Economics Bulletin

Volume 40, Issue 2

Productivity growth, malthus delusion, and unified growth theory

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Abstract

This short paper studies a simple Malthusian model with perpetually growing productivity. The model is calibrated to match the British growth record from 1270 to 1650, and, then, to 1870. Results are as follow: (i) Both the Black Death and perpetually growing productivity have explanatory power for preindustrial prosperity. (ii) For the extended Malthusian model to match the data from 1270 to the mid-17th century, the production technology must be extremely labor intensive. (iii) The model cannot capture the growth acceleration after the mid-17th century even with unrealistic parameter values. These results imply that the British economy was in a distinct Post-Malthusian regime in the post-1650 period, and they substantiate the strong relevance of Unified Growth Theory to the British economic development over the very long run.

The author is grateful to an anonymous referee and to his colleagues at Hacettepe University for their comments and suggestions on an earlier version of this paper. All of the remaining errors are the author's own.
Citation: M Aykut Attar, (2020) "Productivity growth, malthus delusion, and unified growth theory", *Economics Bulletin*, Volume 40, Issue 2, pages 1112-1121
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Submitted: September 29, 2019. **Published:** April 29, 2020.

1. Introduction

The transition from stagnation to growth and the causes of the first Industrial Revolution have been traditional subject matters in economic history. In recent decades, growth economics has become a truly historical field of inquiry with an increasing attention paid to these issues in unified growth models (e.g., Galor and Weil, 2000; Hansen and Prescott, 2002; Lucas, 2002; Galor, 2005, 2011). In the meantime, economic historians have developed new narratives and compiled new datasets from archival records to find new answers to their old questions (e.g., Mokyr, 2002; Clark, 2007; Allen, 2009; Clark, 2010; Broadberry et al., 2010; McCloskey, 2010; Broadberry et al., 2015).

Malthus and the Malthusian model play significant roles in both of these research programs. In unified growth models, a Malthusian regime with poverty emerges as an equilibrium outcome if initial productivity is sufficiently low. For economic historians, whether Malthus was right or not has been a focal point at least since Postan (1966).[1]

In the last decade or so, the Malthusian model has been at the center stage of a particular controversy among economic historians working on the British economy. Two newly compiled gross domestic product (GDP) estimates for the British economy by Clark (2010) and Broadberry et al. (2010) are in considerable conflict for the period before the 18th century and especially for the early 14th century. Clark (2010) estimates similar real GDP per capita figures for the mid-15th and the early 19th centuries and interprets the long swing of real GDP per capita between these episodes as a direct evidence of a Malthusian trap. But the data compiled by Broadberry et al. (2010, 2015) indicate that real GDP per capita estimates for the early 14th century were considerably lower than those estimated for the early 18th century. Even a slowly growing real GDP per capita, according to the proponents of the anti-Malthusian view, should lead to the rejection of the Malthusian model.[2]

In a recent paper, Lagerlöf (2019) demonstrates that a Malthusian model can successfully account for slow-growth trends in real GDP per capita if land productivity exhibits accelerating growth. He simulates a Malthusian model with realistic demographic features and persistent technology shocks. He investigates the model fit for several countries including Britain by using the GDP data of Broadberry et al. (2010). Hence, his paper "*proposes a more nuanced conclusion regarding the validity of the Malthusian model.*" (Lagerlöf, 2019, p. 222)

In another recent paper, Madsen et al. (2019) introduce sustained productivity growth into an otherwise standard Malthusian model as well. They estimate speeds of convergence to the Malthusian steady-state for 17 countries by using GDP and population data covering the 900-1870 period. Their results show that economies in the sample are estimated to converge to the Malthusian steady-state in less than three decades, thereby leading them to conclude that Malthus was right.

[1]Starting with the pioneering work of Lee (1973), a large literature has been investigating the empirical validity of the Malthusian model for England. Recent overviews are presented by Møller and Sharp (2014), Lagerlöf (2019), and Madsen et al. (2019). Two largely agreed-upon views in this literature are (i) the validity of the preventive check mechanism, and (ii) the end of the Malthusian era in the mid-17th century.

[2]The controversy has been initiated when Persson (2008) wrote a harsh critique of the Malthusian interpretation of Clark (2007) and identified his position as the "Malthus Delusion." For the most recent and illuminating discussion of the controversy, see Lagerlöf (2019).

This short paper extends a simple Malthusian model with various types of productivity growth and simulates it for alternative horizons for England. The main purpose is to see whether and for which particular episodes such an extended model can explain the evolution of real GDP per capita in preindustrial England. In this respect, the paper aims to complement Lagerlöf (2019) and Madsen et al. (2019) (i) by investigating the implications of different types of productivity growth and (ii) by focusing exclusively on England.

The paper uses the output-based estimates of Broadberry et al. (2010) for real GDP. This data runs from 1270 to 1870 at annual frequency. The source of population data is also Broadberry et al. (2010). The algorithm calibrates the fixed parameters that shape the pace of productivity growth in the economy.

The benchmark results indicate that (i) a Malthusian model without productivity growth fails to explain preindustrial prosperity after the early 1500s, and (ii) a Malthusian model extended with productivity growth can explain preindustrial prosperity in England after the early 1500s but only at the expense of completely missing the evolution of real GDP per capita between the Black Death and the early 1500s. Results also indicate that, for the model to explain the movements of real GDP per capita from 1270s to the mid-1650s, the aggregate production technology must be extremely labor intensive. Finally, when the model horizon is extended to 1870, even this extremely labor intensive technology fails to account for preindustrial prosperity after the late 17th century.

This paper contributes to the related literature in three ways. First, the paper studies the implications of endogenous productivity growth as well, not only of exogenous productivity growth as in Lagerlöf (2019) and Madsen et al. (2019). Specifically, endogenous productivity growth rate is an increasing function of population as in Boserup (1965) and Kremer (1993). Results show that whether growth is endogenous or not is not trivial; the Boserup-Kremer specification is the only one that implies per-capita GDP growth after the early 17th century.

Second, contrary to the multi-country analyses of Lagerlöf (2019) and Madsen et al. (2019), the paper focuses exclusively on the British economy—the economy whose growth record over the very long run has led to the Malthus Delusion controversy. Concentrating on British economic growth allows us to see exactly for which historical episodes does the extended Malthusian model perform well.

Finally, this paper complements the studies showing that British economic history strongly supports the unified growth models (Klemp, 2012; Møller and Sharp, 2014; Madsen and Murtin, 2017). The paper demonstrates that, without a further regime switch that accelerates productivity growth, the Boserup-Kremer mechanism is not strong enough to explain British economic growth in the very long run.

The outline of the paper is as follows: Section 2 introduces the model economy. Section 3 explains the computational strategy. Section 4 presents the results. Section 5 concludes with a short discussion of results.

2. Model

Imagine an economy in discrete time $t \in \{0, 1, ...\}$. This economy produces a single good using land and labor. The production function is

$$Y_t = X_t L_t^{\alpha}, \tag{1}$$

with $\alpha \in (0, 1)$, where Y_t denotes the volume of output, L_t denotes population under the assumption that all individuals work, and X_t is a productivity term that shifts the production frontier. This is a variable that reflects the effects of several factors on Y_t, e.g., land expansion, technological progress, capital accumulation, organizational improvements, and increasing working hours. Output per capita, denoted by y_t, satisfies

$$y_t = \frac{Y_t}{L_t} = X_t L_t^{-(1-\alpha)}. \tag{2}$$

The main objective here is to understand the role of productivity growth. The analysis builds upon three different hypothetical scenarios regarding the growth rate of X_t:

$$X_{t+1} = \begin{cases} \zeta + X_t, & \text{arithmetic growth } (\zeta > 0) \\ (1+\eta)X_t, & \text{exponential growth } (\eta > 0) \\ (1+\gamma L_t)X_t, & \text{Boserup-Kremer growth } (\gamma > 0) \end{cases} \tag{3}$$

In the first scenario, productivity is described by arithmetic growth that one can loosely associate with the Malthusian view that, while population increases in a geometrical ratio, subsistence increases in an arithmetical one (Malthus, 1798, p. 4). The second one is the exponential growth formulation where the percentage growth rate is fixed at $\eta > 0$. Finally, the third scenario is the one where productivity growth rate increases with population level as suggested by Boserup (1965) and formalized by Kremer (1993).[3]

The law of motion for L_t closes the model. Letting n_t denote the gross growth rate of population, we have

$$L_{t+1} = n_t L_t, \tag{4}$$

and the Malthusian preventive and positive check mechanisms dictate that n_t is a strictly increasing function of y_t such that

$$n_t = n(y_t) : \mathbb{R}_{++} \to \mathbb{R}_{++} \qquad n'(y) > 0. \tag{5}$$

Notice that, if $n(y_t)$ is a known function, (2), (3), and (4) completely describe the evolution of the Malthusian economy given the initial values $X_0 > 0$ and $L_0 > 0$ of state variables. In fact, these equations imply

$$\frac{L_{t+1}}{L_t} = n\left(\frac{X_t}{L_t^{1-\alpha}}\right). \tag{6}$$

A Malthusian steady-state with constant L_t and constant y_t exists if X_t is fixed in the long run. Besides, this Malthusian steady-state is asymptotically stable since n_t is a decreasing function of L_t.

[3] Galor (2011, p. 147) describes the logic of the Boserup-Kremer dynamism through five distinct channels: "(i) *the supply of innovative ideas*, (ii) *the demand for innovations*, (iii) *the rate of technological diffusion*, (iv) *the degree of specialization in the production process and thus the extent of 'learning by doing,' and* (v) *the scope for trade and thus the extent of technological imitation and adoption.*"

3. Data and Computation

The computational analysis of the model described above necessitates two observed variables, population and GDP.[4] For both of these variables, the data source is Broadberry et al. (2010). These authors provide both of these variables as indexed sequences at annual frequency covering the years from 1270 to 1870.

There are two difficulties regarding the computation of equilibrium paths. The first one is the specification of $n(y_t)$. Is it linear or nonlinear? Is it concave or convex? Does it have inflection points? A model of fertility choice would imply $n(y_t)$ as an explicit solution.[5] But non-trivial modifications would be required to imply a satisfactory model-data match in population because of the 14th century demographic crisis, i.e., the Black Death, and the 17th century fluctuations in mortality. Since the sole purpose here is to isolate the role of productivity growth, the effective remedy adopted here is to feed the model directly with population data. This in effect means we are working with a *surrogate population growth function* that perfectly explains the evolution of population.[6]

The second difficulty concerns the date after which preindustrial England was no longer so Malthusian. Would it be the date after which population growth is no longer responding positively to real GDP per capita or real wages? Would it be the date after which real GDP per capita shows some (remarkable) growth? Would it be the date after which there is no demographic crisis characterized by a decline in population? The related literature on this issue offers a wide array of possibilities, such as 1800, 1830s, or mid-1600s, either by taking growth accelerations as reference points or by directly testing the validity of the Malthusian checks for England as summarized by Møller and Sharp (2014, Tab. 5). Based on the evolution of real wages, Clark (2005) argues that the mid-1600s should be taken as the period where the escape from the Malthusian stagnation started. If real wages keep increasing in a particular episode and if this increase is observed along with a non-decreasing or increasing level of population, then the economy must be out of the Malthusian trap. Since the last three population crises in England with decreases larger than 2% per annum occurred at 1635, 1666, and 1681, the year 1650 is taken as a reference point, and the computational analysis is performed for all years running from 1635 to 1665 for all specifications.

There are two parameters to be calibrated. The first one is the production function parameter α, and the other is either one of productivity growth parameters ζ, η, and γ depending on the specification under consideration. For the former, a carefully calibrated value of $\alpha = 0.54$ from Bar and Leukhina (2010) is used as a benchmark. These authors construct a two-sector Malthus-to-Solow model with land and capital accumulation, and

[4] This paper does not use the real wage as a measure of living standards since available real wage constructions are in line with the Malthusian view for England. Hence, this paper engages with the Malthus delusion controversy by evaluating the limits of the Malthusian model in accounting for the slow-growth trend in real GDP per capita.

[5] In a typical Malthusian model with explicitly defined preferences, for instance, Ashraf and Galor (2011) obtain a closed-form solution such as $n_t = \gamma y_t/\rho$ where γ is a preference parameter and ρ is the unit (consumption) cost of having children.

[6] Since both n_t and y_t are observed, one can imagine working with a population growth function $n(y_t, \epsilon_t)$ where the error term ϵ_t is calibrated to match n_t for each t. This is similar to the practice adopted by Jones (2001) in his Appendix A.3.1.

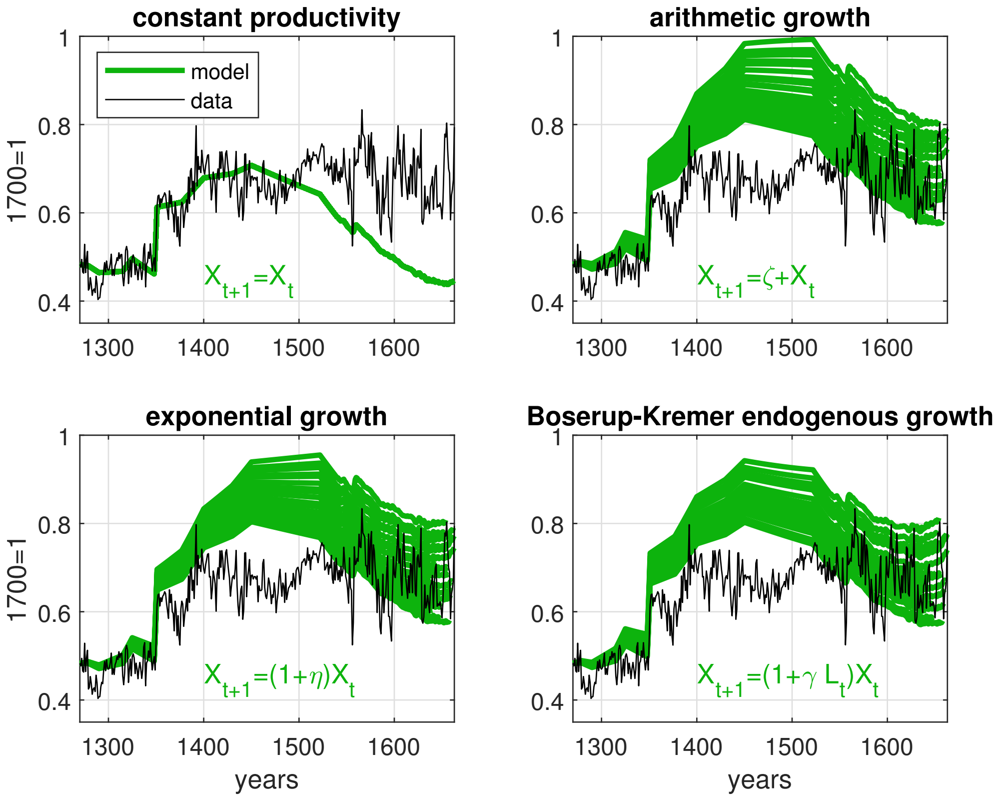

Figure 1: Real GDP per capita: Benchmark ($\alpha = 0.54$)

arrive at this value of α for the Malthus technology.[7]

The parameter that determines the pace of growth in each specification is calibrated to match a given end-point value of real GDP per capita. Hence, for any of the parameters ζ, η, and γ, a different value is assigned to equate the simulated y_t to its observed counterpart for each terminal point between 1635 and 1665. The computation algorithm is described in the appendix.[8]

4. Results

This section documents the results of the analysis in three figures originating from three different computations of the model. Figure 1 pictures the benchmark scenario characterized by $\alpha = 0.54$. Figure 2 pictures the paths of GDP per capita for a significantly more labor intensive production function, i.e., the one with $\alpha = 0.80$. Finally, Figure 3 shows the results when the terminal date is extended to 1870 while the production function parameter is kept at $\alpha = 0.80$. This third exercise uses ζ, η, and γ values obtained for the 1635-1665 period under $\alpha = 0.80$ and investigates the validity of these specifications for the 1650-1870 period.

Figure 1 shows two things: First, as seen in the upper left panel, the Malthusian model without productivity growth fails in explaining preindustrial prosperity in England after the

[7]The corresponding value adopted by Lagerlöf (2019) for the same parameter is equal to 0.6.

[8]The replication files are located at `https://www.dropbox.com/s/j33e63bqu96v794/Delusion.rar?dl=0`.

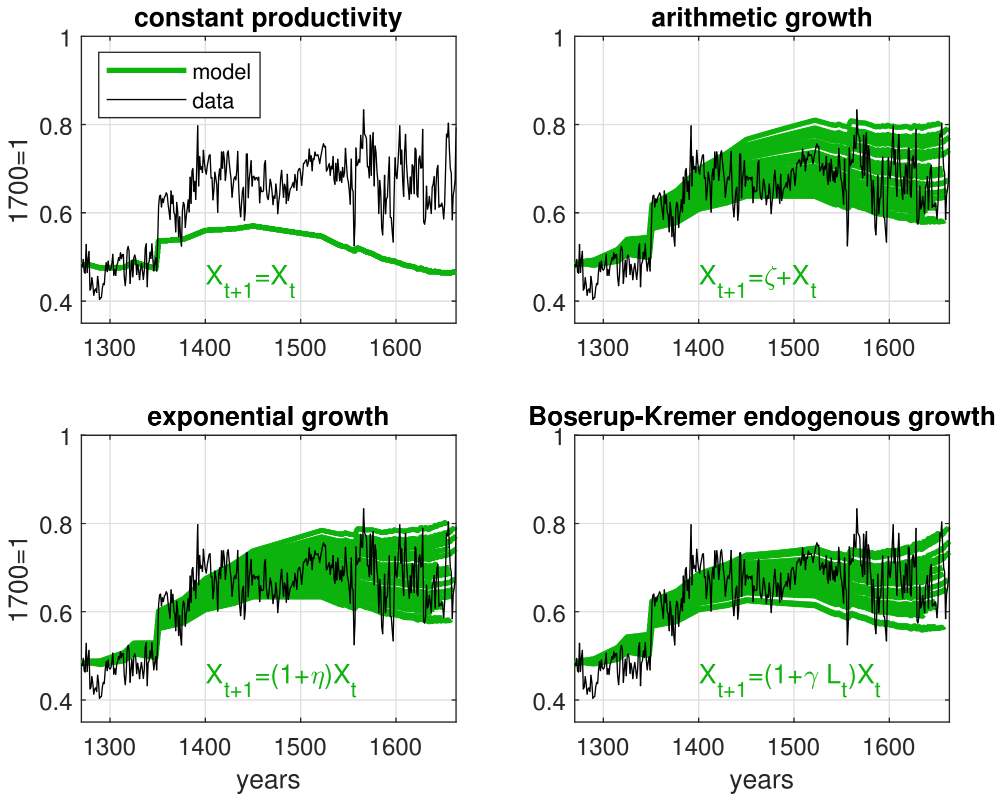

Figure 2: Real GDP per capita: $\alpha = 0.80$

early 1500s. This is a direct consequence of the Malthusian system. Besides, since we feed the model directly with population data, the upper left panel shows that the Black Death alone does not account for preindustrial prosperity in England after the early 1500s.

Second, the other three panels of the figure show that productivity growth does explain preindustrial prosperity in England after the early 1500s but only at the expense of completely missing its evolution between the Black Death and the early 1500s.

Figure 2 demonstrates that the Malthusian model with productivity growth returns a highly satisfactory match with the data when the production function parameter satisfies $\alpha = 0.80$. However, production is now extremely labor intensive, implied by a value of α that is about 1.5 times larger than its benchmark value.

A higher value of α corresponds to factor-eliminating technical change as in Peretto and Seater (2013); production is less intensive with respect to the fixed factors such as land. With a lower share of land, the drag effect of population growth on the growth of GDP per capita is weaker; $y_{t+1}/y_t = (X_{t+1}/X_t)n_t^{-(1-\alpha)}$. The curvature parameter α is equal to the ratio of the marginal productivity of labor (MPL) to real GDP per capita as well. Hence, a more labor intensive technology corresponds to higher values of MPL for any given level of y_t. But why does this imply a successful match with the data in the 15th century? Because the calibration algorithm that targets y_t at the end of the Malthusian era now requires lower levels of productivity to match the 15th century evolution of GDP per capita. In other words, a relaxation of the land constraint relative to the benchmark returns a slower growth of productivity during the Malthusian era.

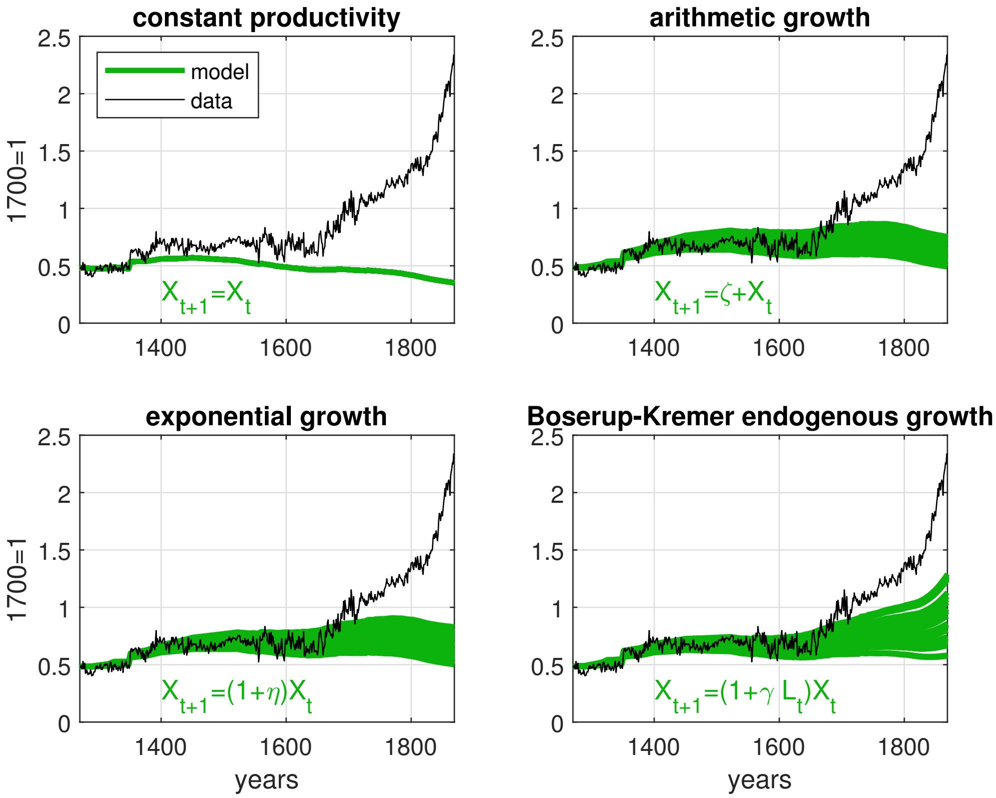

Figure 3: Real GDP per capita: Longer Run with $\alpha = 0.80$

Figure 3 indicates that, even if we take $\alpha = 0.80$ as a good approximation of reality, the model performs remarkably poorly in accounting for the evolution of GDP per capita after the late 17th century. Among the three alternative formulations of productivity growth, the closest one that exhibits minuscule productivity growth in the 18th and 19th centuries is the Boserup-Kremer dynamism depicted in the lower right panel. But even the largest simulated value is off-the-target by about 40% in 1800, and the mismatch grows from 1800 to 1870.

5. Conclusion

What do we learn from the simulation exercises presented above? First and the foremost, the Black Death alone cannot be the only driver of preindustrial prosperity in England. Thus, recent works by Lagerlöf (2019) and Madsen et al. (2019) correctly identify productivity growth as an essential driver of real-world Malthusian systems. This is contrary to Voigtländer and Voth (2009) who have found that, for the European economies and for the 1500-1700 period, the shifts in fertility and mortality schedules explain preindustrial riches, not productivity growth.

Second, since the empirical match until the mid-1600s is somewhat sensitive to the curvature of the production function, a more sophisticated production structure must be developed and calibrated to fully illuminate whether a Malthusian model with productivity growth successfully accounts for the observed slow-growth in preindustrial England. The need to develop realistic unified growth models to understand preindustrial economies has also been underlined by Lagerlöf (2019) but for a different reason concerning the demographic structure of the existing models.

Finally, the extended Malthusian model that returns the best match for the period until the mid-1600s fails in explaining the growth acceleration between the mid-1600s and 1870. This clearly suggests that a single Malthusian regime cannot account for the entirety of the 1270-1870 period in England. This is in perfect accordance both with the *multiple regimes view* in the unified growth literature and with empirical studies that identify a *Post-Malthusian regime* with growth acceleration in England (Klemp, 2012; Møller and Sharp, 2014).

A final remark concerning the future of research on the very long-run patterns of British economic development is now in order. In 2005, Galor (2005) presented the canonical model of the UGT. In 2006, Lagerlöf (2006) presented a quantitative analysis of the model with a useful parameterization. Since then, numerous empirical studies confirmed that the UGT captures the very long-run patterns of the British economy from 1270 onwards (Madsen and Murtin, 2017). The next achievement in this line of research should be the one that takes a unified growth model to data with rigorous strategies for the calibration and estimation of structural parameters for all regimes.

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Appendix: The Computation Algorithm

Let $\{Y_t^{\text{obs}}\}_t$ and $\{L_t^{\text{obs}}\}_t$ denote GDP and population sequences from Broadberry et al. (2010). Let $T^{\text{end}} \in \{1635, 1636, ..., 1665\}$ denote the terminal point. The pseudocode for the computation algorithm is as follows:

Load $\{Y_t^{\texttt{obs}}\}_t$ and $\{L_t^{\texttt{obs}}\}_t$

Compute $\{y_t^{\texttt{obs}}\}_t$ using $y \equiv Y/L$

Set $\alpha \in (0,1)$

Calibrate X_{1270} using $X_{1270} = y_{1270}^{\texttt{obs}} \left(L_{1270}^{\texttt{obs}}\right)^{1-\alpha}$

  for $T^{\texttt{end}} \in \{1635, 1636, ..., 1665\}$

    for $t \in \{1270, 1271, ..., T^{\texttt{end}}\}$

      for $\{\texttt{Arithmetic}, \texttt{Exponential}, \texttt{Boserup-Kremer}\}$

        Choose $\begin{cases} \zeta > 0 \\ \eta > 0 \\ \gamma > 0 \end{cases}$ to minimize $|y_{T^{\texttt{end}}}^{\texttt{obs}} - y_{T^{\texttt{end}}}^{\texttt{model}}|$

      end

    end

  end