Economics Bulletin

Volume 40, Issue 2

The Fundamental Surplus or the Fundamentality of Vacancy Posting Costs?

Mehrab Kiarsi
School of Economics, Henan University

Abstract

In a recent influential paper, Ljungqvist and Sargent (2017) suggest that beneath the large responses of unemployment to movements in productivity in the various proposed reconfigurations of the standard Mortensen and Pissarides model is simply the small size of the fundamental surplus fraction. I show that the fundamental surplus fraction is small if and only if the per-vacancy posting cost is small.

Citation: Mehrab Kiarsi, (2020) "The Fundamental Surplus or the Fundamentality of Vacancy Posting Costs?", *Economics Bulletin*, Volume 40, Issue 2, pages 1011-1016
Contact: Mehrab Kiarsi - mkiarsi@gmail.com.
Submitted: September 13, 2019. **Published:** April 15, 2020.

1 INTRODUCTION

In a recent influential paper, Ljungqvist and Sargent (2017) suggest that beneath the large responses of unemployment to movements in productivity in the various proposed reconfigurations of the Mortensen and Pissarides model to generate empirically observed fluctuations in unemployment is simply the small size of what they call the fundamental surplus fraction. The fundamental surplus fraction is an upper bound on the fraction of a job's output that the invisible hand can allocate to vacancy creation. A high elasticity of market tightness requires a small fundamental surplus fraction. They advocate the fundamental surplus as the single intermediate channel through which economic forces generating a high elasticity of market tightness with respect to productivity must operate.[1]

This paper shows that in the standard Mortensen-Pissarides (MP) framework with Nash bargaining, and the models considered by Ljungqvist and Sargent (2017), by definition, the fundamental surplus fraction is small if and only if the per-vacancy posting cost is small. The relationship comes from the free entry assumption of vacancies in the standard MP framework, that, for a common calibration, makes it impossible to have a low per-vacancy posting cost without not having a high unemployment income, and therefore, a small fundamental surplus fraction. The intuition behind the importance of small vacancy posting costs to have a volatile labor market activity is simple and intuitive: they make it easy and almost costless for firms to hire and fire workers, and therefore, induce a quite volatile labor market activity.

2 The Central Equations

At the center of the analysis of Ljungqvist and Sargent (2017) are the free entry of vacancies assumption and the Nash bargaining wage equation. The free-entry condition gives rise to the following standard job creation condition in matching frameworks

$$\frac{c}{q_t} = y_t - w_t + (1-s)\,\Xi_t\{\beta\frac{C_t}{C_{t+1}}\frac{c}{q_t}\}, \tag{1}$$

where c is the per-vacancy posting cost, q_t is the probability of filling a vacancy, y_t is the marginal product of labor, w_t is the real wage, s is the exogenous separation rate, and $\Xi_t\beta\frac{C_t}{C_{t+1}}$ is the stochastic discount factor with which firms discount profit flows.

To derive the Nash-bargained wage equation, suppose that V_t^E and V_t^U denote, respectively, the values to a worker of being matched with a firm and being unemployed:

$$V_t^E = w_t + E_t\left(\beta\frac{C_t}{C_{t+1}}\right)\left\{(1-s)\,V_{t+1}^E + s\left[(1-p_{t+1})\,V_{t+1}^U + p_{t+1}V_{t+1}^E\right]\right\}, \tag{2}$$

$$V_t^U = z + E_t\left(\beta\frac{C_t}{C_{t+1}}\right)\left\{(1-p_{t+1})\,V_{t+1}^U + p_{t+1}V_{t+1}^E\right\}, \tag{3}$$

where p_t is the job finding probability, and z is unemployment income.

Suppose Υ_t denote the value to the firm of a worker, then we have the following expression for the surplus

[1] The fundamental surplus also now constitutes chapter 30 of Ljungqvist and Sargent's famous graduate textbook "Recursive Macroeconomic Theory, Fourth Edition, The MIT Press, 2018".

to a firm of the marginal worker:

$$\Upsilon_t = y_t - w_t + (1-s)\, E_t(\beta \frac{C_t}{C_{t+1}})\Upsilon_{t+1}, \tag{4}$$

where $\Upsilon_t = \frac{c}{q_t}$. Suppose that $\phi \in (0,1)$ denotes a worker's bargaining power. Then in generalized Nash bargaining, the parties choose w_t to maximize

$$\left(V_t^E - V_t^U\right)^{\phi} \Upsilon_t^{1-\phi},$$

that results in the following standard wage equation:

$$w_t = \phi y_t + (1-\phi) z + \phi\,(1-s)\, c E_t \beta \frac{C_t}{C_{t+1}} \frac{p_{t+1}}{q_{t+1}}, \tag{5}$$

2.1 The centrality of per-vacancy posting cost c

In order to see the tight relationship between the per-vacancy posting cost c, and the fundamental surplus fraction $(1-z)$, we need to focus on the steady-state versions of conditions (1) and (5), which in terms of w are:

$$w = y - [1 - \beta\,(1-s)] \frac{c}{q\,(\theta)}, \tag{6}$$

$$w = \phi y + (1-\phi) z + \beta \phi c\,(1-s)\, \frac{p\,(\theta)}{q\,(\theta)}, \tag{7}$$

where, $\theta = \vartheta/u$ stands for the market tightness. Note that, ϑ is the number of vacancies posted, and u is the unemployment rate. To have a unique solution for w we must have:

$$\phi y + (1-\phi) z + \beta \phi c\,(1-s)\, \frac{p\,(\theta)}{q\,(\theta)} = y - [1 - \beta\,(1-s)] \frac{c}{q\,(\theta)}. \tag{8}$$

An inspection of expression (8) reveals that it contains only one endogenous variable, the market tightness θ. From the definition of market tightness we have that $\theta = \vartheta/u$. This, in itself, does not impose any restriction on the magnitude of θ. However, because of the nature of the matching function $M\,(u, \vartheta) = A u^{\alpha} \vartheta^{1-\alpha}$, and the fact that it exhibits constant returns to scale, the job-finding and vacancy-filling rates are defined, respectively, as $p = A\theta^{1-\alpha}$ and $q = A\theta^{-\alpha}$. Since $0 < p, q < 1$, the inequality $\left(\frac{1}{A}\right)^{-1/\alpha} < \theta < \left(\frac{1}{A}\right)^{1/1-\alpha}$ must hold, and $\theta = \frac{p}{q}$ must also be the case. The inequality holds only if $A < 1$. Moreover, given the definitions of p and q, the matching parameter A is uniquely pinned down to

$$A = (p)^{\alpha} (q)^{1-\alpha}. \tag{9}$$

Given the fact that the job-finding and vacancy-filling probabilities must be above zero and below one, it is not a good strategy to solve expression (8) in terms of θ since we may end up getting a probability above one or at zero for some calibrations of the parameters. Moreover, in this case, we cannot also anymore pin down A by (9) and have to assign an arbitrary value to it. It is more reasonable to calibrate p and q to some acceptable values and pin down θ by the ratio $\frac{p}{q}$ and A by (9).

Note also that with the exception of c and z, all the other parameters that appear in (8) are bounded to change in a narrow range, and there is a consensus in the literature regarding the values that each of these parameters can take. Therefore, it remains only two reasonable options to solve for the steady-state equilibrium condition (8): c and z. Given the value of one, the value of the other is determined by condition (8). Solving for z yields:

$$z = y - \frac{\beta\left(1-s\right)}{\left(1-\phi\right)}\left\{\phi\theta + \left(\frac{1}{\beta\left(1-s\right)} - 1\right)\frac{\theta^{\alpha}}{A}\right\}c, \tag{10}$$

where it is quite clear that there is a strict negative relationship between z and c; that is, z is at its highest possible level ($y = 1$), when $c = 0$.

2.2 Market tightness elasticity

In the literature, the elasticity of market tightness with respect to productivity ($\eta_{\theta,y}$) is used as the main measure of labor market volatility. Implicit differentiation of expression (8) yields

$$\frac{d\theta}{dy} = \frac{\left(1-\phi\right)}{\beta\phi\left(1-s\right)c + \frac{\alpha}{A}c\theta^{\alpha-1}\left[1-\left(1-s\right)\beta\right]}, \tag{11}$$

where $\frac{d\theta}{dy}$ gives the derivative of market tightness with respect to the marginal product of a worker, y, that in the steady state is just one. Thus, the elasticity of market tightness is:

$$\eta_{\theta,y} = \frac{d\theta}{dy}\frac{y}{\theta} = \frac{\left(1-\phi\right)}{\beta\phi\left(1-s\right)\theta + \frac{\alpha}{A}\theta^{\alpha}\left[1-\left(1-s\right)\beta\right]}\frac{1}{c}. \tag{12}$$

The above expression makes it clear that ruling out the uninteresting and unreasonable case that the market tightness $\theta \to 0$ (which means that the job-finding probability $p \to 0$, the vacancy-filing probability $q \to \infty$, and vacancies $\vartheta \to 0$), the magnitude of $\eta_{\theta,y}$ is determined by the second term that is only a function of c. Therefore, to generate high volatility in market tightness a low value for c is essential. A very small c features a very large value for the second term of expression (12), and therefore, a very large market tightness elasticity. More precisely, as $c \longrightarrow 0$, the market tightness elasticity $\eta_{\theta,y} \longrightarrow +\infty$.

2.3 Relationship to the Fundamental Surplus

Another way of writing Eq. (12) is to eliminate $(1-\phi)$ in the numerator of (12) by using condition (11), which results in:

$$\frac{dq}{da} = \frac{\beta\phi\left(1-s\right)\theta + \frac{1}{A}\theta^{\alpha}\left[1-\left(1-s\right)\beta\right]}{\beta\phi\left(1-s\right)\theta + \frac{\alpha}{A}\theta^{\alpha-1}\left[1-\left(1-s\right)\beta\right]}\left(\frac{y}{y-z}\right), \tag{13}$$

therefore,

$$\eta_{q,a} = \frac{\beta\phi\left(1-s\right)\theta + \left[1-\left(1-s\right)\beta\right]/q}{\beta\phi\left(1-s\right)\theta + \alpha\left[1-\left(1-s\right)\beta\right]/q}\left(\frac{y}{y-z}\right). \tag{14}$$

Using the same notation as Ljungqvist and Sargent (2017), let's call the first term in expression (14) $\Upsilon^{Nash} = \frac{\beta\phi(1-s)\theta+[1-(1-s)\beta]/q}{\beta\phi(1-s)\theta+\alpha[1-(1-s)\beta]/q}$, that, as Ljungqvist and Sargent (2017) explain, is bounded from above by $1/\alpha$. Hence, based on Ljungqvist and Sargent's interpretation, it is the second factor, or more precisely, $y - z$, that they call the fundamental surplus, that is critical in generating large volatility in labor market tightness, and therefore, unemployment and vacancies. To have large fluctuations in unemployment the inverse of the fundamental surplus fraction ($\frac{y}{y-z}$) must be large. Given that $y = 1$, the only way to get a high value for the inverse of the fundamental surplus is to have a high unemployment benefit z. Note that expression (14) by definition is essentially the same as equation (12). Eq. (10) clearly shows that, given a reasonable above zero value for the market tightness θ, z achieves its maximum, or the fundamental surplus is at its minimum, if $c \to 0$. Thus, by definition, a small fundamental surplus means a small per-vacancy posting cost. To see it more clearly, rewrite expression (10) as follows:

$$y - z = \left\{ \frac{1 - \beta (1 - s) (1 - \phi p)}{(1 - \phi) q} \right\} c. \tag{15}$$

The above expression shows $y - z$ as the product of two terms. The first term on the right hand side of (15) is a function of the subjective discount factor β, the separation rate s, workers' bargaining power ϕ, and the job-finding and vacancy filling rates, p and q, respectively. All these parameters take a value between zero and unity and there is a consensus regarding the values they can take in the literature. For example, at a quarterly basis β is mainly set at 0.99, s between 0.1 and 0.14, ϕ between 0.4 and 0.7, and p and q between 0.6 and 0.9. Thus, given reasonable calibrations of these parameters the term in the curly brackets in (15) is fairly around 1. Ljungqvist and Sargent (2017) throughout their paper calibrate these parameters such that at a quarterly basis they are as follows: $\beta = 0.988$, $s = 0.1$, $\phi = 0.5$, $p = q = 0.9$. Taken together, these reasonable parameters pin down the term in curly brackets to 1.1354. Thus, to have a small $y - z$, the per-vacancy posting cost c must be small. For example, it is well established that in the canonical MP model with Nash bargaining z must be more than 0.95 to have enough empirically relevant volatility in unemployment. Given Ljungqvist and Sargent's (2017) calibration, expression (15) tells us that c must be less than 0.04 for z more than 0.95. In the next subsections, I show that this is the case in all the models discussed by Ljungqvist and Sargent (2017).

2.4 The case of Hall's sticky wages

Now, assume that wages are set a la Hall, where a constant wage $\widehat{w}$ inside the Nash bargaining set is paid to workers. In this case, expression (8) changes to

$$\widehat{w} = y - [1 - \beta (1 - s)] \frac{c}{q}. \tag{16}$$

Implicit differentiation of expression (16) and using the definition of the elasticity of market tightness with respect to productivity y yield

$$\eta_{\theta,y}^{\widehat{w}} = \frac{y}{[1 - \beta (1 - s)] \frac{\alpha}{q}} \frac{1}{c}. \tag{17}$$

Again, it is very clear that, given a reasonable calibration for β, α, ρ, and q, the only way to get a large market tightness elasticity is through having a small vacancy posting cost c. Ljungqvist and Sargent (2017) derive $\eta_{\theta,y}^{\widehat{w}}$ differently. To highlight the importance of the fundamental surplus, they use expression (16) to rearrange the denominator of the first equality of equation (17) to get

$$\eta_{\theta,y}^{\widehat{w,SL}} = y/\alpha(y - \widehat{w}), \tag{18}$$

where $y - \widehat{w}$ is the fundamental surplus fraction ($y = 1$ in the steady state). Ljungqvist and Sargent (2017) mention that the smaller is the fundamental surplus the higher the inverse of the fundamental surplus, thus the higher would be $\widehat{\eta_{\theta,y}^{w,SL}}$. However, the fundamental surplus is small if and only if c is small. If c is high, there is no way to get a small fundamental surplus fraction.

2.5 The cases of layoff taxes, fixed match costs, a financial accelerator, and AOB model

Now, I discuss the other cases (in addition to the case of Nash-bargained wages and Hall's fixed wage that I already discussed above) considered by Ljungqvist and Sargent (2017). In all these cases a small fundamental surplus fraction is a necessary result of a small per-vacancy posting cost. I simply repeat below the relevant equations in Ljungqvist and Sargent (2017) in terms of the notations of this paper, and for each case rewrite it in terms of the per-vacancy posting cost and the fundamental surplus fraction.

Let's start with the case of layoff taxes. Suppose that the government imposes a layoff tax τ on each layoff and returns the tax revenues as lump sum transfers to workers. In this case, expression (8), that its equivalent version in Ljungqvist and Sargent (2017) is expression (22), is as follows

$$y - z - \beta (1 - s) \tau = \frac{[1 - \beta (1 - s)] + \phi p}{(1 - \phi) q} c. \tag{19}$$

It is very clear from (19) that the fundamental surplus fraction $y - z - \beta (1 - s) \tau$ is low if and only if c is low. In fact, another way of inducing a small c is to raise τ instead of assuming a high unemployment benefit z.

Now, let's consider the case of a fixed match cost, introduced by Pissarides [2009]. Interestingly, Pissarides does mention the importance of the vacancy posting costs of the firm to the size of the market tightness elasticity. In addition to the vacancy posting cost, he assumes that there is a fixed matching cost. Then he shows that as the fixed matching cost increases the per-vacancy posting cost decreases and the market tightness elasticity increases. Indeed the importance of a fixed match cost is in its ability to induce a smaller per-vacancy posting cost, and so unemployment benefits can be sat at a relatively low value. Otherwise, it does not matter, per se. To see this, equation (20) below represents the adjusted version of expression (8) with fixed match costs H, which is equation (81) in Ljungqvist and Sargent (2017):

$$\frac{(1 - \phi) (y - z - [1 - \beta (1 - s)] H)}{[1 - \beta (1 - s)] + \phi \theta} = c. \tag{20}$$

It is quite clear from expression (20) that, all else equal, a small vacancy cost c by definition necessarily results in a low fundamental surplus fraction $(y - z - [1 - \beta (1 - s)] H)$.

Another case discussed by Ljungqvist and Sargent (2017) is the case of a financial accelerator. With a financial accelerator, the adjusted equation (8), that is equation (43) in Ljungqvist and Sargent (2017), is

$$\frac{q [y - \widehat{w} - [1 - \beta (1 - s)] \beta K (\sigma)]}{1 - \beta (1 - s)} - (1 - \beta) K (\sigma) = c, \tag{21}$$

where $K (\sigma)$ is a function of credit market tightness σ and $\widehat{w}$ is a constant wage. Again, the fundamental surplus $[y - \widehat{w} - [1 - \beta (1 - s)] \beta K (\sigma)]$ is small if c is small.

The last case that I discuss in this subsection is the model of alternating-offer wage bargaining. In this case, the expression for equilibrium market tightness, which is equation (36) in Ljungqvist and Sargent (2017), is

$$y - \beta s \psi - z = (1 + \beta s) \frac{[1 - \beta (1 - s)]}{q} c, \tag{22}$$

where ψ is a cost of delay that firms incur. Expression (22) shows clearly that, in this case too, the fundamental surplus $y - \beta s \psi - z$ is small if c is small.

3 Conclusion

This article shows that the fundamental surplus fraction, advocated by Ljungqvist and Sargent (2017) as the central channel in generating big responses of unemployment to productivity changes is small if and only if the vacancy posting costs are small.

References

Christopher A. Pissarides. The unemployment volatility puzzle: Is wage stickiness the answer? *Econometrica*, 77(5):1339–1369, 2009. 2.5