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Mixed duopoly in price competition under the optimal privatization rate

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Abstract

This study examines social welfare in a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in price after the government sets the optimal degree of privatization for the partially privatized firm. Comparing social welfare when the timing of decision making is different, we present the following results. When the degree of substitutability of goods is low, social welfare in the Stackelberg equilibrium is the largest when a partially privatized firm is the leader. By contrast, when the degree is high, the social welfare in the Bertrand equilibrium is the largest. Unlike the results presented in quantity competition, the Stackelberg equilibrium when a partially privatized firm is the follower never achieves the largest social welfare.

1. Introduction

We examine social welfare in a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in price after the government sets the optimal degree of privatization for the partially privatized firm. Comparing the social welfare when the timing of decision making is different, we determine which equilibrium achieves the largest social welfare. Notably, this study is closely related to Hamada (2020), which compared social welfare within a mixed duopoly during quantity competition. Hamada (2020) demonstrated that social welfare is equal in the Cournot equilibrium and Stackelberg equilibrium when a partially privatized firm is the leader, and it is the largest in the Stackelberg equilibrium when a partially privatized firm is the follower.[1] One of unresolved issues in the previous research is whether the results on the order of social welfare changes when we consider price competition in a mixed duopoly.

In the oligopoly theory, it is well known that the choice of strategic variables, quantity or price, drastically affects the equilibrium results in oligopoly competition, as shown in the Cournot or Bertrand equilibrium. Recently, Méndez-Naya (2015) examined the situation in which a partially-privatized firm and a private firm compete in quantity or price simultaneously or sequentially and presented the endogenous order of moves by firms in a mixed duopoly. His study demonstrated that the equilibrium of the endogenous timing game greatly differs depending on whether firms compete in quantity or price. However, the degree of privatization for a partially-privatized firm is exogenously given. If the government sets the optimal degree of privatization, we need to evaluate social welfare under the different degrees of privatization to compare social welfare. Although we abstracted the endogenous timing from the analysis, in quantity competition, we have already shown the results in our previous studies. In this companion study, we compare social welfare when a partially-privatized firm and a private firm engage in price competition under the optimal degree of privatization. Assuming that the government can determine the optimal degree of privatization for the partially privatized firm before a partially privatized firm and a private firm compete in a market, we examine which type of equilibrium leads to higher social welfare.

Comparing the social welfare when the timing of decision making is different, we present the following results. When the degree of substitutability of goods is low, the social welfare in the Stackelberg equilibrium when a partially privatized firm is the leader is the largest. By contrast, when the degree is high, the social welfare in the Bertrand equilibrium is the largest. Unlike the results presented in quantity competition, the Stackelberg equilibrium when a partially privatized firm is the follower never achieves the largest social welfare amount.

The remainder of this paper is organized as follows: Section 2 presents a model in which a partially privatized firm and a private firm engage in price competition simultaneously or sequentially. Section 3 derives the equilibrium results in price competition and presents the main results. Section 4 presents the concluding remarks.

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[1] To avoid repetition of the references already presented by Hamada (2020), we omit a detailed introduction of the literature already mentioned herein.

2. The model

We consider a mixed duopoly in which a partially privatized firm and a private firm compete within a differentiated goods market. The partially privatized firm and private firm are indexed by firms 0 and 1, respectively. Firm 0 maximizes the weighted average of social welfare and profit, and firm 1 maximizes its profit. Both firms produce differentiated goods and engage in duopolistic competition. q_i denotes firm i's output, $i = \{0, 1\}$.

We consider an economy that consists of a mixed duopoly market of differentiated goods and a perfectly competitive market of numeraire goods. The utility function of the representative consumer is assumed to be additively separable and linear in the numeraire goods. Following Singh and Vives (1984), the utility function is quadratic, strictly concave, and symmetric with respect to q_0 and q_1 as follows:

$$U(q_0, q_1) = q_0 + q_1 - \frac{1}{2}\left(q_0^2 + q_1^2 + 2bq_0q_1\right). \tag{1}$$

$b \in (0, 1]$ denotes the degree of substitutability between the two goods. From the utility function (1), we obtain demand functions and inverse demand functions as follows:

$$p_i = 1 - q_i - bq_j, j \neq i, \tag{2}$$

$$q_i = \frac{1 - b - p_i + bp_j}{1 - b^2} = d - fp_i + gp_j, \tag{3}$$

where $d \equiv 1/(1 + b) > 0$, $f \equiv 1/(1 - b^2) > 0$, and $g \equiv b/(1 - b^2) > 0$. Consumer surplus is $CS \equiv U(q_0, q_1) - p_0q_0 - p_1q_1 = \frac{1}{2}(q_0^2 + q_1^2 + 2bq_0q_1)$.

Both firms have identical technologies with increasing marginal costs. Firm i's cost function is quadratic as follows: $C(q_i) = F + \frac{1}{2}q_i^2$, where F is the fixed cost. For brevity and without loss of generality, we assume $F = 0$. Firm i's profit function is as follows:

$$\pi_i = p_iq_i - \frac{q_i^2}{2}. \tag{4}$$

Producer surplus is $PS \equiv \pi_0 + \pi_1 = p_0q_0 + p_1q_1 - \frac{1}{2}(q_0^2 + q_1^2)$. Social welfare is the sum of consumer surplus and producer surplus, that is, $W \equiv CS + PS = q_0 + q_1 - q_0^2 - q_1^2 - bq_0q_1$.

Following Matsumura (1998), a partially privatized firm aims to maximize the weighted average of social welfare and its own profit. Thus, its objective function is as follows:

$$\Omega = (1 - \alpha)W + \alpha\pi_0, \tag{5}$$

where $\alpha \in [0, 1]$ denotes the degree of privatization for the partially privatized firm, which determines the weight of the firm's profit in the objective function. When $\alpha = 0$, it is fully nationalized, and when $\alpha = 1$, it is fully privatized. A private firm aims to maximize its own profit, and a government aims to maximize social welfare. The government can determine the optimal degree of privatization α^* to maximize social welfare.

The timing of the game has the following two stages: In the first stage, the government sets the optimal degree of privatization for the partially privatized firm. In the second stage, each firm sets the price level. The solution concept follows the subgame perfect Nash equilibrium.

In the following analysis, we consider the following three scenarios: (i) the simultaneous-move equilibrium, that is, the Bertrand equilibrium; (ii) the Stackelberg equilibrium when a partially privatized firm is the leader; and (iii) the Stackelberg equilibrium when a partially privatized firm is the follower.

3. Price competition

The partially privatized firm and private firm choose their prices p_0 and p_1 to maximize their objective, namely (5) and (4), respectively.[2] Solving the first-order conditions for both firms, we obtain their reaction functions as follows:[3]

$$\frac{\partial \Omega}{\partial p_0} = 0 \ \Rightarrow \ p_0 = r_0(p_1) \equiv \frac{(1-b)[1-b+(1+b-b^2)\alpha] + b(3-b^2-\alpha)p_1}{2+(1-2b^2)\alpha}, \quad (6)$$

$$\frac{\partial \pi_1}{\partial p_1} = 0 \ \Rightarrow \ p_1 = r_1(p_0) \equiv \frac{(2-b^2)(1-b+bp_0)}{3-2b^2}. \quad (7)$$

The first derivatives are $r_0'(p_1) = \frac{b(3-b^2-\alpha)}{2+(1-2b^2)\alpha} > 0$ and $r_1'(p_0) = \frac{b(2-b^2)}{3-2b^2} > 0$.

3.1 Bertrand equilibrium

Solving the simultaneous equations (6) and (7) with respect to p_0 and p_1, we obtain the Bertrand equilibrium price as follows:

$$\left(p_0^B(\alpha), p_1^B(\alpha)\right) = \left(\frac{3-2b^2-b^3+b^4+(1-b^2)(3-2b)\alpha}{6-4b^2+b^4+3(1-b^2)\alpha}, \frac{(2-b^2)[2-b+(1-b^2)\alpha]}{6-4b^2+b^4+3(1-b^2)\alpha}\right). \quad (8)$$

Table 1 summarizes the equilibrium variables other than price.[4]

Table 1: Bertrand equilibrium

firm 0's output	$q_0^B(\alpha)$	$\frac{3-2b-b^2+b^3+b(1-b^2)\alpha}{6-4b^2+b^4+3(1-b^2)\alpha}$
firm 1's output	$q_1^B(\alpha)$	$\frac{2-b+(1-b^2)\alpha}{6-4b^2+b^4+3(1-b^2)\alpha}$
firm 0's profit	$\pi_0^B(\alpha)$	$\frac{[3-2b-b^2+b^3+b(1-b^2)\alpha][3+2b-3b^2-3b^3+2b^4+(6-5b)(1-b^2)\alpha]}{2[6-4b^2+b^4+3(1-b^2)\alpha]^2}$
firm 1's profit	$\pi_1^B(\alpha)$	$\frac{(3-2b^2)[2-b+(1-b^2)\alpha]^2}{2[6-4b^2+b^4+3(1-b^2)\alpha]^2}$
social welfare	$W^B(\alpha)$	$\frac{B_0+(1-b^2)(17-10b-3b^2+3b^4-2b^5+b^5)\alpha+(1+2b)(2-b)(1-b^2)^2\alpha^2}{[6-4b^2+b^4+3(1-b^2)\alpha]^2}$
firm 0's objective	$\Omega^B(\alpha)$	$\frac{2B_0+B_1\alpha-B_2\alpha^2-(4+b^2)(1-b^2)^2\alpha^3}{2[6-4b^2+b^4+3(1-b^2)\alpha]^2}$

$B_0 \equiv 17-8b-18b^2+8b^3+9b^4-4b^5-2b^6+b^7 > 0$, $B_1 \equiv 9-4b-20b^2+8b^3+b^4+3b^6-2b^7$,

$B_2 \equiv 2(1-b^2)(6-b-2b^2+2b^3) > 0$.

We derive the optimal degree of privatization α^{B*} to maximize social welfare. Since $W^B(\alpha)$ does not necessarily satisfy concavity with respect to α, the second-order condition of social welfare is not satisfied. However, since $W^B(\alpha)$ is a strictly monotonically decreasing function of α in the range of $\alpha \in [0,1]$, the optimal degree of privatization is the corner solution, $\alpha^{B*} = 0$. Thus, the government chooses full nationalization and firm 0's objective is equal to social welfare, that is, $W^{B*} = \Omega^{B*}$. Substituting $\alpha^{B*} = 0$ into (8) and Table 1, we obtain the variables in the Bertrand equilibrium in Table 2.

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[2] Firm i's profit and social welfare are, respectively, $\pi_i = \frac{(1-b-p_i+bp_j)(-1+b+(3-2b^2)p_i-bp_j)}{2(1-b^2)^2}$ and $W = \frac{(1-b)^2(b+p_0+p_1)-(p_0^2+p_1^2)+b(3-b^2)p_0p_1}{(1+b)^2(1-b)^2}$.

[3] By assumption, the second-order conditions for maximization are necessarily satisfied.

[4] The superscripts B, l, and f denote the equilibrium variables in the Bertrand equilibrium, the Stackelberg equilibrium when the partially privatized firm is the leader, and the Stackelberg equilibrium when the partially privatized firm is the follower, respectively.

Table 2: Bertrand equilibrium when α is optimal

firm 0's output	q_0^{B*}	$\frac{3-2b-b^2+b^3}{6-4b^2+b^4}$
firm 1's output	q_1^{B*}	$\frac{2-b}{6-4b^2+b^4}$
firm 0's price	p_0^{B*}	$\frac{3-2b^2-b^3+b^4}{6-4b^2+b^4}$
firm 1's price	p_1^{B*}	$\frac{(2-b)(2-b^2)}{6-4b^2+b^4}$
firm 0's profit	π_0^{B*}	$\frac{(3-2b-b^2+b^3)(3+2b-3b^2-3b^3+2b^4)}{2(6-4b^2+b^4)^2}$
firm 1's profit	π_1^{B*}	$\frac{(3-2b^2)(2-b)^2}{2(6-4b^2+b^4)^2}$
social welfare	W^{B*}	$\frac{B_0}{(6-4b^2+b^4)^2}$
firm 0's objective	Ω^{B*}	$\frac{B_0}{(6-4b^2+b^4)^2}$

$B_0 \equiv 17 - 8b - 18b^2 + 8b^3 + 9b^4 - 4b^5 - 2b^6 + b^7 > 0$

3.2 Stackelberg equilibrium when the partially privatized firm is the leader

Considering that the private firm reacts following (7), the partially privatized firm sets the price to maximize its objective. From the first-order condition, we obtain the equilibrium output of the partially privatized firm:

$$\frac{\partial \Omega}{\partial p_0} = (1-\alpha)\Big(\frac{\partial W}{\partial p_0} + \frac{\partial W}{\partial p_1}r_1'\Big) + \alpha\Big(\frac{\partial \pi_0}{\partial p_0} + \frac{\partial \pi_0}{\partial p_1}r_1'\Big) = 0$$

$$\Rightarrow p_0(\alpha) = \frac{9 - 2b - 7b^2 + 2b^4 + (9 - 4b - 8b^2 + 3b^3 + b^4)\alpha}{18 - 16b^2 + 4b^4 + 9\alpha - 8b^2\alpha + b^4\alpha}. \tag{9}$$

Substituting (9) into $p_1 = r_1(p_0)$, we obtain the private firm's equilibrium price:

$$p_1(\alpha) = \frac{(2-b^2)[(2-b)(3-b^2) + (3-2b^2)\alpha]}{18 - 16b^2 + 4b^4 + 9\alpha - 8b^2\alpha + b^4\alpha}. \tag{10}$$

Table 3 summarizes the equilibrium variables other than price.

Table 3: Stackelberg equilibrium when the partially privatized firm is the leader

firm 0's output	$q_0^l(\alpha)$	$\frac{9-4b-6b^2+2b^3+b^4+b(1-b^2)\alpha}{2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha}$
firm 1's output	$q_1^l(\alpha)$	$\frac{(2-b)(3-b^2)+(3-2b^2)\alpha}{2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha}$
firm 0's profit	$\pi_0^l(\alpha)$	$\frac{[9-4b-6b^2+2b^3+b^4+b(1-b^2)\alpha][9-8b^2-2b^3+3b^4+(18-9b-16b^2+7b^3+2b^4)\alpha]}{2[2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha]^2}$
firm 1's profit	$\pi_1^l(\alpha)$	$\frac{(3-2b^2)[(2-b)(3-b^2)+(3-2b^2)\alpha]^2}{2[2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha]^2}$
social welfare	$W^l(\alpha)$	$\frac{C_0[9-8b^2+2b^4+(9-8b^2+b^4)\alpha]+C_2\alpha^2}{[2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha]^2}$
firm 0's objective	$\Omega^l(\alpha)$	$\frac{D_0+D_1\alpha-D_2\alpha^2-D_3\alpha^3}{2[2(9-8b^2+2b^4)+(9-8b^2+b^4)\alpha]^2}$

$C_0 \equiv 17 - 8b - 10b^2 + 4b^3 + b^4 > 0$, $C_2 \equiv 18 + 9b - 34b^2 - 17b^3 + 22b^4 + 9b^5 - 5b^6 - b^7 > 0$,

$D_0 \equiv 2(9 - 8b^2 + 2b^4)(17 - 8b - 10b^2 + 4b^3 + b^4) > 0$, $D_1 \equiv 81 - 36b - 126b^2 + 32b^3 + 58b^4 - 10b^6 - 4b^7 + b^8$,

$D_2 \equiv 2(54 - 9b - 84b^2 + 17b^3 + 42b^4 - 9b^5 - 7b^6 + b^7) > 0$, $D_3 \equiv (4 - 3b^2)(9 - 8b^2 + b^4) > 0$.

Since the second-order condition is satisfied, the optimal degree of privatization α^{l*} is obtained as follows:

$$\frac{dW^l}{d\alpha} = \frac{dq_0^l}{d\alpha} + \frac{dq_1^l}{d\alpha} - 2q_0^l\frac{dq_0^l}{d\alpha} - 2q_1^l\frac{dq_1^l}{d\alpha} - b\Big(q_0^l\frac{dq_1^l}{d\alpha} + \frac{dq_0^l}{d\alpha}q_1^l\Big)$$

$$= -\frac{(27 - 18b - 33b^2 + 22b^3 + 11b^4 - 6b^5 - b^6)^2\alpha}{[2(9 - 8b^2 + 2b^4) + (9 - 8b^2 + b^4)\alpha]^3} = 0 \Rightarrow \alpha^{l*} = 0. \tag{11}$$

Thus, in the Stackelberg case, similar to the case in the Bertrand equilibrium, the partially privatized firm is fully nationalized. Firm 0's objective completely corresponds with social welfare, that is, $W^{l*} = \Omega^{l*}$. Substituting $\alpha^{l*} = 0$ into (9), (10), and Table 3, we obtain the variables in this Stackelberg equilibrium in Table 4.

Table 4: Stackelberg equilibrium when α is optimal

firm 0's output	q_0^{l*}	$\frac{9-4b-6b^2+2b^3+b^4}{2(9-8b^2+2b^4)}$
firm 1's output	q_1^{l*}	$\frac{(2-b)(3-b^2)}{2(9-8b^2+2b^4)}$
firm 0's price	p_0^{l*}	$\frac{9-2b-7b^2+2b^4}{2(9-8b^2+2b^4)}$
firm 1's price	p_1^{l*}	$\frac{(2-b)(2-b^2)(3-b^2)}{2(9-8b^2+2b^4)}$
firm 0's profit	π_0^{l*}	$\frac{(9-8b^2-2b^3+3b^4)(9-4b-6b^2+2b^3+b^4)}{8(9-8b^2+2b^4)^2}$
firm 1's profit	π_1^{l*}	$\frac{(3-2b^2)(2-b)^2(3-b^2)^2}{8(9-8b^2+2b^4)^2}$
social welfare	W^{l*}	$\frac{17-8b-10b^2+4b^3+b^4}{4(9-8b^2+2b^4)}$
firm 0's objective	Ω^{l*}	$\frac{17-8b-10b^2+4b^3+b^4}{4(9-8b^2+2b^4)}$

3.3 Stackelberg equilibrium when the private firm is the leader

Considering that the partially privatized firm reacts following (6), the private firm sets the price to maximize its profit. From the first-order condition, we obtain the private firm's equilibrium price:

$$\frac{\partial \pi_1}{\partial p_1} = d - (2f - gr_0')p_1 + gp_0 + (d - fp_1 + gp_0)(f - gr_0') = 0$$

$$\Rightarrow \ p_1(\alpha) = \frac{[2 - b + (1 - b^2)\alpha][4 - b^2 + 2(1 - b^2)\alpha]}{(2 - b^2)(6 - b^2) + 4(3 - 3b^2 + b^4)\alpha + (3 - 4b^2)\alpha^2}. \tag{12}$$

Substituting (12) into $p_0 = r_0(p_1)$, that is, (6), we obtain the equilibrium price of the partially privatized firm:

$$p_0(\alpha) = \frac{6 - 4b^2 + b^3 + (1 - b)(9 + 5b - 4b^2 - 3b^3 + b^4)\alpha + (1 - b)(3 + b - 3b^2)\alpha^2}{(2 - b^2)(6 - b^2) + 4(3 - 3b^2 + b^4)\alpha + (3 - 4b^2)\alpha^2}. \tag{13}$$

Table 5 summarizes the equilibrium variables other than price.[5]

Now, we derive the optimal degree of privatization α^{f*}. The second-order condition of $W^f(\alpha)$ is satisfied because it is strictly concave with respect to α. However, $W^f(\alpha)$ can be a strictly monotonically increasing or decreasing function, with respect to α, depending on the parameters. Thus, in some cases, the optimal degree of privatization is the corner solution, $\alpha^{f*} = 0$ or 1. Unfortunately, since the first-order condition of $W^f(\alpha)$ is a fourth- or higher-degree polynomial equation with respect to α, we cannot derive α^{f*} explicitly. Instead, the numerical calculation derives α^{f*} such that when $b \leq 0.45$, $\alpha^{f*} = 0$; when $b \geq 0.58$, $\alpha^{f*} = 1$. Otherwise, α^{f*} is the interior solution. In all these cases, $\alpha^{f*} \geq \alpha^{B*} = \alpha^{l*} = 0$.[6]

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[5] $E_0 \equiv 68 - 32b - 80b^2 + 40b^3 + 24b^4 - 12b^5 - 2b^6 + b^7 > 0$, $E_1 \equiv 136 - 72b - 184b^2 + 94b^3 + 104b^4 - 54b^5 - 16b^6 + 8b^7 > 0$, $E_2 \equiv 93 - 36b - 158b^2 + 65b^3 + 77b^4 - 29b^5 - 24b^6 + 10b^7 + b^8$, $E_3 \equiv 25 + 2b - 57b^2 - 6b^3 + 36b^4 + 8b^5 - 3b^6 - 8b^7 + 2b^8$, $E_4 \equiv (1 - b)(2 + 5b - 2b^2 - 12b^3 - 2b^4 + 6b^5)$. $F_0 \equiv 136 - 64b - 160b^2 + 80b^3 + 48b^4 - 24b^5 - 4b^6 + 2b^7 > 0$, $F_1 \equiv 172 - 80b - 272b^2 + 136b^3 + 172b^4 - 92b^5 - 27b^6 + 14b^7 > 0$, $F_2 \equiv 22 - 24b - 72b^2 + 44b^3 + 38b^4 - 24b^5 - 22b^6 + 16b^7$, $F_3 \equiv 55 - 4b - 88b^2 + 18b^3 + 70b^4 - 24b^5 - 32b^6 + 16b^7 + 2b^8 > 0$, $F_4 \equiv 28 - 2b - 56b^2 + 8b^3 + 30b^4 - 8b^5 - 8b^6$, $F_5 \equiv 4 - 9b^2 + 2b^4 + 4b^6 > 0$.

[6] In the Appendix, Table A.1 presents the numerical calculation of the optimal privatization rates and social welfare. Figure A.1 presents the relationship between b and α^{f*}.

Table 5: Stackelberg equilibrium when the private firm is the leader

firm 0's output	$q_0^f(\alpha)$	$\dfrac{6-4b-2b^2+b^3+(3-2b^2+2b^3)\alpha+b(1-2b^2)\alpha^2}{(2-b^2+\alpha)[6-b^2+(3-4b^2)\alpha]}$
firm 1's output	$q_1^f(\alpha)$	$\dfrac{2-b+(1-b^2)\alpha}{6-b^2+(3-4b^2)\alpha}$
firm 0's profit	$\pi_0^f(\alpha)$	$\dfrac{6+4b-6b^2+b^3+(15-8b-16b^2+8b^4-2b^5)\alpha+(6-5b-8b^2+8b^3)\alpha^2}{2(2-b^2+\alpha)[6-b^2+(3-4b^2)\alpha]}q_0^f$
firm 1's profit	$\pi_1^f(\alpha)$	$\dfrac{[2-b+(1-b^2)\alpha]^2}{2(2-b^2+\alpha)[6-b^2+(3-4b^2)\alpha]}$
social welfare	$W^f(\alpha)$	$\dfrac{E_0+E_1\alpha+E_2\alpha^2+E_3\alpha^3+E_4\alpha^4}{(2-b^2+\alpha)^2[6-b^2+(3-4b^2)\alpha]^2}$
firm 0's objective	$\Omega^f(\alpha)$	$\dfrac{F_0+F_1\alpha+F_2\alpha^2-F_3\alpha^3-F_4\alpha^4-F_5\alpha^5}{2(2-b^2+\alpha)^2[6-b^2+(3-4b^2)\alpha]^2}$

3.4 Welfare comparison

We compare social welfare in the above-mentioned three cases. By the numerical simulation, $W^{B*} > W^{f*}$ is necessarily satisfied. In the Appendix, Figure A.2 also indicates that if b is small, social welfare when the partially privatized firm is the leader is the highest, and if b is large, social welfare in the simultaneous move is the highest. We summarize the following proposition:

Proposition 1. *Consider that the government chooses the optimal degree of privatization. If the degree of substitutability is less than a certain threshold, the Stackelberg equilibrium when the partially privatized firm is the leader achieves the largest social welfare. If it exceeds the threshold, the Bertrand equilibrium achieves the largest social welfare. More specifically, if $b < \underline{b}$, $W^{l*} > W^{B*} > W^{f*}$; if $b \in (\underline{b}, \overline{b})$, $W^{B*} > W^{l*} > W^{f*}$; if $b > \overline{b}$, $W^{B*} > W^{f*} > W^{l*}$.*

Proposition 1 implies that which equilibrium the government prefers depends on the degree of substitutability. If the degree is low, the government prefers the Stackelberg equilibrium when a partially privatized firm is the leader. Otherwise, the government prefers the simultaneous-move Bertrand equilibrium. The result of social welfare comparison in price competition is in stark contrast with that in quantity competition. When firms compete in quantity, the Stackelberg equilibrium when a partially privatized firm is the follower achieves the largest social welfare because the firm has the second-mover advantage. By contrast, in price competition, this type of Stackelberg equilibrium never achieves the largest social welfare.

The difference exists because in price competition, the optimal degree of privatization is the corner solution in many cases. When substitutability is relatively low, the government sets $\alpha^* = 0$ in all equilibria. It causes the price competition between a fully nationalized firm and a private firm. As Bárcena-Ruiz (2007) showed, when the degree of privatization is zero, the Stackelberg equilibrium when a public firm is the leader achieves the largest social welfare. By contrast, when substitutability is sufficiently close to unity, the government chooses full nationalization in the Bertrand equilibrium and the Stackelberg equilibrium when the private firm is the follower. However, it chooses full privatization in the Stackelberg equilibrium when the private firm is the leader. Depending on the succeeding equilibrium, the privatized firm's purpose that the government sets changes drastically. Before privatization, the government prefers the fully nationalized public firm to have the first-mover advantage.

After full privatization, the government prefers the simultaneous-move Bertrand competition to obtain high social welfare through harsh competition.

4. Concluding remarks

This study examines a mixed duopoly in differentiated products in which a partially privatized firm and a private firm simultaneously or sequentially compete in price after the government sets the optimal degree of the privatization for the partially privatized firm. Comparing the extent of social welfare when the timing of decision making is different, we present the following results. When the degree of substitutability of goods is low, the social welfare in the Stackelberg equilibrium when a partially privatized firm is the leader is the largest. By contrast, when the degree is high, the social welfare in the Bertrand equilibrium is the largest. Unlike the results presented in quantity competition, the Stackelberg equilibrium when a partially privatized firm is the follower never achieves the largest social welfare.

Finally, we discuss the future perspectives of our study. First, we do not compare social welfare when firms compete in quantity and price. The immediate challenge is to consider the welfare comparison between quantity and price competition. Second, we do not endogenize the timing of decision making. If we consider the observable delay game developed by Hamilton and Slutsky (1990), we can revisit the endogenous timing by firms in a mixed oligopoly model when the government sets the optimal degree of privatization. As the endogenization of the timing is one of the recent interesting topics on mixed oligopoly, extending our analysis to endogenize the firms' timing of decision making would be discussed in future research.

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Appendix

Table A.1 presents the numerical calculation of the optimal privatization rates and social welfare. Figure A.1 presents the relationship between b and α^{f*}. Figure A.2 presents social welfare, W^{B*}, W^{l*}, and W^{f*}.

Table A.1: The optimal privatization rates and social welfare

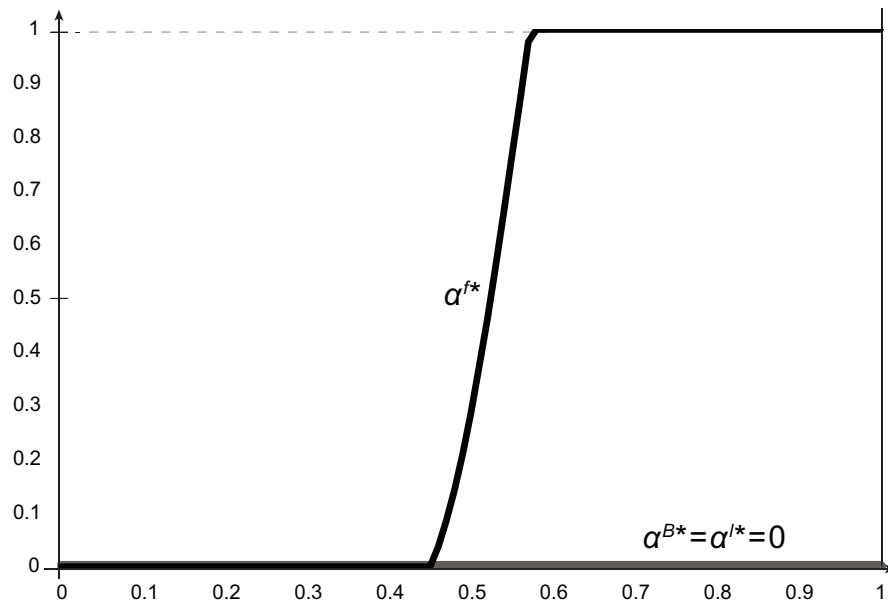| b	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
α^{B*}	0	0	0	0	0	0	0	0	0	0	0
α^{l*}	0	0	0	0	0	0	0	0	0	0	0
α^{f*}	0	0	0	0	0	0.2867	1	1	1	1	1
W^{B*}	0.4722	0.4512	0.4325	0.4156	0.4005	0.3868	0.3746	0.3635	0.3534	0.3437	0.3333
W^{l*}	0.4722	0.4513	0.4326	0.4155	0.3995	0.3838	0.3675	0.3493	0.3270	0.2964	0.25
W^{f*}	0.4722	0.4507	0.4305	0.4114	0.3932	0.3767	0.3652	0.3559	0.3470	0.3381	0.3281

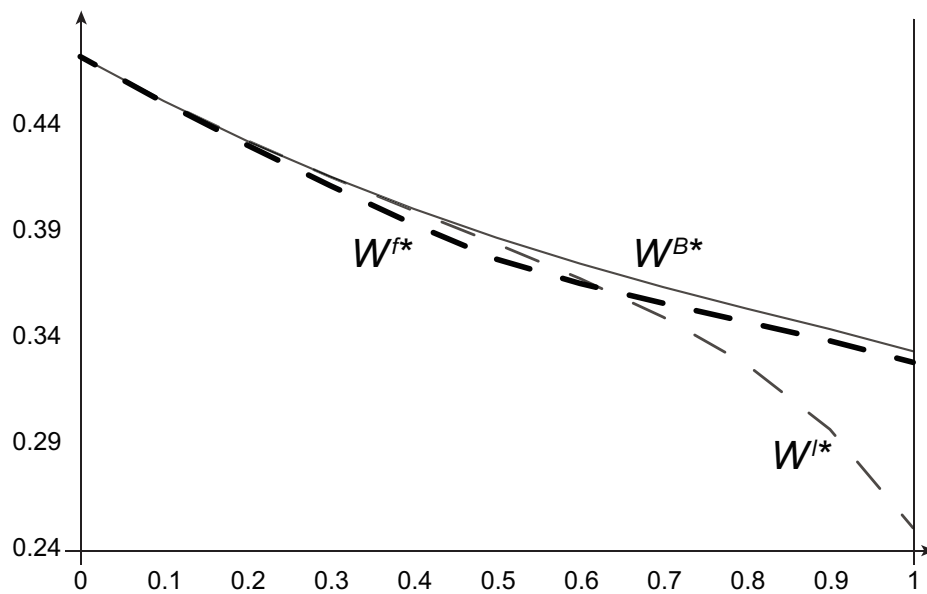Figure A.1: The optimal degree of privatization ($\alpha^{f*} \geq \alpha^{B*} = \alpha^{l*} = 0$)

Figure A.2: Social welfare (W^{B*}, W^{l*}, and W^{f*})