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Stochastic Expected Inequity-Averse Choice

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Abstract

This note studies stochastic inequity-averse choice behavior, capturing ex-post fairness and ex-ante fairness. By studying deliberate randomization, we characterize the stochastic choice behavior stemming from Saito [Social Preferences under Risk: Equality of Opportunity versus Equality of Outcome, American Economic Review, 103 (7): 3084-3101]. We find that the violations of first order stochastic dominance (FOSD) and Regularity occur, and that ex-ante fairness can lead to preference reversal phenomena.

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1. Introduction

This note contributes to the study of stochastic choice in social contexts. It is challenging to identify the motivation behind altruistic or prosocial behavior. Our related work by Hashidate and Yoshihara (2021) develops a stochastic choice model of *additive perturbed utility* (Fudenberg et al., 2015), and compares stochastic inequity-averse behavior with stochastic image-conscious behavior.[1] This note complements Hashidate and Yoshihara (2021) by studying inequity-averse preferences beyond their model, *additive perturbed inequity-averse utility* (APU(IA)).

In this study, we examine inequity aversion in stochastic choice. In particular, we focus on Saito (2013), a seminal axiomatic model in inequity aversion, including *ex-post fairness* and *ex-ante fairness*.[2] We find that stochastic behavior stemming from *inequity-averse preferences* is not consistent with *first-order stochastic dominance* (FOSD) and *Regularity*, a well-known property in stochastic choice.[3] Moreover, stochastic expected inequity-averse choice can lead to preference reversal phenomena in stochastic choice, which is different from APU(IA).

We characterize Saito (2013)'s stochastic choice behavior (Corollary 1), called stochastic expected inequity-averse choice (henceforth, stochastic EIA) by using the framework of *deliberate randomization*, which is introduced by Machina (1985), and generalized by Cerreia-Vioglio et al. (2019). The notion of *ex-ante fairness* can be regarded as an example of *deliberate randomization*. Cerreia-Vioglio et al. (2019) restrict deliberate randomization in terms of FOSD, but inequity-averse preferences can deviate from FOSD (Observation 1). We provide a weaker version of stochastic dominance relation (Observation 2). We also show that stochastic EIA choice behavior deviates from *Regularity* (Observation 3). Moreover, we provide an example of preference reversal phenomena stemming from *ex-ante fairness* (Observation 4).

2. The Model

Let $I = \{1, 2\}$ be the set of individuals, where 1 is the decision maker, and 2 is the other (passive) agent.[4] We assume that the set of payoffs is $\mathbb{R}_+$. A vector $\boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2_+$ is called an *allocation* of payoffs among individuals, yielding payoff $x_i \in \mathbb{R}$ for each $i \in I$. Let $X \subseteq \mathbb{R}^2_+$ be the *compact* set of allocations. Let $X_+ := \{\boldsymbol{x} \in X | x_1 \geq x_2\}$, and $X_- := \{\boldsymbol{x} \in X | x_1 \leq x_2\}$, respectively.

Let Δ be the set of *lotteries* over X, where we call *lotteries* probability distributions over X with finite support. The elements in Δ are denoted by $p, q \in \Delta$.

[1] Image-conscious behavior stems from social pressures; that is, decision makers care about how their behavior is perceived by other agents. This motivation is different from outcome-based social preferences such as inequity aversion.

[2] For inequity-averse preferences, see Fehr and Schmidt (1999) and Fudenberg and Levine (2012). The former studies *ex-post fairness*, the equity of ex-post payoffs, while the latter studies *ex-ante fairness*, the equity of ex-ante expected payoffs for self-interested individuals. In experimental economics, it is widely recognized that much attention needs to be paid not only to *ex-post fairness*, but also *ex-ante fairness* (Brock et al. 2013, Sandroni et al. 2013, Miao and Zhong, 2018, and Andreoni et al. 2020).

[3] This property requires that the choice probability of an alternative decreases as the size of choice sets increases in the sense of set inclusions.

[4] We can extend the n-th agents' case.

A choice set, that is, *menu*, is a non-empty subset of X. Let $\mathcal{A}$ be the collection of all non-empty *finite* subsets of X. The elements in $\mathcal{A}$ are denoted by $A, B, C \in \mathcal{A}$.

We study a stochastic choice rule ρ that maps a *menu* A to a probability distribution over the *allocations* in A, denoted by $\rho(A)$. Formally, let us denote a stochastic choice rule by $\rho : \mathcal{A} \to \Delta(X)$, where $\Delta(X)$ is the set of probability distributions over X with finite support. Given a menu $A \in \mathcal{A}$ with $\boldsymbol{x} \in A$, let us denote the probability that an allocation $\boldsymbol{x}$ is chosen from the menu A by $\rho(\boldsymbol{x}, A)$. For example, take a menu $A = \{\boldsymbol{x}, \boldsymbol{y}\}$, and then we have $\rho(A) = (\rho(\boldsymbol{x}, A), \rho(\boldsymbol{y}, A))$.

Define a binary relation $\succsim_\rho$ over Δ induced by ρ, that captures a ranking of *deliberate randomization*. Let $\overline{\rho(A)} = \sum_{\boldsymbol{x}\in A} \rho(\boldsymbol{x}, A)\boldsymbol{x}$ for each $A \in \mathcal{A}$. For each $A \in \mathcal{A}$, let $\mathrm{co}(A)$ be the convex hull of A.

Definition 1. We say p is *stochastically preferred to* q, that is,

$$p \succsim_\rho q \text{ if there exists } A \in \mathcal{A} \text{ such that } p = \overline{\rho(A)} \text{ and } q \in \mathrm{co}(A). \tag{1}$$

We present the key assumption of this study. By considering the axioms of Saito (2013), the properties of the binary relation $\succsim$ over Δ, we describe *deliberate randomization* as a binary relation induced by a stochastic choice function ρ, denoted by $\succsim_\rho$ over Δ. The key axiom connects the two binary relations.

The *deliberate randomization* ranking described by $\succsim_\rho$ is a subset of $\succsim$ satisfying the axioms of Saito (2013).

Assumption 1. $\succsim_\rho \subseteq \succsim$ where $\succsim$ satisfies the axioms of Saito (2013).

This assumption is testable. Consider the pair of preference relations over lotteries and stochastic choice $\langle \succsim, \rho \rangle$. We can test whether decision makers satisfy Saito (2013)'s axioms. Moreover, we can test whether their stochastic choice behavior is rationalizable.

Define the stochastic EIA choice in the following.

Definition 2. A stochastic choice rule ρ is a stochastic EIA choice if there exists a tuple (α, β, γ), where $\alpha \geq 0$, $\beta \geq 0$, and $\gamma \in [0, 1]$ such that $\succsim_\rho$ induced by ρ are represented by the function V on Δ defined by

$$V(\rho) = \gamma u_{IA}\Big(\sum_{\boldsymbol{x}\in A} x\rho(\boldsymbol{x})\Big) + (1-\gamma)\sum_{\boldsymbol{x}\in A} u_{IA}(\boldsymbol{x})\rho(\boldsymbol{x}) \tag{2}$$

where $u_{IA}(\boldsymbol{x}) = x_1 - \alpha \max\{x_2 - x_1, 0\} - \beta \max\{x_1 - x_2, 0\}$, and ρ is represented by

$$\rho_{\text{Saito}}(A) = \arg\max_{\rho\in\Delta(A)} \Big(\gamma u_{IA}\Big(\sum_{\boldsymbol{x}\in A} x\rho(\boldsymbol{x})\Big) + (1-\gamma)\sum_{\boldsymbol{x}\in A} u_{IA}(\boldsymbol{x})\rho(\boldsymbol{x})\Big). \tag{3}$$

We state the result by applying Saito (2013).

Corollary 1. *The following statements are equivalent:*

(a) $\succsim_\rho$ *induced by* ρ *satisfies Assumption 1.*

(b) ρ *is a stochastic EIA choice* (ρ_{Saito}).

Proof. The necessity part is obvious; thus, it has been omitted, and we show the sufficiency part. First, we show that the binary relation induced by ρ and denoted by $\succsim_\rho$ is represented by a continuous function $V : \Delta \to \mathbb{R}$. This statement holds because $\succsim$ is a continuous weak order (expected utility representation theorem without *Independence*), and $\succsim_\rho$ is a subset of $\succsim$. Second, we show that $\succsim$ is represented by

$$V(\rho) = \gamma u_{IA}\Big(\sum_{\boldsymbol{x}\in A} x\rho(\boldsymbol{x})\Big) + (1-\gamma)\sum_{\boldsymbol{x}\in A} u_{IA}(\boldsymbol{x})\rho(\boldsymbol{x}).$$

The proof is in Saito (2013)'s theorem (p. 3089). For each $A \in \mathcal{A}$, by definition, $V(\overline{\rho(A)}) \geq V(q)$ for all $q \in \mathrm{co}(A)$; that is, ρ exhibits $\succsim_\rho$-maximal behavior. Hence, we obtain the desired utility representation:

$$\rho_{\mathrm{Saito}}(A) = \arg\max_{\rho\in\Delta(A)}\Big(\gamma u_{IA}\Big(\sum_{\boldsymbol{x}\in A} x\rho(\boldsymbol{x})\Big) + (1-\gamma)\sum_{\boldsymbol{x}\in A} u_{IA}(\boldsymbol{x})\rho(\boldsymbol{x})\Big),$$

for each menu $A \in \mathcal{A}$. □

3. Discussions

3.1 Violations of FOSD

Cerreia-Vioglio et al. (2019) introduce the acyclic condition on *deliberate randomization* (Machina, 1985) called *Rational Mixing*. This condition is related to non-EU preferences that satisfy FOSD.[5]

In inequity-averse preferences, the decision maker may not satisfy FOSD because their tastes are monotone in *equal* allocations only. Thus, in stochastic choice, stochastic inequity-averse choice behavior may not satisfy *Rational Mixing*; that is, we need a weaker version of acyclic conditions.[6]

Observation 1. *Stochastic EIA Choice (ρ_{Saito}) is not consistent with FOSD.*

We provide an example. Even though $\boldsymbol{x} \leq \boldsymbol{y}$ holds, the stochastic EIA exhibits

$$\rho_{\mathrm{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}\}) \geq \rho_{\mathrm{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}\}).$$

Example 1. *Let $\boldsymbol{x} = (5,4)$ and $\boldsymbol{y} = (6,9)$. For Saito (2013)'s preference, let $\alpha = 1$, $\beta = 0.6$, and $\gamma = 0.5$. Then, we can easily calculate it, and obtain the following.*[7]

$$V(\rho) = \begin{cases} 3 + 2.2\rho & \rho \leq \frac{3}{4} \\ 5.4 - \rho & \rho > \frac{3}{4}. \end{cases} \tag{4}$$

Hence, V is maximized at $\rho = 0.75$. We obtain the following optimal stochastic choice.

$$\rho_{\mathrm{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}\}) = 0.75 \text{ and } \rho_{\mathrm{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}\}) = 0.25.$$

[5]Here, the definition of FOSD follows from Dean and Ortoleva (2017). For any $p, q \in \Delta$, we say that p *first-order stochastically dominates* q if $p(\{x|\delta_{\boldsymbol{x}} \succsim \delta_{\boldsymbol{z}}\}) \geq q(\{x|\delta_{\boldsymbol{x}} \succsim \delta_{\boldsymbol{z}}\})$. Following Mas-Colell et al. (1995), if p *first-order stochastic dominates* q, then for any *non-decreasing* function $u : X \to \mathbb{R}$, $\int u(\boldsymbol{x})dp(\boldsymbol{x}) \geq \int u(\boldsymbol{x})dq(\boldsymbol{x})$.

[6]See Hashidate and Yoshihara (2021).

[7]See the online appendix (A.4) in detail.

We consider a weaker version of FOSD in outcome-based social preferences.

Definition 3. (Fairness-Based FOSD): For any $p, q \in \Delta$, we say that p *fairness-based first-order stochastically dominates* q if $p(\{x|\delta_{\boldsymbol{x}} \succsim \delta_{\boldsymbol{z}}\}) \geq q(\{x|\delta_{\boldsymbol{x}} \succsim \delta_{\boldsymbol{z}}\})$ where $\boldsymbol{x}, \boldsymbol{z} \in X_+$ or $\boldsymbol{x}, \boldsymbol{z} \in X_-$ with $\boldsymbol{x} \geq \boldsymbol{z}$.

Observation 2. *Stochastic EIA Choice (ρ_{Saito}) is consistent with Fairness-Based FOSD.*

$\rho_{\text{Saito}}(\boldsymbol{x}) \geq \rho_{\text{Saito}}(\boldsymbol{y})$ holds whenever $\boldsymbol{x}, \boldsymbol{y} \in X_+$ (or $\boldsymbol{x}, \boldsymbol{y} \in X_-$) with $\boldsymbol{x} \geq \boldsymbol{y}$, because ex-post fairness concerns are captured by a piece-wise linear form with (α, β). Notice that, in the case of $|I| = 2$, the indifference curves of the Fehr and Schmidt (1999)'s model kink at the 45-degree line (*fair allocations*). Then, in the areas of X_+ or X_-, the indifference curves are linear. Hence, there is no incentive for randomization.

Ex-ante fairness concerns do not matter in Observation 2 because there is no incentive for randomization. The preference for randomization stemming from ex-ante fairness can occur when the allocations of $\boldsymbol{x}$ and $\boldsymbol{y}$ are opposite ($\boldsymbol{x} \in X_+$ and $\boldsymbol{y} \in X_-$, etc.).

3.2 Violations of Regularity

Stochastic EIA choices (ρ_{Saito}) are not consistent with *Regularity*. The axiom requires that for any $A, B \in \mathcal{A}$ with $A \subseteq B$, if $\boldsymbol{x} \in A$, then $\rho(\boldsymbol{x}, A) \geq \rho(\boldsymbol{x}, B)$.

Observation 3. *Stochastic EIA Choice (ρ_{Saito}) is not consistent with Regularity.*

Consider the following counter example:

Example 2. *Let $\boldsymbol{x} = (3,4), \boldsymbol{y} = (4,1)$, and $\boldsymbol{z} = (5,0)$. For Saito (2013)'s preference, let $\alpha = 1$, $\beta = 0.6$, and $\gamma = 0.5$. Then, we obtain the following.*[8]

$$V(\rho) = \begin{cases} 2.2 + 0.6\rho & \rho \leq \frac{3}{4} \\ 4.6 - 2.6\rho & \rho > \frac{3}{4}. \end{cases} \tag{5}$$

Hence, V is maximized at $\rho = \frac{3}{4}$. Thus, we obtain the optimal stochastic choice in the following way.

$$\rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}\}) = 0.75 \text{ and } \rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}\}) = 0.25.$$

Moreover, by adding $\boldsymbol{z} \in X$ to the menu $\{\boldsymbol{x}, \boldsymbol{y}\}$, we can verify that V is maximized at $\rho = (\rho(\boldsymbol{x}), \rho(\boldsymbol{y}), \rho(\boldsymbol{z})) = (\frac{5}{6}, 0, \frac{1}{6})$. Thus, we have

$$\rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}) \approx 0.83 \text{ and } \rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}) = 0,$$

and $\rho_{\text{Saito}}(\boldsymbol{z}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}) \approx 0.17$.

We have $\rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}\}) = 0.75 < 0.83 \approx \rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\})$, which is not consistent with *Regularity*. This behavioral pattern can occur due to *equity of opportunity*, that is, the *ex-ante fairness* concern. When the third allocation $\boldsymbol{z}$ is added to the menu $\{\boldsymbol{x}, \boldsymbol{y}\}$, the randomization between $\boldsymbol{x}$ and $\boldsymbol{z}$ is beneficial in terms of *ex-ante fairness* (see Figure 2 in the online appendix (A.4)).

[8] See the online appendix (A.4) in detail.

3.3 Preference Reversals in Stochastic Choice

Stochastic EIA choices (ρ_{Saito}) can lead to preference reversals. We say that a stochastic choice rule ρ exhibits a preference reversal in stochastic choice if there exist $A, B \in \mathcal{A}$ with $\boldsymbol{x}, \boldsymbol{y} \in A \cap B$ such that $\rho(\boldsymbol{x}, A) \leq \rho(\boldsymbol{y}, A)$ and $\rho(\boldsymbol{x}, B) > \rho(\boldsymbol{y}, B)$.[9]

Observation 4. *Stochastic EIA Choice (ρ_{Saito}) can lead to preference reversals in stochastic choice.*

Consider the following example:

Example 3. *Let $\boldsymbol{x} = (3,4), \boldsymbol{y} = (4,2)$, and $\boldsymbol{z} = (5,8)$. For Saito (2013)'s preference, let $\alpha = 1$, $\beta = 0.6$, and $\gamma = 0.6$. Then, we obtain the following.*[10]

$$V(\rho) = \begin{cases} 2.8 + 0.16\rho & \rho \leq \frac{2}{3} \\ 4.72 - 2.72\rho & \rho > \frac{2}{3}. \end{cases} \tag{6}$$

Thus, we obtain the optimal stochastic choice:

$$\rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}\}) \approx 0.67 > 0.33 \approx \rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}\}).$$

Moreover, by adding $\boldsymbol{z} \in X$ to the menu $\{\boldsymbol{x}, \boldsymbol{y}\}$, we can verify that V is maximized at $\rho = (\rho(\boldsymbol{x}), \rho(\boldsymbol{y}, \rho(\boldsymbol{z})) = (0, \frac{3}{5}, \frac{2}{5})$. Thus, we have

$$\rho_{\text{Saito}}(\boldsymbol{x}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}) = 0 < 0.6 = \rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}),$$

and $\rho_{\text{Saito}}(\boldsymbol{z}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}) = 0.4$.

We have $\rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}\}) \approx 0.33 < 0.6 = \rho_{\text{Saito}}(\boldsymbol{y}, \{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\})$, which is not consistent with *Regularity*. This behavioral pattern can lead to not only the violation of the *Regularity*, but also a *preference reversal*. This stems from *ex-ante fairness* concerns. When the third allocation $\boldsymbol{z}$ is added to the menu $\{\boldsymbol{x}, \boldsymbol{y}\}$, the randomization between $\boldsymbol{y}$ and $\boldsymbol{z}$ is beneficial in terms of *ex-ante fairness* (see Figure 3 in the online appendix (A.4)).

References

Andreoni, J. D., Aydin, B. Barton, B. D. Bernheim, and J. Naecker (2020) "When Fair isn't Fair: Understanding Choice Reversals involving Social Preferences" *Journal of Political Economy* **128**(5), 1673-1711.

Brock, J. M., A. Lange, and E. Y. Ozbay (2013) "Dictating the Risk: Experimental Evidence on Giving in Risky Environments," *American Economic Review* **103**(1), 415-37.

Cerreia-Vioglio, S., D. Dillenberger, P. Ortoleva, and G. Riella, G. (2019) "Deliberately Stochastic" *American Economic Review* **109**(7), 2425-45.

[9] We say that $\boldsymbol{x}$ is the stochastically best in A if $\rho(\boldsymbol{x}, A) > \rho(\boldsymbol{x}', A)$ for all $\boldsymbol{x}' \in A \setminus \{\boldsymbol{x}\}$. We focus on the case that $\boldsymbol{x}$ is the stochastically best in A, and $\boldsymbol{y}$ is the stochastically best in B.

[10] See the online appendix (A.4) in detail.

Dean, M., and P. Ortoleva (2017) “Allais, Ellsberg, and Preferences for Hedging” *Theoretical Economics* **12**(1), 377-424.

Fehr, E., and K. M. Schmidt, K. M. (1999) “A Theory of Fairness, Competition, and Cooperation” *Quarterly Journal of Economics* **114**(3), 817-868.

Fudenberg, D., R. Iijima, and T. Strzalecki, T. (2015) “Stochastic Choice and Revealed Perturbed Utility” *Econometrica* **83**(6), 2371-2409.

Fudenberg, D., and D. K. Levine (2012) “Fairness, Risk Preferences and Independence: Impossibility Theorems” *Journal of Economic Behavior and Organization* **81**(2), 606-612.

Hashidate, Y., and K. Yoshihara (2021) “Stochastic Choice and Social Preferences: Inequity Aversion versus Shame Aversion” TCER Working Paper E-155.

Machina, M. J. (1985) “Stochastic Choice Functions generated from Deterministic Preferences over Lotteries” *Economic Journal* **95**(379), 575-594.

Miao, B., and S. Zhong (2018) “Probabilistic Social Preference: How Machina’s Mom randomizes Her Choice” *Economic Theory* **65**(1), 1-24.

Saito, K. (2013) “Social Preferences under Risk: Equality of Opportunity versus Equality of Outcome” *American Economic Review* **103**(7), 3084-3101.

Sandroni, A., S. Ludwig, and P. Kircher (2013) “On the Difference between Social and Private Goods” *The B.E. Journal of Theoretical Economics* **13**(1), 151-177.