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Involuntary unemployment with indivisible labor supply under perfect competition

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Abstract

We show the existence of involuntary unemployment without the assumption of wage rigidity under perfect competition with indivisible labor supply and decreasing or constant returns to scale technology. We derive involuntary unemployment by considering consumers' utility maximization and firms' marginal cost pricing behavior using an overlapping generations model with three generations including childhood generation.

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1 Introduction

According to Otaki (2009), the definition of involuntary unemployment consists of two elements: (i) the nominal wage rate is set above the reservation nominal wage rate; and (ii) the employment level and economic welfare never improve by lowering the nominal wage rate. Umada (1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. In this paper, we consider consumers' utility maximization and firms' marginal cost pricing behavior in an overlapping generations (OLG) model under perfect competition according to Otaki (2010) and Otaki (2015), and demonstrate the existence of involuntary unemployment without the assumption of wage rigidity. A key point in our analysis is the indivisibility of the labor supply. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if the labor supply is divisible and it is small, no unemployment exists[2].

In the next section we analyze the relationship between the indivisibility of the labor supply and the existence of involuntary unemployment under decreasing or constant returns to scale technology. We show that under decreasing returns to scale the real wage rate is decreasing in the employment and the reservation real wage rate for individuals is constant given an expected inflation rate. If the real wage rate is larger than the reservation real wage rate, an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they become equalized. We consider a three-generation (or three-period) OLG model with a childhood period, and pay-as-you go pension system for the older generation. In this model an increase in the employment (by the so-called real balance effect) due to a reduction in the nominal wage rate might be small or even negative (please see Section 3).

2 Indivisible labor supply and involuntary unemployment

2.1 Consumers

We consider a three-period (0: childhood, 1: younger or working, and 2: older or retired) OLG model under perfect competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), Otaki (2009), and Otaki (2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is one good which is produced under perfect competition with decreasing or constant returns to scale technology.

[1] Lavoie (2001) presented a similar analysis.

[2] About the indivisible labor supply Hansen (1985) is an important study. He studies the existence of unemployed workers and fluctuations in the rate of unemployment over the business cycle with indivisible labor supply. To treat an indivisible labor supply in a representative agent model he assumes that people choose lotteries rather than hours worked. Each person chooses a probability of working, then a lottery determines whether or not he actually works. There is a contract between firms and individuals that commits the individual to work the predetermined number of hours with the probability which is chosen by an individual. The contract is being traded, so the individual is paid whether he works or not. The firm provides complete unemployment insurance to the workers.

In this paper we do not consider a representative worker from the entire workforce, including the employed and unemployed. We consider utility maximization by distinguishing between the employed and the unemployed individuals ($\delta = 1$ or $\delta = 0$). Unemployed individuals are not insured.

2. Consumers consume the good during the childhood period (Period 0). This consumption is covered by borrowing money from the younger generation. They must repay their debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply one unit of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the good using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. They are covered by taxes on employed consumers of the younger generation.

5. Consumers determine their consumption in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

We use the following notation.

c_i: consumption of an individual in Period i, $i = 1, 2$.
D: consumption of an individual in Period 0, which is constant.
p_i: the price of the good in Period i, $i = 1, 2$.
$\rho = \frac{p_2}{p_1}$: (expected) inflation rate (plus one).
p_0: the price of the good in Period 0.
W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\hat{D}$: consumption in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for consumers of the younger generation when they retire.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
L: total employment.
β: disutility of labor, $\beta > 0$.
L_f: population of labor or employment during the full-employment state.
$y(L)$: labor productivity, which is deceasing or constant with respect to the employment, $y' \leq 0$.

δ is the definition function. If a consumer is employed, $\delta = 1$; if he is unemployed, $\delta = 0$.
We assume that L_f is constant, that is, there is no population growth.
We define the employment elasticity of the labor productivity as follows,

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

We assume that $-1 < \zeta \leq 0$, and ζ is constant. Decreasing (constant) returns to scale means that $\zeta < 0$ ($\zeta = 0$).

Since the taxes for unemployed consumers' debts are paid by employed consumers, D and Θ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D. \tag{1}$$

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers, Q and Ψ satisfy the following relationship.

$$L_f Q = L\Psi. \tag{2}$$

The utility of consumers of one generation over the three periods is

$$U(c_1, c_2, D, \delta, \beta) = c_1^{\alpha} c_2^{1-\alpha} D^{\gamma} - \delta\beta,\ 0 < \alpha < 1,\ 0 < \gamma < 1.$$

The budget constraint is

$$p_1 c_1 + p_2 c_2 = \delta W + \Pi - D - \delta\Theta + \hat{Q} - \delta\Psi + (1 - \delta)R.$$

p_2 is the expected price of the good in Period 2. Since $R = D$,

$$p_1 c_1 + p_2 c_2 = \delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi.$$

The Lagrange function is

$$\mathscr{L} = c_1^{\alpha} c_2^{1-\alpha} D^{\gamma} - \delta\beta - \lambda[p_1 c_1 + p_2 c_2 - (\delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi)].$$

λ is the Lagrange multiplier. The first order conditions are

$$\alpha c_1^{\alpha-1} c_2^{1-\alpha} D^{\gamma} = \lambda p_1,\ (1 - \alpha) c_1^{\alpha} c_2^{-\alpha} D^{\gamma} = \lambda p_2.$$

These conditions are re-written as:

$$\alpha c_1^{\alpha} c_2^{1-\alpha} D^{\gamma} = \lambda p_1 c_1,\ (1 - \alpha) c_1^{\alpha} c_2^{1-\alpha} D^{\gamma} = \lambda p_2 c_2.$$

Then, we obtain

$$c_1^{\alpha} c_2^{1-\alpha} D^{\gamma} = \lambda\,(p_1 c_1 + p_2 c_2) = \lambda(\delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi),$$

$$p_1 c_1 = \alpha(\delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi),\ \ p_2 c_2 = (1 - \alpha)(\delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi).$$

Consumers' indirect utility is written as follows:

$$V = \frac{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}}{p_1^{\alpha} p_2^{1-\alpha}} D^{\gamma}(\delta W + \Pi - \delta D - \delta\Theta + \hat{Q} - \delta\Psi) - \delta\beta, \tag{3}$$

with

$$\lambda = \frac{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}}{p_1^{\alpha} p_2^{1-\alpha}} D^{\gamma}.$$

The reservation nominal wage rate W^r is a solution of the following equation.

$$\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{p_1^\alpha p_2^{1-\alpha}}D^\gamma\left(W^r+\Pi-D-\Theta+\hat{Q}-\Psi\right)-\beta=\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{p_1^\alpha p_2^{1-\alpha}}D^\gamma(\Pi+\hat{Q}).$$

This yields

$$W^r=\frac{p_1^\alpha p_2^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\beta+D+\Theta+\Psi. \tag{4}$$

The labor supply is indivisible. If $W>W^r$, the total labor supply is L_f. If $W<W^r$, it is zero. If $W=W^r$, consumers are indifferent to employment and unemployment, and no involuntary unemployment exists, even if $L<L_f$. The indivisibility of the labor supply might be because minimum standards of civilized life exists, even in an advanced economy (see Otaki (2015))[3].

Let $\rho=\frac{p_2}{p_1}$. This is the expected inflation rate (plus one). The reservation real wage rate is

$$\omega^r=\frac{W^r}{p_1}=\frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}\beta+\frac{1}{p_1}\left(D+\Theta+\hat{Q}+\Psi\right). \tag{5}$$

If the value of ρ is given, ω^r is constant. Otaki (2007, 2009, 2015) assumes that the (nominal or real) wage rate is equal to the reservation (nominal or real) wage rate at the equilibrium. However, there exists no mechanism to equalize them. We do not assume that the wage rate is equal to the reservation wage rate. This is the most significant difference between this study and Otaki(2007, 2009, 2015).

2.2 Firms

The total demand for the good by younger generation consumers is

$$c^1=\frac{\alpha(WL+L_f\Pi-LD-L\Theta+L_f\hat{Q}-L\Psi)}{p_1}=\frac{\alpha(WL+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ)}{p_1}.$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers. Similarly, their total demand for the good in the second period is

$$c^2=\frac{(1-\alpha)(WL+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ)}{p_2}.$$

Let $\bar{c}^2$ be the demand for the good by the older generation. Then

$$\bar{c}^2=\frac{(1-\alpha)\left(\bar{W}\bar{L}+L_f\bar{\Pi}-L_f\bar{D}+L_fQ-L_f\bar{Q}\right)}{p_1},$$

[3] He says "it is natural to assume that people do not reduce working hours when they are threatened by poverty. Such propensity to work is preserved even in advanced economies for people to maintain minimum standards of civilized life even though working adjoins non-negligible disutility. This lends legitimacy to the concept of the indivisibility of working hours per capita" (Sec. 2.3.2 in Otaki (2015)).

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. Note that Q is the pay-as-you-go pension for the older generation. Let

$$M = (1 - \alpha)(\bar{W}\bar{L} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}). \tag{6}$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in the older generation's Period 1. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_fQ. \tag{7}$$

Their demand for the good is written as $\frac{M}{p_1}$. Government expenditure as well as the consumption of childhood, younger and older generation constitute the national income. Let c be the total demand for the good, and Y be the effective demand defined by

$$Y = \alpha(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M.$$

Then,

$$c = \frac{Y}{p_1}.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pension and unemployment benefits (see Otaki (2007), Otaki (2015) about this demand function). Now, we assume that G is financed by seigniorage similarly to Otaki (2007) and Otaki (2009). In the later section, we will consider the government's budget constraint with respect to taxes.

Let x and z be the output and employment of a firm, respectively. We have $x = y(z)z$. Thus,

$$\frac{dz}{dx} = \frac{1}{y(z) + y'z} = \frac{1}{(1+\zeta)y(z)}.$$

The profit of the firm is

$$\pi = p_1x - \frac{x}{y(z)}W.$$

The condition for profit maximization under perfect competition is

$$p_1 - \frac{y(z) - xy'\frac{dz}{dx}}{y(z)^2}W = p_1 - \frac{1 - y'z\frac{dz}{dx}}{y(z)}W = p_1 - \frac{1}{y(z) + y'z}W = p_1 - \frac{1}{(1+\zeta)y(z)}W = 0.$$

Therefore,

$$p_1 = \frac{1}{(1+\zeta)y(z)}W.$$

This represents marginal cost pricing. Since at the equilibrium $x = c$ and $z = L$, we obtain

$$p_1 = \frac{1}{(1+\zeta)y(L)}W. \tag{8}$$

With decreasing (constant) returns to scale, $-1 < \zeta < 0$ ($\zeta = 0$).

2.3 Involuntary unemployment

The real wage rate is

$$\omega = \frac{W}{p_1} = (1+\zeta)y(L).$$

Under decreasing (constant) returns to scale, since ζ is constant, ω is decreasing (constant) with respect to L. Aggregate supply of the good is equal to

$$WL + L_f\Pi = p_1 Ly(L).$$

Aggregate demand is

$$\alpha(WL+L_f\Pi-L_fD+L_f\hat{Q}-L_fQ)+G+L_f\hat{D}+M = \alpha[p_1Ly(L)-L_fD+L_f\hat{Q}-L_fQ]+G+L_f\hat{D}+M.$$

Since they are equal,

$$p_1Ly(L) = \alpha[p_1Ly(L) - L_fD + L_f\hat{Q} - L_fQ] + G + L_f\hat{D} + M,$$

or

$$p_1Ly(L) = \frac{\alpha(-L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M}{1-\alpha}.$$

In real terms[4]

$$Ly(L) = \frac{\alpha(-L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M}{(1-\alpha)p_1}. \tag{9}$$

L is obtained as a solution to (9). It cannot be larger than L_f. However, it may be strictly smaller than $L_f (L < L_f)$. Then, involuntary umemployment exists. Under decreasing returns to scale the real wage rate, $\omega = (1+\zeta)y(L)$, is decreasing with respect to L. Since the reservation real wage rate ω^r is constant, if $\omega > \omega^r$ an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they become equalized.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers[5],

$$G = T,$$

aggregate demand is

$$\begin{aligned}&\alpha(WL + L_f\Pi - G - L_fD + L_f\hat{Q} - L_fQ) + G + L_f\hat{D} + M\\ =&\alpha[p_1Ly(L) - G - L_fD + L_f\hat{Q} - L_fQ] + G + L_f\hat{D} + M.\end{aligned}$$

Then, we obtain[6]

$$Ly(L) = \frac{\alpha(-L_fD + L_f\hat{Q} - L_fQ) + (1-\alpha)G + L_f\hat{D} + M}{(1-\alpha)p_1}.$$

[4] $\frac{1}{1-\alpha}$ is a multiplier.
[5] Of course, only employed consumers pay the taxes. T denotes the total tax payment.
[6] This equation means that the balanced budget multiplier is 1.

2.4 Discussion summary

The real wage rate depends on the employment elasticity of the labor productivity and the employment level. But the employment level does not depend on the real wage rate. The real aggregate demand and the employment level are determined by the value of

$$\frac{\alpha(-L_f D + L_f \hat{Q} - L_f Q) + G + L_f \hat{D} + M}{p_1}. \tag{10}$$

If employment is smaller than the labor population, then involuntary unemployment exists.

Under decreasing returns to scale the real wage rate is decreasing with respect to the employment and the reservation real wage rate remains constant. When the real wage rate is larger than the reservation real wage rate, an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they become equalized.

2.5 The case of full-employment

If $L = L_f$, full-employment is realized. Then, (9) is re-written as

$$L_f y(L_f) = \frac{\alpha(-L_f D + L_f \hat{Q} - L_f Q) + G + L_f \hat{D} + M}{(1-\alpha)p_1}. \tag{11}$$

Since L_f is constant, this is an identity not an equation. On the other hand, (9) is an equation not an identity. (11) should be re-written as

$$\frac{\alpha(-L_f D + L_f \hat{Q} - L_f Q) + G + L_f \hat{D} + M}{(1-\alpha)p_1} \equiv L_f y(L_f). \tag{12}$$

This yields:

$$p_1 = \frac{1}{(1-\alpha)L_f y(L_f)}[\alpha(-L_f D + L_f \hat{Q} - L_f Q) + G + L_f \hat{D} + M].$$

Then, the nominal wage rate is determined by:

$$W = (1+\zeta)y(L_f)p_1.$$

2.6 Steady state

Consider a steady state where the employment L is constant. If $L < L_f$, involuntary unemployment exists even at the steady state. We permit $\rho \neq 1$. If $\rho = 1(\neq 1)$, consumers correctly predict that the price is constant (rises or falls). Let T be the tax revenue which is not necessarily equal to G. Then,

$$p_1 L y(L) = \alpha[p_1 L y(L) - T - L_f D + L_f \hat{Q} - L_f Q] + G + L_f \hat{D} + M.$$

At the steady sate, $\hat{D} = \rho D$ and $\hat{Q} = \rho Q$. Thus,

$$p_1 L y(L) = \alpha[p_1 L y(L) - T - L_f D + (\rho - 1)L_f Q] + G + \rho L_f D + M.$$

The savings of the younger generation including the pay-as-you-go pension must be equal to ρM. Therefore,

$$(1-\alpha)[p_1 Ly(L) - T - L_f D + (\rho - 1)L_f Q] = G - T + (\rho - 1)(L_f D + L_f Q) + M = \rho M.$$

This means that:

$$G = T + (\rho - 1)(M - L_f D - L_f Q).$$

If $M > L_f D + L_f Q$, in order to maintain the steady state with a falling price ($\rho < 1$) (rising price ($\rho > 1$)) a budget surplus (deficit) is required. If $M < L_f D + L_f Q$, we obtain the inverse results.

3 Effects of a decrease in the nominal wage rate

In this paper's model, no mechanism determines the nominal wage rate. For example, when the nominal value of G increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If, when G increases, the price rises considerably, then the output might not increase and involuntary unemployment might not decrease. If the price does not rise or rises only slightly, involuntary unemployment decreases.

Finally we examine the effects on employment of a decrease in the nominal wage rate. A decrease in the nominal wage rate induces a decrease in the price of the good (see (8)), and it does not directly rescue involuntary unemployment. Proposition 2.1 in Otaki (2016) says

> Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate fall in the price level. In other words, a fall in the nominal wage never rescues workers who are involuntarily unemployed.

However, *indirect effects* on aggregate demand due to a fall in the nominal wage rate may exist. We assume that a reduction in the nominal wage rate and the price is not predicted by consumers. If the price of the good falls, the real value of the older generation's savings increases. But, at the same time, a decrease in the price of the good increases the real value of the younger generation consumers' debts.

The real values of the following variables will be maintained even when both the nominal wage rate and the price fall.

G/p_1: the government expenditure.

$\hat{D}/p_1$: consumption in the childhood period of a next generation consumer.

Q/p_1: pay-as-you-go pension for an older generation consumer.

$\hat{Q}/p_1$: pay-as-you-go pension for a younger generation consumer when he retires.

The nominal value of D and that of $M - L_f Q$, which is the older generation's net savings, does not change. Therefore, from (10), whether the fall in the nominal wage rate increases or decreases the effective demand depends on whether

$$M - L_f Q - \alpha L_f D \tag{13}$$

is positive or negative. This is the real balance effect. If D or Q is large, (13) is negative, and the fall in the nominal wage rate increases involuntary unemployment[7].

[7] As the referee points out, our argument does not completely rule out the possibility that real balance effects could

4 Concluding Remarks

In this paper we have examined the existence of involuntary umemployment using a three-generation OLG model under perfect competition with decreasing or constant returns to scale. We considered the case of an indivisible labor supply, and we assumed that the good is produced only by labor.

In future research, we want to analyze involuntary unemployment in the following situations.

1. Divisible labor supply.

   Even if labor supply is divisible, unless it is somewhat large, involuntary unemployment may still exist.

2. Monopolistic competition with constant or increasing returns to scale technology.

3. Goods are produced by capital and labor, and there is investment of firms.

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reduce unemployment, but we believe that the likelihood is sufficiently small. The discussion in this section is from the different perspectives of the real balance effect for which the argument was fought by Pigou (1943) and Kalecki (1944).

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