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Convergence or confusion? A study of world economic growth

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Abstract

This paper takes a closer look at the typical growth convergence regression of Barro (1991), Mankiw, Romer and Weil (1992), Sala-i-Martin (1996), and others. By interpreting the two components of the regression coefficient separately, i.e. the correlation coefficient and the ratio of standard deviations, we distinguish between "time-series" convergence and "cross-section" convergence, and consequently the relationship between β− and σ−convergence. And, using data from the latest Penn World Table database (version 9.1), we investigate the convergence or the lack-of-convergence in samples of countries representing the "World", OECD and Sub-Saharan Africa. The implications of this study for the neoclassical growth model are also discussed.

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1. Introduction

In this paper, we revisit the cross-section regression approach in the neoclassical growth literature on convergence, among which Barro (1991) , Barro and Sala-i-Martin (1992), Mankiw, Romer and Weil (1992), and Sala-i-Martin (1996) are the main representative studies. As stated by Durlauf (1996), alternative perspectives on convergence often arise as a combination of differences in (1) the definition of convergence, (2) the class of theoretical growth models of interest, and (3) the econometric methods employed. Hence, research on convergence remains active and important.[1]

Often, convergence is understood to mean "catch-up" i.e. the case where relatively poorer countries grow faster than their richer counterparts thereby reducing the income gap of countries over time. On the other hand, *cross-section* convergence refers to reduced cross-sectional income dispersion among countries over time. The two, time-series and cross-section convergence, are clearly different. Moreover, it is easy to show that cross-section convergence is possible without time-series convergence, and vice versa. In this study, by decomposing the cross-section regression coefficient of the bivariate model into its two components, namely, the correlation coefficient (r) and the ratio of standard deviations between two periods, (SD_2/SD_1), we distinguish between *time-series* and *cross-section* convergence, and then explain what this means when considered together in the typical neoclassical growth regression.[2] The distinction between convergence in the time-series and cross-sectional senses is important when interpreting convergence over time in cross-section regressions using a lagged dependent variable as an explanatory variable.

2. Decomposing the regression coefficient

The empirical literature on economic growth often features the following regression equation (1) and its variants, where a negative coefficient "b" is presented as evidence for "convergence" (i.e. the growth rate is inversely related to some initial income).[3]

$$\gamma_i = a + b \times ln(y_{1i}) + u_i \tag{1}$$

where γ_i is the growth rate of real income of country i between some initial time period, t_1, and a future period t_2 (i.e. $\gamma_i = ln(y_{2i}) - ln(y_{1i})$, or the difference in the natural log of real income of country i), and u_i is the usual stochastic error term. Note that t_1 is a period that comes before t_2.

Now, suppose we have observations for various countries on real income for any two time periods, t_1 and t_2, then we could rewrite equation (1) as,

$$ln(y_{2i}) = a + (b + 1) \times ln(y_{1i}) + u_i \tag{2}$$

[1]For a summary, see Aghion and Durlauf (2005), Durlauf and Blume (2010), and Johnson and Papageorgiou (2018).

[2]For a more formal treatment, see Bernard and Durlauf (1996) who study the complex relationship between cross-section and time-series convergence.

[3]There are of course other ways to measure convergence than regression, e.g. Walheer (2016).

The OLS estimate of $(b+1)$ is equal to:[4]

$$b + 1 = r \times \left(\frac{SD_2}{SD_1}\right) \tag{3}$$

where r is the correlation coefficient between $ln(y_{1i})$ and $ln(y_{2i})$, and SD_t are the cross-sectional standard deviations of $ln(y_{ti})$ for $t = 1, 2$.

The OLS estimate of "b" is, of course,

$$b = r \times \left(\frac{SD_2}{SD_1}\right) - 1 \tag{4}$$

2.1 Interpretation of "b"

It is useful here to note that the empirical literature on convergence distinguishes between $\beta-$, $\sigma-$ and $\gamma-$convergence. $\beta-$convergence or *unconditional* convergence is based on the primary definition of convergence relating initial income to subsequent growth, and where a negative "b" (in equation 1 above) is taken to imply that lower-income economies grow faster than their higher-income counterparts.[5] On the other hand, $\sigma-$convergence, another common statistical measure of convergence (Friedman 1992; Quah 1993; Andrade *et al.* 2004), refers to the cross-sectional distribution of income, where a reduction in the variance over time is interpreted as convergence, i.e. $\sigma^2(ln(y_{1i})) - \sigma^2(ln(y_{2i})) > 0$. The variance measure in $\sigma-$convergence however does not capture the possibility of rank changes or mobility of individual countries within the distribution of income levels over time. Hence $\gamma-$convergence introduced by Boyle and McCarthy (1997) directly examines inter-temporal mobility using Kendall's index of rank concordance or Kendall's W to capture changes in the ranking of income levels.[6]

Given equation 4 above, for $\beta-$convergence $(b < 0)$ we require that $r \times SD_2/SD_1 < 1$. If $r = 1$, then $SD_2 > SD_1$ would mean $\beta-$divergence. But if we instead have $SD_2 < SD_1$ or $\sigma-$convergence, then we will also have $\beta-$convergence. However, with $r < 1$, we can have $\beta-$convergence even without $\sigma-$convergence (if $SD_2 > SD_1$) as long as the product $r \times SD_2/SD_1 < 1$. Put differently, $\sigma-$divergence does not lead to $\beta-$divergence, and here lies the critical ambiguity. We address the four possible cases more precisely:

CASE 1: $r < 1$ and $SD_1 > SD_2$

Here we have $\sigma-$ or *cross-section* convergence, that is, decreasing cross-sectional income dispersion over time.

[4] For simplicity of notation and argument, we decide not to distinguish between the true parameter value and its estimate.

[5] Unsurprisingly, one typically does not find evidence for unconditional convergence, which to be fair, is not what conventional growth theories would predict anyway. Rather researchers investigate "conditional" convergence in the form $\gamma_i = a + b \times ln(y_{1i}) + c \times Z_i + u_i$, where Z_i is a set of growth determinants assumed to affect growth, in addition to a country's initial income (see Mankiw *et al.* 1992).

[6] Boyle and McCarthy (1997) use $\gamma = \left(\frac{\sigma^2(y_{t2}+y_{t1})}{\sigma^2(y_{t1}\times 2)}\right)$, where $\sigma^2(y_t)$ is the corresponding variance of the ranks of real per capita income.

Correlation less than 1, i.e. "$r < 1$", simply corresponds to the "regression fallacy" (or as Galton describes it, "regression towards mediocrity"). That is, in the context of growth convergence, this implies that countries with lower incomes in some previous period, t_1, would rebound with higher incomes in the latter period, t_2, while those with higher incomes in t_1 will revert to mediocrity in t_2. Such variation simply arises due to inevitable chance from mother nature and should not be subject to any other interpretation. Hence, time-series convergence is nothing but a myth in the sense that any two random variances has in general a correlation coefficient less than 1, and moreover since the correlation coefficient is direction-free, to suggest time convergence equally means time divergence by running time backwards. As correctly identified by Friedman (1992) and Quah (1993), one should therefore avoid the incorrect idea that such mean reversion implies that its variance is declining, i.e. Galton's fallacy.

Put differently, with most data, correlation is not perfect, and reversal in rankings over time are inevitable. Since r is typically less than 1 (in the right-hand side of equation 4), the only meaningful quantity in $\beta-$convergence is SD_2/SD_1, i.e. $\sigma-$convergence.

CASE 2: $r < 1$ and $SD_1 < SD_2$

In this case, there is an absence of $\sigma-$ or cross-section convergence. In fact, we might say there is cross-section *divergence*. However, $\beta-$convergence is still possible and hence we can have time-series convergence without $\sigma-$convergence. This is consistent with Fuceri (2005) who shows that $\sigma-$convergence is only a *sufficient* (but not necessary) condition for the existence of $\beta-$convergence. The implication is that the absence of $\sigma-$convergence cannot be taken as implying the absence of $\beta-$convergence.

CASE 3: $r = 1$ and $SD_1 > SD_2$

In this case, we have $\sigma-$convergence and $\beta-$convergence, but with r not different to 1, time-series convergence (nor divergence) cannot be claimed.

CASE 4: $r = 1$ and $SD_1 < SD_2$

Lastly, in this case, there is neither cross-section nor time-series convergence. Obviously, $\beta-$convergence is also absent.

It should be clear that $\beta-$convergence implied by a negative value of b in the growth empirics is ambiguous and can be misleading. Simply making inferences based on the estimated value of b cannot provide a definite answer about convergence over time or economic *catch-up*, because the question is still left begging whether indeed cross-section and time-series convergence is being implied. Hence, for correct inference, the necessary decomposition of "b" is required, and accordingly, the appreciation of the time-ambiguity of Pearson's correlation coefficient r when considering time-series convergence.

3. Empirical results

In this section, we present estimations based on the above decomposition using real per capita GDP at constant 2011 prices as a measure of income from the latest Penn World Table database version 9.1 (Feenstra, Inklaar and Timmer, 2015) for various countries from 1950 to 2017. Figures 1 to 3 present the empirical estimation for the World, OECD countries and Sub-Sahara African (SSA) countries, respectively, for 3 different reference/initial years $t_1 = 1960, 1970$ and 1990 with t_2 running from $t_1 + 1$ up until $t_2 = 2017$, corresponding to panels top, middle and bottom, respectively.

It is illustrative to compare the figures. Representing the World (sample of 111 countries) in the top panel of Figure 1 with reference year 1960, the correlation coefficient, r, steadily decreases by about 20 percent, while the ratio of standard deviations remains larger than one at least consistently from around 1970s. Hence the period after 1970 (and this is confirmed by the middle panel based on initial year 1970 with a sample of 156 countries) thereby corresponds to case 2 of our discussion above, i.e. $r < 1$ and $SD_1 < SD_2$. For the World, we find evidence for the absence of cross-section convergence (although this is somewhat mitigated in the 2010s). A closer look shows that for the World the b coefficient is only negative after about 2010 (or slightly earlier judging by the bottom panel with reference year 1990 and a sample of 180 countries), thereby indicating $\beta-$convergence was generally absent prior to the early 2000s. Furthermore, $\sigma-$convergence was also largely absent before 2010. Hence, despite the negative and falling correlation coefficient, especially by the middle panel of 156 countries with reference year 1970 (negative b since 1980s), arguably there is little or no support for time-series convergence for the World.

From Figure 2, which presents the corresponding empirical estimations for the set of OECD countries, we find similarly that r decreases steadily but by 40 percent, much more drastically than compared to the sample for the World, thereby, as we may expect, suggesting stronger time-series convergence among OECD countries relative to the sample of the World.[7] One noticeable difference (except for the bottom panel with 1990s as the reference year) is that the ratio of standard deviations was less than one and decreasing (except for the decades of the 1980s and 1990s when it was somewhat constant). We find that OECD countries fit case 1, $r < 1$ and $SD_1 > SD_2$, as discussed above.[8]

Many Sub-Sahara African (SSA) countries experienced *negative* real per capita income growth since the early-1970s to at least the early-2000s.[9] The neoclassical bivariate regression should be treated with caution as cross-section or $\sigma-$divergence and negative growth rates in the case of SSA countries do not sit well with the somewhat misleading $\beta-$ convergence shown in Figure 3. Rather than true catch-up, the SSA experience reflects a different kind of time-series convergence, a "race to the bottom", especially during what has now been coined Africa's lost decades.

[7] Note however that Spearman's correlation coefficient does not fall as quickly.

[8] Although not shown here, as with Maasoumi *et al.* (2007), we compared the OECD with non-OECD countries, with the latter falling into case 2.

[9] See Vila-Artadi *et al.* (2003).

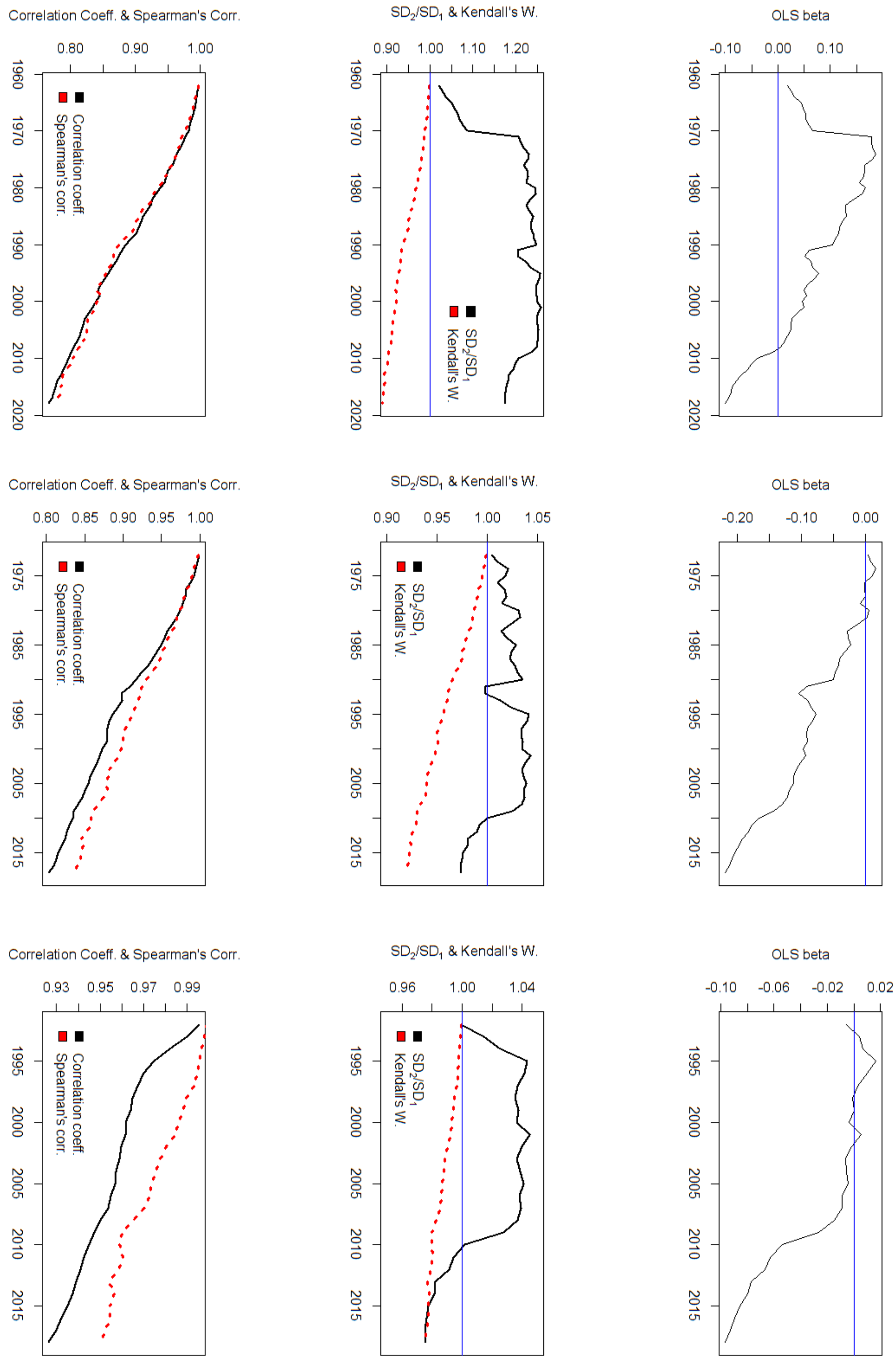

Figure 1: The World (n=111, 156, 180)

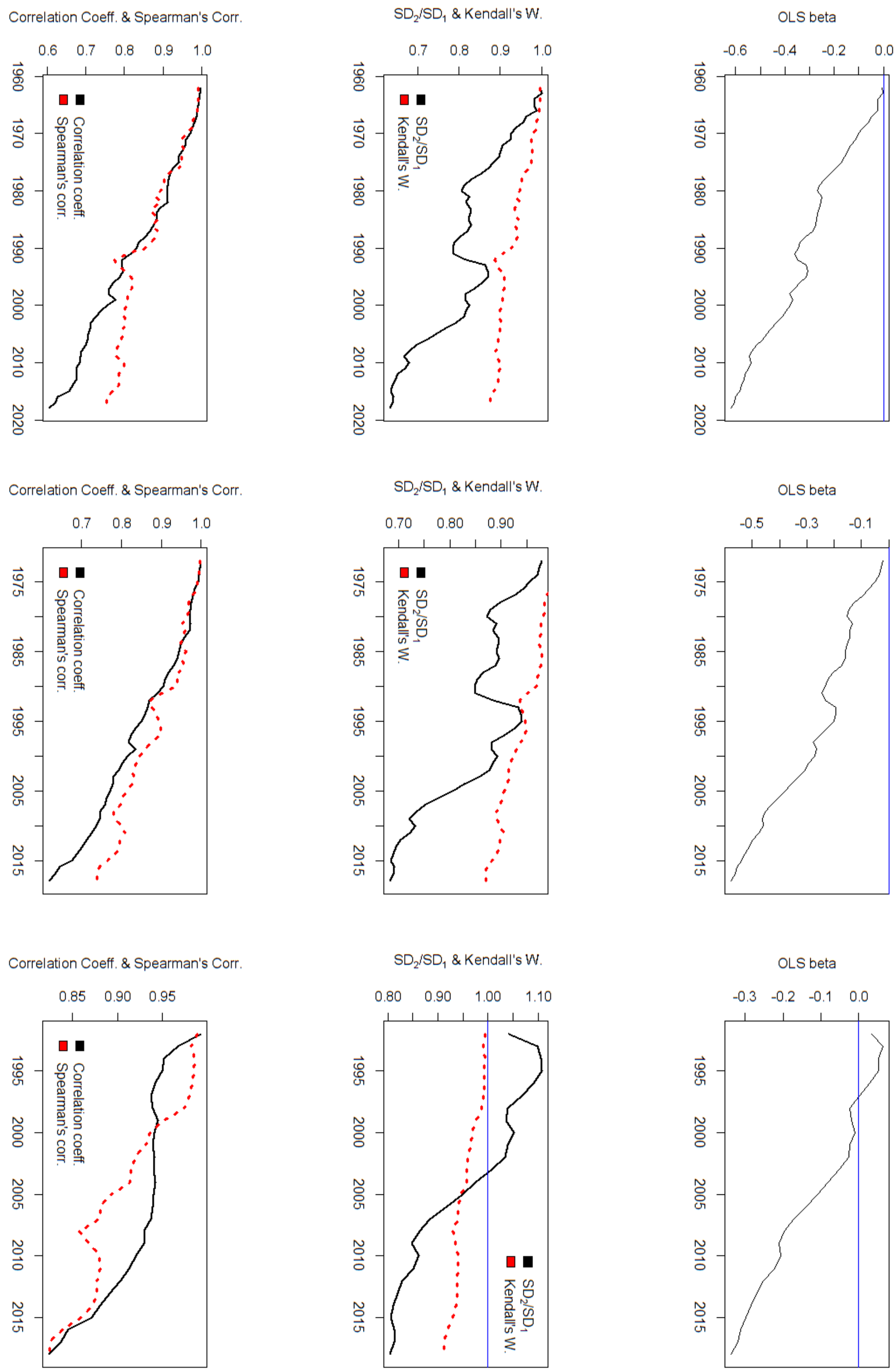

Figure 2: OECD (n=28, 30, 35)

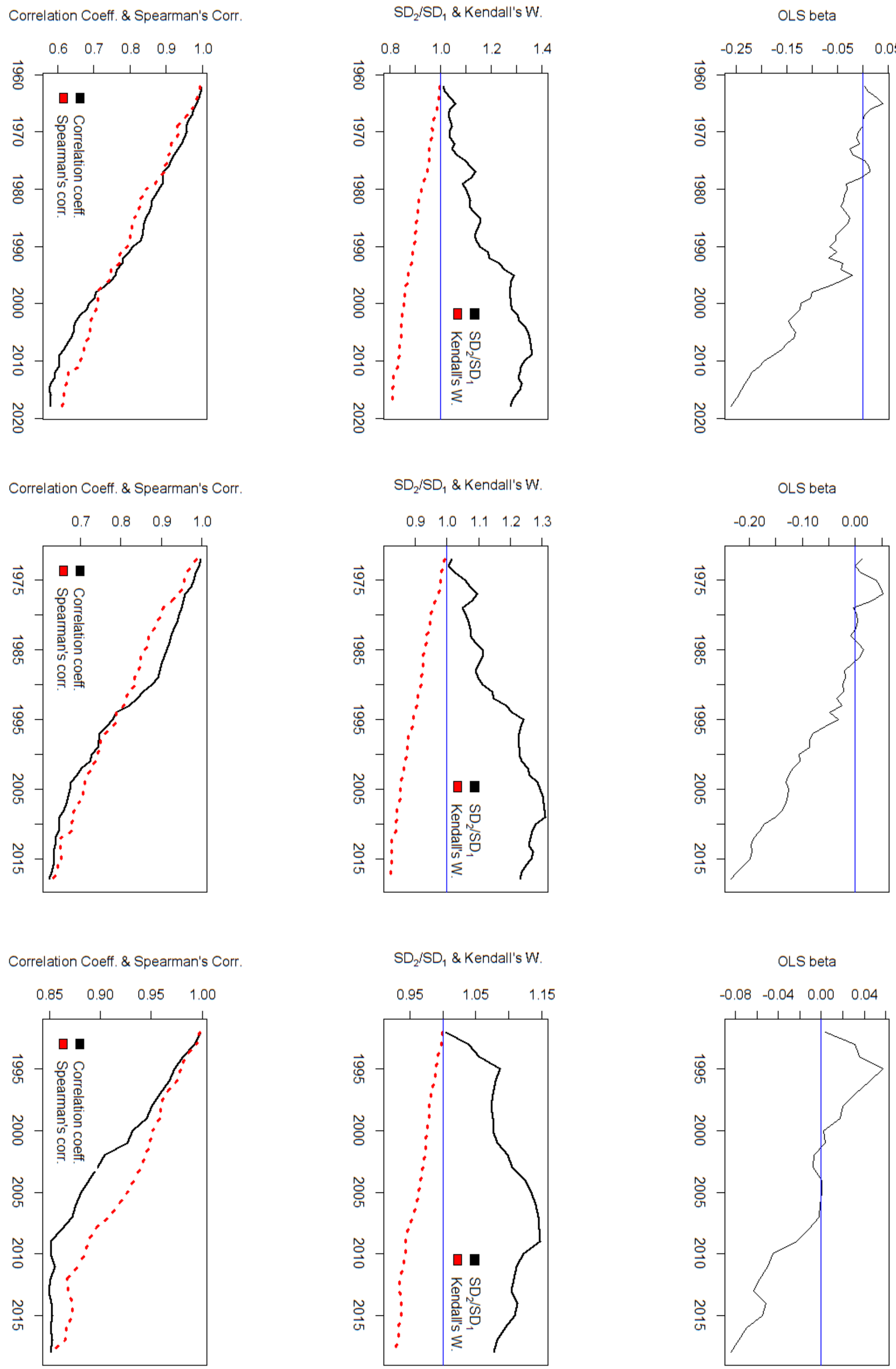

Figure 3: Sub-Saharan Africa (n=39, 45, 45)

4. Conclusion

Following Daulauf's observations as stated in the introduction, this paper throws caution when interpreting "convergence" in the neoclassical growth model, by decomposing the typical growth regression coefficient "b" into its time-series and cross-sectional components. Essentially, using income data for a sample of countries in two time periods (t_1 before t_2), by decomposing the regression coefficient into the Pearson's correlation coefficient, r, and the ratio of standard deviations, SD_2/SD_1, we argue that ignoring the non-directional nature of the correlation coefficient, one can easily and mistakenly interpret a negative "b" as *time-series* convergence. Such ambiguity of $\beta-$convergence in the neoclassical growth model as time-series convergence or catch-up over time is highlighted in this paper.

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