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A New Keynesian Phillips Curve With Staggered Contracts and Indexation

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Abstract

We develop a New Keynesian Phillips curve based on a combination of staggered price contracts and indexation to past inflation. This Phillips curve links current inflation dynamics to past inflation with a positive weight, as well as current and lagged expectations of inflation and output, giving a possible alternative explanation for recent empirical findings on the role of expectations in the determination of inflation

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1 Introduction

There is a debate on the role of expectations in inflation dynamics. A long literature (see Woodford, 2003 for early presentations) has claimed that the presence of non rational price setters helps to reproduce the role of past inflation in the determination of current inflation that is observed in the data. Fuhrer (2017) have argued more recently that on the contrary, the inertia of aggregate variables is well captured by the inertia of expectations, both for inflation and output, that is found in expectations surveys, leaving a small role only for lagged inflation.

We propose a framework that allows both elements to play a flexible role in inflation dynamics. To do that, we develop a New Keynesian Phillips curve that is based on a staggered pricing structure proposed by Taylor (1980), in which we introduce a partial indexation of prices to lagged inflation, as in Woodford (2003). The staggered prices structure offers a role for inertia of expectations in the determination of current inflation. While it has been shown that in a purely forward-looking models, this lagged inflation has always a negative coefficient (Yao, 2009, Whelan, 2007), we show that introducing price indexation transforms this influence into a positive one. We also show that if we remove expectational error termes, this Phillips curve is similar to the inflation equation proposed by Fuhrer and Moore (1995), which contained several lags and leads of inflation to explain the current inflation rate.

2 A New Keynesian Model with Staggered Contracts and Indexation

We develop a staggered prices model in the sense of Taylor (1980). Prices are set in advance for a length of N periods. N different cohorts of price setters of equal size coexist at any given time. Each price contract has a duration of N periods. When a new contract is set, the firms chooses rationally the price to set initially, and then during the rest of the life of the contract, this initial price is updated to the last know inflation rate (see Woodford, 2003). Compared to Taylor (1980), prices are no longer fully predetermined and fixed since they can be modified each period to adjust to last period inflation.

2.1 Price setting under indexed staggered contracts

We consider a standard linearized model of price setting in which the profit-maximizing flexible price at time t, noted p_t^* is equal to[1]:

$$p_t^* = p_t + \phi y_t \tag{1}$$

where p_t is the general price level, $\phi > 0$ a parameter measuring real rigidities and strategic complementarities, and y_t the real marginal costs of firms[2]. Each cohort of firms can set a new price at a different period. When a firm can set a new price at period t, it does it on order to minimize the distance between the price effectively set and the profit maximizing price p^* during the N periods (including the current one) of the duration of the price contract. We note $x_{t,t+i}$, $i \in [0, ..., N-1]$ the value of the price effective during period $t+i$ for a contract set in period t. The initial value of the contract is $x_{t,t} = x_t$. In the model of Taylor (1980), prices are fixed all over the duration of the contract and then we have $x_{t,t+i} = x_t$, $\forall i \in [0, ..., N-1]$. If we introduce indexation to past inflation in the sense of Woodford (2003), during each period of the contract consecutive to the initial period, a fraction γ of the value of the previous period inflation is added to the current value of the price. For example, the value in $t+1$ of a price modified in t is $x_{t,t+1} = x_t + \gamma\pi_t$, with $\pi_t = p_t - p_{t-1}$ the inflation rate in t. More generally, for $0 < i < N$, we have:

[1] See Woodford (2003) or Sheedy (2010)

[2] The model is linear. All variables are in log. Uneder restrictive assumptions, the real marginal cost can be linearly related to the output gap.

$$x_{t,t+i} = x_t + \gamma \sum_{l=0}^{i-1} \pi_{t+l} \tag{2}$$

A firm deciding in t to set the initial value of its new price contract has the following program:

$$\min_{x_t} \sum_{i=0}^{N-1} \beta^i \mathbb{E}_t (x_{t,t+i} - p^*_{t+i}) \tag{3}$$

where $x_{t,t+i}$ is given by (2) for $i \in]0, N]$, and $x_{t,t} = x_t$. The first order condition of this programs leads to:

$$x_t = \frac{\sum_{i=0}^{N-1} \beta^i \mathbb{E}_t p^*_{t+i} - \gamma \sum_{i=0}^{N-2} \beta^{i+1} \sum_{l=0}^{i} \mathbb{E}_t \pi_{t+l}}{\sum_{i=0}^{N-1} \beta^i} \tag{4}$$

To simplify the exposition, without altering the results, we assume that $\beta = 1$. Using equation (1) and given that for $i \geqslant 1$, we have $p_{t+i} = p_t + \sum_{k=1}^{i} \pi_{t+k}$. it is possible to write the previous equation as:

$$x_t = p_t + \frac{1}{N} \left[\sum_{i=1}^{N-1} (N-i) \mathbb{E}_t (\pi_{t+i} - \gamma \pi_{t+i-1}) \right] + \frac{1}{N} \phi \sum_{i=0}^{N-1} \mathbb{E}_t y_{t+i} \tag{5}$$

A price set in t responds to the current price level, to expected inflation and to the expected trajectory of the driving variable y. Without indexation, this responds to the expected path of inflation over the duration of the contract. With full indexation, it only responds to expected variations of inflation.

Since all cohorts of prices have the same size, the price level in the economy is an average of the price contracts at a given date: $p_t = \frac{1}{N} \sum_{j=0}^{N-1} x_{t-j,t}$. Given the value of prices given in (2), we can rewrite the price level in t such as:

$$p_t = \frac{1}{N} \left[\sum_{j=0}^{N-1} x_{t-j} + \gamma \sum_{j=1}^{N-1} (N-j) \pi_{t-j} \right] \tag{6}$$

2.2 The Phillips curve

Combining equation (5) and equation (6), we have:

$$Np_t = \sum_{i=0}^{N-1} p_{t-i} + \frac{1}{N}\sum_{j=0}^{N-1}\mathbb{E}_{t-j}\left(\sum_{i=1}^{N-1}(N-i)(\pi_{t+i-j}-\gamma\pi_{t+i-j-1}) + \phi\sum_{i=0}^{N-1} y_{t+i-j}\right) + \gamma\sum_{j=1}^{N-1}(N-j)\pi_{t-j} \tag{7}$$

Noting that $Np_t - \sum_{i=0}^{N-1} p_{t-i} = (N-1)\pi_t + \sum_{i=1}^{N-1}(N-i-1)\pi_{t-i}$, we obtain the following Taylor hybrid Phillips curve:

$$\pi_t = \eta\sum_{i=1}^{N-1} a_i\pi_{t-i} + \eta\sum_{j=0}^{N-1}\sum_{i=1}^{N-1} b_{ji}\mathbb{E}_{t-j}\pi_{t+i-j} + \eta\phi\sum_{j=0}^{N-1}\sum_{i=0}^{N-1}\mathbb{E}_{t-j}y_{t+i-j} \tag{8}$$

with $\eta = \frac{1}{(N-1)(N+\gamma)}$, $a_i = [\gamma + N(1-\gamma)(1-N+i)]$ and $b_{ji} = [\gamma + (1-\gamma)(N-i)]$. The first term represents the role of lagged inflation. The second term represents the role of current and lagged expectations of inflation. The third term represents the role of current and past expectations of output gap. We then obtain for Taylor staggered contracts a similar equation to the equation [2.6] proposed by Sheedy (2010) for different hazard functions of price adjustment. Here, the number of lags and leads of inflation is finite, and the value of the coefficients on them have an explicit value depending on the length of contracts and the level of indexation to past inflation. Compared to Whelan (2007) and Yao (2009), the presence of indexation can give a positive weight to lagged inflation, which is excluded in the purely forward-looking model. Indeed, when $\gamma = 0$, the weight on past inflation is strictly negative for any term of past inflation. However, when there is full indexation to past prices ($\gamma = 1$), the weight on past inflation is always positive. For partial indexation, $\gamma \in]O, 1[$, there exists a mix of positive and negative weights on past inflation. For example, if $N = 4$, the weight on π_{t-3} is positive for any value $\gamma > O$, the weight on π_{t-2} is positive for $\gamma > 4/5$ and the weight on π_{t-1} is positive for $\gamma > 8/9$.

3 The staggered Phillips curve without lagged expectations

Lagged expectations of inflation represent an interesting feature of staggered prices models since they allow a sluggish response of inflation to shocks, even in the case of a disinflation policy with forward-looking agents (see Musy, 2006). However, it is frequent to remove lagged expectations under the argument that they represent expectational errors that can be neglected (see Roberts, 1995 or Walsh, 2010)[3]. In this section, we follow this approach and we rewrite lagged expectational of current and past terms as expectational errors terms, equal ot 0 on average. We then assume $\mathbb{E}_{t-j}\pi_{t-j+i} - \pi_{t-j+i} = 0$, for $j > i \geq 0$. We also assume that for $i > 0$, $j > 0$, $\mathbb{E}_{t-j}\pi_{t+i} - \mathbb{E}_t\pi_{t+i} = 0$. Under such assumptions, the New Keynesian Phillips curve becomes:

$$\pi_t = \eta_E \sum_{i=1}^{N-1} c_{i,j}\pi_{t-i} + \eta_E \sum_{i=1}^{N-1} d_{i,j}\mathbb{E}_t\pi_{t+i} + \eta_E\phi\left(Ny_t + \sum_{i=1}^{N-1}(N-i)(y_{t-i} + \mathbb{E}_t y_{t+i}\right) \tag{9}$$

with $\eta_E = \frac{2}{N(N-1)(1+\gamma}$, $c_{i,j} = [\gamma(N-i) - (1-\gamma)\sum_{j=0}^{N-1-j} j]$ and $d_{i,j} = [(N-i)\gamma + (1-\gamma)\sum_{j=1}^{N-1} j]$.

When $\gamma = 0$, the weight on past inflation remains strictly negative. When $\gamma = 1$, the Phillips curve is:

$$\pi_t = \frac{1}{N(N-1)}\left[\sum_{i=1}^{N-1}(N-i)\left(\pi_{t-i} + \mathbb{E}_t\pi_{t+i}\right)\right] + \frac{\phi}{N(N-1)}\left[Ny_t + \sum_{i=1}^{N-1}(N-i)(y_{t-i} + \mathbb{E}_t y_{t+i})\right] \tag{10}$$

This Phillips curve with full indexation is close to one proposed by Fuhrer and Moore (1995), equation (25), who have different foundations. Considering expectational errors as equal to 0, they display a two-sided inflation equation of the form:

$$\pi_t = f(L)f(L^{-1})[\pi_t + \phi g^{-1}(L)y_t] \tag{11}$$

where L stands for the lag operator. The number of lags and leads of inflation depends on the length of contracts N. Their assumptions have been strongly criticized by Holden and Driscoll (2003), but it is possible to obtain a similar Phillips curve with a standard assumption of price indexation.

4 Conclusion

Compared to the existing forms in the literature, the Phillips curve developed in this paper has the interesting feature to reproduce a role for sluggish expectations in the inflation an output dynamics, as well as possible positive weights on past inflation, which was not the

[3] Sheedy (2010) proposes a formal discussion of removing these terms.

case in purely forward-looking models. Such a form can be an alternative way to reproduce a role for sticky expectations in inflation dynamics (see Fuhrer, 2017), but keeping the standard assumptions used in the New Keynesian literature.

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