Jagannathaan and Liu (2016), also confirm the importance of long-run risk for understanding the cross section of asset returns.

However, none of the aforementioned studies develop a theoretical stock valuation model that integrates the long-run covariance between dividends and aggregate dividends, using the framework of the conditional-CAPM.

Indeed, following the procedure initiated by Bansal et Yaron (2004) or Bansal et al. (2005), the measure of long-run risk is implicitly obtained from the framework of the consumption-CAPM. The purpose of this paper is to extend this concept to the conditional-CAPM, and then build a corresponding theoretical stock valuation model. In this sense, the motivation of this paper comes from the following observations: (1) the simplicity of the CAPM, or the conditional-CAPM, relative to the intertemporal consumption-CAPM; (2) the failure of the consumption-CAPM in practice; (3) the effect of measurement errors in available consumption data sets (see Campbell and Cochrane, 2000); (4) the validity of a long-run approach for measuring risk; (5) the importance of

---

1 For a good description of the standard CAPM of Sharpe (1964) andLintner (1965), or the conditional-CAPM of Jagannathan and Wang (1996), see Campbell (2017, Chapter 3).

2 For a good introduction of the consumption-CAPM of Rubinstein (1976), Lucas (1978), and Breeden (1979), see Campbell (2017) again.

3 As noted by Campbell and Cochrane (2000, p. 2864), "The canonical consumption-based model has failed perhaps the most important test of all, the test of time. Twenty-five years after the development of the consumption-based model, almost all applied work in finance still uses portfolio-based models to correct for risk, to digest anomalies, to produce cost of capital estimate, and so forth."
stock valuation in finance; (6) the absence of a theoretical stock valuation model that explicitly integrates the long-run covariance between a firm’s dividends and aggregate dividends, without assuming that aggregate consumption equals aggregate dividends.\(^4\)

Using the conditional-CAPM, we demonstrate that the intrinsic value of a stock is directly proportional to its current dividends, positively related to its long-run dividend growth rate, and negatively related to its long-run dividend beta, measured by the conditional covariance between dividends and aggregate dividends, over many periods.

The remainder of the paper proceeds as follows. Section 2 presents the model.\(^5\) Section 3 provides the paper conclusion.

2. The model

In this section, we develop our theoretical stock valuation model. First, we express the valuation problem in terms of dividends. Second, we introduce the basic risk-return relationship predicted by the CAPM in a conditional form. Third, we extend over many periods to integrate a long-run risk measure into the intrinsic value of a stock. Our development is similar to the procedure employed by Bergeron (2019)\(^6\). However, in this paper, we do not use the intertemporal framework of the consumption-CAPM, the assumption of log-normality, or recursive preferences.

2.1 Asset prices and stochastic discount factor

Over an infinite horizon, at time \(t\) (\(t = 0, 1, 2, \ldots, T - 1\)), the fundamental equation of asset pricing can be written as follows (see, for example, Campbell, 2017, Chapter 4):

\[
P_{it} = E_t \sum_{s=1}^{\infty} M_{t+s} \bar{D}_{i,t+s},
\]

where \(P_{it}\) is the price of asset \(i\) at time \(t\), \(\bar{D}_{i,t+s}\) is the dividends of asset \(i\) at time \(t + s\), and \(M_{t+s}\) is the stochastic discount factor (SDF) between time \(t\) and \(t + s\) (\(s = 1, 2, 3, \ldots, \infty\)). Here, the SDF is quite general relying primarily on the law of one price. It just represents some random variable that generates prices from payoffs. We can use it without implicitly assuming anything about utility function or aggregate consumption (as noted by Cochrane, 2005, at page 63).

\(^4\) In the consumption-CAPM, aggregate consumption equals aggregate dividends; but this assumption is inconsistent with empirical facts (see, Campbell and Cochrane, 2000, p. 2876).

\(^5\) In Section 2, the tilde (\(\sim\)) indicates a random variable. Operators \(E_t\), \(VAR_t\), and \(COV_t\) refer respectively to mathematical expectations, variance and covariance, where index \(t\) implies that we consider the available information at time \(t\).

\(^6\) See also Bergeron (2013a and 2013b).
Since the dividend value of asset $i$ at time $t, D_{it}$, is known, it can thus be passed through the conditional expectation operator of equation (1), to exhibit:

$$P_{it} = D_{it}E_t \sum_{s=1}^{\infty} \bar{M}_{t+s} \bar{D}_{i,t+s} / D_{it},$$

or if we assemble the elements of the summation term:

$$P_{it} = D_{it}E_t[\bar{F}_{it}] = D_{it}\theta_{it},$$

where parameter $\theta_{it}$ is such that $\theta_{it} \equiv E_t[\bar{F}_{it}], and where variable $\bar{F}_{it}$ is defined in this manner:

$$\bar{F}_{it} \equiv \sum_{s=1}^{\infty} \bar{M}_{t+s} \bar{D}_{i,t+s} / D_{it}.$$  

If the sequence of variables $\bar{F}_{it}$ (for $t = 0, 1, 2, ..., T - 1$) is independent and identically distributed (iid), as in Bergeron (2019), then:

$$P_{it} = D_{it}\theta_i.$$  

Given the available information at time $t$, we can thus write:

$$\bar{P}_{l,t+1} = \bar{D}_{l,t+1}\theta_i,$$  

where $\bar{P}_{l,t+1}$ represents the random price of asset $i$ at time $t + 1$. The last relationship reveals that the random future prices of a long-lived asset (such as a stock) are directly related to the corresponding dividends, or if we prefer, that prices and dividends are perfectly cointegrated. If we only consider the information set available at time $t = 0$, then equation (4) indicates that $\bar{P}_{l,t} = \bar{D}_{l,t}\theta_i$, for $t = 1, 2, ..., T - 1$. We will utilize this result in the next subsection.

### 2.2 The conditional-CAPM

The standard static CAPM reveals that the excess return of an asset is directly proportional to its systematic risk (the beta). In a dynamic economy, Jagannathan and Wang (1996) assume that the CAPM holds in a conditional sense, period by period (see equation 2, at page 8). We extend this approach to the long-run. More precisely, we assume that, given the available information at time $t = 0$, next asset returns, and subsequent asset returns, imply the following equilibrium relationship, for $t = 0, 1, 2, ..., T - 1$:

$$E_0[\bar{R}_{l,t+1}] = R_{F,t+1} + (E_0[\bar{R}_{m,t+1}] - R_{F,t+1})\beta_{it},$$
where $\bar{R}_{i,t+1}$ is the returns of asset $i$ between time $t$ and $t + 1$, $R_{F,t+1}$ is the risk-free rate of return between time $t$ and $t + 1$, $\bar{R}_{m,t+1}$ is the returns of the market portfolio $m$ between time $t$ and $t + 1$, and $\beta_{it}$ is the beta of asset $i$ measured in the following manner:

$$\beta_{it} = \text{COV}_0[\bar{R}_{m,t+1}, \bar{R}_{i,t+1}] / \text{VAR}_0[\bar{R}_{m,t+1}] .$$

Given the available information at time $t = 0$, the returns of asset $i$ are defined in this manner $\bar{R}_{i,t+1} = \frac{\bar{p}_{t+1} + \bar{b}_{t+1}}{\bar{p}_{it}} - 1$, for $t = 0$, and like this $\bar{R}_{i,t+1} = \frac{\bar{p}_{t+1} + \bar{b}_{t+1}}{\bar{p}_{it}} - 1$, for $t = 1, 2, ..., T - 1$.

Likewise, integrating equations (4) and (5) into the definition of returns indicates that:

$$\bar{R}_{i,t+1} = \frac{\theta_i \bar{b}_{t+1} + \bar{b}_{t+1}}{\theta_i \bar{b}_{it}} - 1, \text{ for } t = 0, \text{ and } \bar{R}_{i,t+1} = \frac{\theta_i \bar{b}_{t+1} + \bar{b}_{t+1}}{\theta_i \bar{b}_{it}} - 1, \text{ for } t = 1, 2, ..., T - 1 .$$

Also, integrating equations (4) and (5) into the expectation operators of equation (6), shows, after simple manipulations, that (index $m$ indicates the market portfolio):

$$E_0[(1 + \bar{g}_{i,t+1})(1 + \theta_i^{-1}) - 1] = R_{F,t+1} + (E_0[(1 + \bar{g}_{m,t+1})(1 + \theta_m^{-1}) - 1] - R_{F,t+1}) \beta_{it} , \quad (7)$$

where $\bar{g}_{i,t+1}$ is the dividend growth rates of asset $i$ between time $t$ and $t + 1$, defined as follows: $\bar{g}_{i,t+1} = \frac{\bar{p}_{it+1}}{\bar{p}_{it}} - 1$, for $t = 0$, and $\bar{g}_{i,t+1} = \frac{\bar{p}_{it+1}}{\bar{p}_{it}} - 1$, for $t = 1, 2, ..., T - 1$.

Considering the definition of parameter $\beta_{it}$, and the basic properties of mathematical expectation, we can write:

$$E_0[1 + \bar{g}_{i,t+1}](1 + \theta_i^{-1}) - 1 = R_{F,t+1} + (E_0[1 + \bar{g}_{m,t+1}](1 + \theta_m^{-1}) - 1 - R_{F,t+1}) \beta_{it} , \quad (8)$$

$$\times \text{COV}_0[\bar{g}_{m,t+1}; \bar{g}_{i,t+1}](1 + \theta_i^{-1}) / \text{VAR}_0[\bar{g}_{m,t+1}](1 + \theta_m^{-1}) .$$

Rearranging equation (8) shows that:

$$E_0[1 + \bar{g}_{i,t+1}] = (1 + R_{F,t+1})/(1 + \theta_i^{-1}) + (E_0[1 + \bar{g}_{m,t+1}](1 + \theta_m^{-1}) - 1 - R_{F,t+1}) \beta_{it} , \quad (9)$$

$$\times \text{COV}_0[\bar{g}_{m,t+1}; \bar{g}_{i,t+1}]/\text{VAR}_0[\bar{g}_{m,t+1}](1 + \theta_m^{-1}) ,$$

or, if we assemble the last elements of equation (9):

$$E_0[1 + \bar{g}_{i,t+1}] = (1 + R_{F,t+1})/(1 + \theta_i^{-1}) + \lambda_t \beta_{it}^d , \quad (10)$$

where
\[ \lambda_t \equiv (E_0[1 + \bar{g}_{m,t+1}](1 + \theta_m^{-1}) - 1 - R_{F,t+1})/(1 + \theta_m^{-1}) > 0, \]
\[ \beta_{it}^d \equiv COV_0[\bar{g}_{m,t+1}, \bar{g}_{l,t+1}]/VAR_0[\bar{g}_{m,t+1}]. \]

The resulting coefficient \( \beta_{it}^d \) is similar to the key determinant of risk proposed by Abel (1999) and Bergeron (2013a, p. 552); however, these two models are based on consumption. Here, coefficient \( \beta_{it}^d \) represents the short-run dividend beta of asset \( i \), at time \( t \), calculated by the conditional covariance between the dividend growth rates of the asset and the growth rates of the aggregate market dividends. It measures the sensitivity of an asset's dividends to aggregate dividends.\(^7\)

Besides, it is easy to demonstrate that parameter \( \lambda_t \) is positive, if we accept that investors are risk-averse, and if we recognize that prices and dividends are positive. Indeed, for the market portfolio, we know that: \( E_0[\bar{R}_{m,t+1}] = E_0[1 + \bar{g}_{m,t+1}](1 + \theta_m^{-1}) - 1. \) Given that the market expected return is superior to the risk-free rate of return, and because \( \theta_m^{-1} = D_{mt}/P_{mt} \), then, in accordance to equation (10), parameter \( \lambda_t \) is positive.

Equation (10) represents an equilibrium condition for one period. It could be extended over several periods.

### 2.3 Long-run risk

In the long-run, the relationship between an asset’s expected dividend growth rate and its sensitivity to aggregate dividends can be obtained by summing from time zero \( (t = 0) \) to time \( T - 1 \) \( (t = T - 1) \), that is to say:

\[
\sum_{t=0}^{T-1} E_0[1 + \bar{g}_{l,t+1}] = \sum_{t=0}^{T-1} (1 + R_{F,t+1})/(1 + \theta_i^{-1}) + \sum_{t=0}^{T-1} \lambda_t \beta_{it}^d. \tag{11}
\]

Multiplying by the scalar value \( \sum_{t=0}^{T-1} \lambda_t \) on each side of equation (11), yields:

\[
\sum_{t=0}^{T-1} E_0[1 + \bar{g}_{l,t+1}] = \sum_{t=0}^{T-1} (1 + R_{F,t+1})/(1 + \theta_i^{-1}) + \sum_{t=0}^{T-1} \lambda_t \sum_{t=0}^{T-1} w_t \beta_{it}^d, \tag{12}
\]

where \( w_t \equiv \lambda_t/\sum_{t=0}^{T-1} \lambda_t \), with \( 0 < w_t < 1 \). Considering the stationarity assumption associated to parameter \( \theta_i \), we have:

---

\(^7\) In this paper, the terms market portfolio dividends, aggregate market dividends, and aggregate dividends, are interchangeable.
Dividing by \( T \) on each side of equation (13) indicates that:

\[
1 + \bar{g}_i = (1 + \bar{R}_F)/(1 + \theta^{-1}) + \bar{\lambda}\bar{\beta}_i^d.
\]

where

\[
\bar{g}_i \equiv \frac{\sum_{t=0}^{T-1} E_0[\tilde{g}_{i,t+1}]}{T}, \quad \bar{R}_F \equiv \frac{\sum_{t=0}^{T-1} R_{F,t+1}}{T},
\]

Estimators \( \bar{g}_i, \bar{R}_F, \) and \( \bar{\lambda} \) correspond to arithmetic averages (over \( T \) periods), while \( \bar{\beta}_i^d \) corresponds to a weighted average (over the same periods). Particularly, \( \bar{g}_i \) represents the long-run expected dividend growth rate of asset \( i \), and \( \bar{\beta}_i^d \) represents the long-run dividend beta of asset \( i \).

Rearranging equation (14), we can write:

\[
(1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d)/(1 + \bar{R}_F) = 1/(1 + \theta^{-1}).
\]

Taking the inverse on each side of equation (15), and isolating \( \theta^{-1} \), we have:

\[
1/[(1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d)/(1 + \bar{R}_F)] - 1 = \theta^{-1}.
\]

At time \( t = 0 \), equation (4) indicates that: \( \theta^{-1} = D_{i0}/P_{i0} \). Thus, integrating equation (4) into equation (16) allows us to write:

\[
1/[(1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d)/(1 + \bar{R}_F)] - 1 = D_{i0}/P_{i0},
\]

or, after simple algebraic manipulations:

\[
\frac{1 + \bar{R}_F}{1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d} - \frac{1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d}{1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d} = \frac{D_{i0}}{P_{i0}}.
\]

Consequently, we can easily isolate the equilibrium price of a stock to obtain a simple formula, expressed without consumption values. Indeed, considering the current dividends of stock \( i, D_{i0}, \) the corresponding current equilibrium price of stock \( i, P_{i0}, \) is such that:

\[
P_{i0} = \frac{1 + \bar{g}_i - \bar{\lambda}\bar{\beta}_i^d}{\bar{R}_F + \bar{\lambda}\bar{\beta}_i^d - \bar{g}_i} D_{i0}.
\]

Equation (19) represents our main result. It suggests that the current value of a stock is directly proportional to its current dividend payment, positively related to its long-run
dividend growth rate, and negatively related to its long-run dividend beta, obtained from
the long-run covariance between dividends and aggregate market dividends. Since the
relationship between the intrinsic value of the stock and its corresponding long-run
dividend beta is negative ($\lambda_t > 0$), this beta is viewed as a rightful measure of risk, in the
long-run. This result suggests that the theoretical framework of the conditional-CAPM can
be used to examine the effect of long-run risk on firm values. This result also suggests that
the long-run covariance between dividends and aggregate dividends represents a relevant
measure of risk.

Besides, if we accept that the long-run dividend beta corresponds to an appropriate
measure of risk, then equation (14) reveals that the dividend growth rate of a stock is
linearly and positively related to risk (in the long-run). This point of view is consistent with
intuition and with several studies on the relation between dividends and risk.\footnote{See, for example, Abdoh and Varela (2017).} Our
contribution, here, is to characterize this theoretical relationship with a long-run risk
measure obtained from the conditional-CAPM. Another point of interest, for our model,
can be expressed in this way: if asset fundamental value is determined by fundamental
economic variables such as dividend cash flows, then it makes sense to measure risk
directly from dividends. Our measure of risk and our corresponding valuation model are
consistent with this premise. The approach incorporates a fundamental dividend risk
measure directly into the final intrinsic value.

3. Conclusion

In this note, we developed a simple theoretical stock valuation model. We first referred to
the law of one price and the corresponding fundamental equation of asset pricing with
stochastic discount factors. Afterward, we introduced the basic risk-return relationship
predicted by the CAPM in a conditional form. Then, we extended over many periods to
integrate a long-run risk measure into the intrinsic value of a stock.

Our main result shows that the current value of a stock is directly proportional to its
current dividend payment, positively related to its long-run dividend growth rate, and
negatively related to its long-run dividend beta, obtained from the long-run covariance
between dividends and aggregate market dividends.

In this manner, the main contribution of this paper, in addition to the development of
a simple theoretical stock valuation model, can be summarized in the following points.
First, the paper extends the concept of long-run risk from the consumption-CAPM to the
conditional-CAPM. Also, it suggests that the theoretical framework of the conditional-
CAPM can be used to examine the effect of long-run risk on firm values. Moreover, it
indicates that the long-run covariance between dividends and aggregate dividends
represents a correct measure of risk, without assuming anything about aggregate
consumption. More specifically, the model indicates the amplitude of the negative effect
of long-run dividend betas on firm values. In addition, the paper reveals that the expected
dividend growth rate of a stock is linearly and positively related to long-run risk, in the context of the conditional-CAPM. Furthermore, it characterizes this relationship between the expected dividend growth rate and the appropriate measure of risk.

As noted by Beeler and Campbell (2012), the long-run concept of risk has attracted a great deal of attention since the important work of Bansal and Yaron (2004). For future research, this note opens the way for new studies on long-run risks without implicitly referring to consumption, the consumption-CAPM, or the restrictive assumption that claims theoretical equivalency between aggregate consumption and aggregate dividends.

References


