Economics Bulletin

Volume 41, Issue 4

A single-parameter generalization of Gini based on the 'metallic' sequences of number theory

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Abstract

The best-known and most-widely studied generalization of the Gini coefficient of inequality is the single-parameter extension due to authors such as David Donaldson, John Weymark, Nanak Kakwani, Shlomo Yitzhaki, and Satya Chakravarti. The 'S-Gini' parametrization is essentially in the form of a scalar employed as an exponent on Gini's income-weight, which is the Borda rank-order. The present note considers an alternative single-parameter generalization in which income-weights are derived from Fibonacci-like sequences of numbers, each sequence being indexed by a non-negative integer. The Gini coefficient is a special case of the resulting series of indices, another of which—the 'Fibonacci' index—is introduced, and shown to be a transfer-sensitive extension of Gini.

I am grateful to two anonymous referees for their very helpful comments on an earlier version of the note. Surviving errors and shortcomings are of course solely my own.
Citation: S Subramanian, (2021) "A single-parameter generalization of Gini based on the 'metallic' sequences of number theory", *Economics Bulletin*, Vol. 41 No. 4 pp. 2309-2319.
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Submitted: September 03, 2021. **Published:** December 29, 2021.

1. Introduction

The best-known and most widely investigated generalization of the Gini coefficient of income inequality, involving a single-parameter extension of the measure, is due to Donaldson and Weymark (1980), and related studies by—among others—Kakwani (1980), Yitzhaki (1983) and Chakravarty (1988). As is well-known, the Gini coefficient is derived from a weighting of incomes based on the Borda rank-order system, and the generalization of Gini just referred to essentially relies on raising the income-weights to higher powers: the power index is the single parameter by means of which the Gini coefficient is generalized to a class of measures whose 'distribution-sensitivity' is an increasing function of the chosen value for the parameter.

In the present note, we shall consider an alternative generalization of the Gini coefficient which is based on a parametric variation of the Borda weighting system that relies on the transformation of rank-orders into corresponding Fibonacci-like sequences of different orders. The resulting class of extended Ginis is found to consist of inequality measures which are functions of the various 'metallic ratios' of number theory—the universal constants known as the 'golden ratio' φ, the 'silver ratio' δ, and a succession of other 'metallic ratios'. These statements are admittedly cryptic, but should become clearer as we proceed.

2. The Gini Coefficient

In everything that follows, we shall derive inequality indices in terms of the 'Atkinson-Kolm-Sen' (Atkinson, 1970; Kolm, 1969; Sen, 1973) approach involving the use of an 'equally distributed equivalent (ede)' income. But first, some preliminary formalities. The basic unit of consideration is an *income distribution*, which is a non-decreasingly ordered *n*-vector of individual incomes $\mathbf{x} = (x_1,..x_i,...,x_n)$ in which x_i is the income of the ith poorest individual in a community of *n* individuals, and $\mu(\mathbf{x}) \equiv (1/n)\sum_{i=1}^{n} x_i$ is the mean income. A *social welfare function* $W(\mathbf{x})$ is taken to be a weighted sum of individual incomes in the vector **x**, with the weights supposed to reflect the evaluator's assessment of the social worth of the incomes in the given distribution. We shall confine attention to *weakly egalitarian* social welfare functions, namely those for which the income-weights employed are non-increasing:

$$W(\mathbf{x}) = \sum_{i=1}^{n} w_i x_i \, , \tag{1}$$

where w_i is the weight placed on the *i*th poorest person's income, and, for all $j,k,\ w_j \geq w_k$ whenever $x_j \leq x_k$. A system of weights which corresponds to this requirement is the Borda rank-order system, in which $w_i = (n+1-i) \forall i = 1,...,n.$ The social welfare function with this weighting system will be called W^G:

$$W^G(\mathbf{x}) = \sum_{i=1}^{n} (n+1-i) x_i . \tag{2}$$

Let $\hat{\mu}^G$ be the equally distributed equivalent income (ede), namely, that level of income which, when equally distributed, yields the same welfare level as the actual distribution under review. In the Atkinson-Kolm-Sen welfare-based approach to inequality measurement, an inequality measure I can be written as the proportionate deviation of the ede income from the mean income: $I = 1 - \hat{\mu}/\mu$. Given (2), it can be easily verified that for the welfare function $W^G, \hat{\mu}^G = \sum_{i=1}^{n}(n+1-i)x_i / \sum_{i=1}^{n}(n+1-i) = \left[\frac{2}{n(n+1)}\right]\sum_{i=1}^{n}(n+1-i)x_i$. The inequality index corresponding to the welfare function W^G can then be written as

$$I^G = 1 - \left[\frac{2}{n(n+1)\mu}\right]\sum_{i=1}^{n}(n+1-i)x_i. \qquad (3)$$

But then I^G in Equation (3) is, precisely, the familiar Gini inequality coefficient G.

Notice that, in deriving the Gini index, we have employed a particular transformation of the elements of the set $M_n = \{1,...,i,...,n\}$: the transformation employed is the rank-order transformation r, where, for all $i \in M_n : r(i) = n+1-i$. The question immediately arises: are there alternative transformations one can consider, which lead to sequences different from the rank-order sequence? Can such sequences be parametrically varied in some straightforward way, thus facilitating the possibility of a parametric extension of the Gini coefficient of inequality? This question is addressed in the subsequent sections of the paper.

3. Metallic Sequences

Given the set of the first n natural numbers $M_n = \{1,...,i,...,n\}$, consider, for all $k = 0,1,2,...$, the mapping f^k on M_n such that $f^k(1) = 1, f^k(2) = \max[f^k(1), kf^k(1)]$, and for all $i \in M_n - \{1,2\} : f^k(i) = kf^k(i-1) + f^k(i-2)$. For illustrative purposes, pegging n at 5, Table 1 details the values of the function $f^k(i)$ for $i = 1,2,3,4,5$, and for four specified values of $k : k = 0,1,2,9$.

Table 1: $f^k(i)$ for $i = 1,2,3,4,5$, for four specified values of $k : k = 0,1,2,9$.

i	1	2	3	4	5
$f^0(i)$	1	1	1	1	1
$f^1(i)$	1	1	2	3	5
$f^2(i)$	1	2	5	12	29
$f^9(i)$	1	9	82	747	6805

Notice from Table 1 that the set of numbers $\{f^1(i)\}$ is just the well-known *Fibonacci* sequence of numbers, the first two of which are 1 and 1 respectively, and any subsequent number is the sum of the preceding two Fibonacci numbers. The set of numbers $\{f^2(i)\}$ is just the well-known *Pell* sequence of numbers, the first two of which are 1 and 2 respectively, and any subsequent number is the sum of twice the preceding number and the one preceding that;…, and so on, down the line. As n goes to infinity, the nth Fibonacci number converges on a number which is proportional to the nth power of the so-called *golden ratio* $\varphi = [1+\sqrt{5}]/2 \approx 1.618$; the nth Pell number converges on a number which is proportional to the nth power of the so-called *silver ratio* $\delta = 1+\sqrt{2} \approx 2.4142$; and for other sequences we have corresponding asymptotic convergences on other ‘metallic’ ratios, which are distinguished irrational mathematical constants. Hence the reference to the f^k functions as generating a class of ‘metallic’ sequences, in each sequence of which numbers are derived from their immediately preceding numbers by the sort of recursive relations earlier defined.

To proceed further, it helps to *reverse* the order of the sequences portrayed in Table 1, as is done in Table 2.

Table 2: $f^k(n+1-i)$ **for** $i = 1,2,3,4,5$**, for four specified values of** $k : k = 0,1,2,9.$

i	1	2	3	4	5
$f^0(n+1-i)$	1	1	1	1	1
$f^1(n+1-i)$	5	3	2	1	1
$f^2(n+1-i)$	29	12	5	2	1
$f^9(n+1-i)$	6805	747	82	9	1

Next, for all k and i, let us define the quantities $S^k(i) \equiv \sum_{j=1}^{n+1-i} f^k(n+1-j)$, and $S^k \equiv \sum_{i=1}^{n} S^k(i)$. For the moment, let us just concentrate on $k = 0$. Then it is easy to see, using Table 2 as a spot reference, that $S^0(1) = 5, S^0(2) = 4, S^0(3) = 3, S^0(4) = 2, S^0(5) = 1,$ and $S^0 = 1+2+3+4+5$; or, in general, $S^0(i) = n+1-i, \forall i = 1,...,n,$ and $S^0 = \sum_{i=1}^{n}(n+1-i) = n(n+1)/2$. Now, given an ordered n-vector of incomes $\mathbf{x} = (x_1,...x_i,...,x_n)$, let us define a social welfare function W^k of order k $(k = 0,1,2,...)$ such that it is a weighted sum of income levels, with the weight w_i^k on the ith poorest person’s income being $S^k(i) \equiv \sum_{j=1}^{n+1-i} f^k(n+1-j)\ \forall i = 1,...,n.$ Returning to $k = 0$, and noting that $S^0(i) = n+1-i, \forall i = 1,...,n,$ it is immediate that the social welfare function W^0 is identical to the social welfare function W^G defined in Equation (2). Consequently, the Atkinson-Kolm-Sen inequality measure associated with the welfare function W^0, call it I^0, must be identical to the the inequality measure—which is just the Gini coefficient—associated with the welfare function W^G. That is to say,

$$I^0 = G. \qquad (4)$$

How does the weighting function change as we move to higher orders of k in the sequence of welfare functions W^k? To see what is involved, consider, for each $k = 0,1,2,...,$ the normalized set of weights $\varpi_i^k \equiv S^k(i)/\sum_{j=1}^{n} S^k(j), \forall i = 1,...,n$. For purposes of illustration, these weights are derived from Table 2 in Table 3.

In Figures 1(a)-1(d), we plot the normalized weights ϖ_i^k for $k = 0,1,2,9$. Figure 1(a) suggests that the normalized weighting function is linear for $k = 0$: the welfare function W^0, as we have seen, leads to the Gini coefficient of inequality. Figures 1(b)-1(d) indicate that for $k \geq 1$, the normalized weighting function is strictly convex; and as *k* increases, the weighting curve becomes more and more convex. As can be seen from Figure 1(d), drawn for *k* = 9, the curve already converges on the classical Rawlsian L-shaped curve, that is, on a weighting structure of (1,0,0,…,0), where all the weight is on the lowest income.

Table 3: The normalized sets of weights $\varpi_i^k \equiv S^k(i)/\sum_{j=1}^{n} S^k(j), \forall i = 1,\ldots,n$, for four specified values of $k : k = 0,1,2,9$.

i	1	2	3	4	5	
$S^0(i) \equiv \sum_{j=i}^{n} f^0(n+1-j)$	5	4	3	2	1	$S^0 = \sum_i S^0(i) =$ 15
$\varpi_i^0 = S^0(i)/S^0$	.3333	.2667	.2000	.1333	.0667	$\sum_i \varpi_i^0 = 1$
$S^1(i) \equiv \sum_{j=i}^{n} f^1(n+1-j)$	12	7	4	2	1	$S^1 = \sum_i S^1(i) =$ 26
$\varpi_i^1 = S^1(i)/S^1$	.4615	.2692	.1539	.0769	.0385	$\sum_i \varpi_i^1 = 1$
$S^2(i) \equiv \sum_{j=i}^{n} f^2(n+1-j)$	49	20	8	3	1	$S^2 = \sum_i S^2(i) =$ 81
$\varpi_i^2 = S^2(i)/S^2$	.6049	.2469	.0988	.0370	.0124	$\sum_i \varpi_i^2 = 1$
$S^9(i) \equiv \sum_{j=i}^{n} f^k(n+1-j)$	7644	839	92	10	1	$S^9 = \sum_i S^9(i) =$ 8586
$\varpi_i^9 = S^9(i)/S^9$	.8903	.0977	.0107	.0012	.00012	$\sum_i \varpi_i^9 = 1$

It is clear that as *k* increases, the inequality measure I^k becomes more and more distribution-sensitive. In particular, and as we shall see later, the Gini coefficient—this is a well known feature of the measure—is not transfer-sensitive: it does not distinguish between transfers at the lower and upper ends of a distribution. Such sensitivity—on an increasing scale—is a feature of the class of inequality indices I^k for values of *k* exceeding 0. As in the case of the Donaldson-Weymark (1980) generalization of Gini, so too in the present generalization, it is the distribution-sensitivity of the inequality measure that is being parameterized by the relevant extensions.

Figure 1: The Graphs of the Normalized Weighting functions $\varpi^k(i)$ [$k = 0,1,2,9$] corresponding to the Numbers in Table 3

Figure 1(a): $\varpi^0(i)$

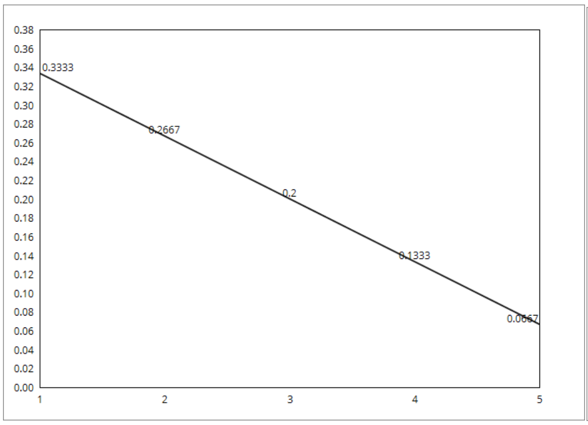


Figure 1(b): $\varpi^1(i)$

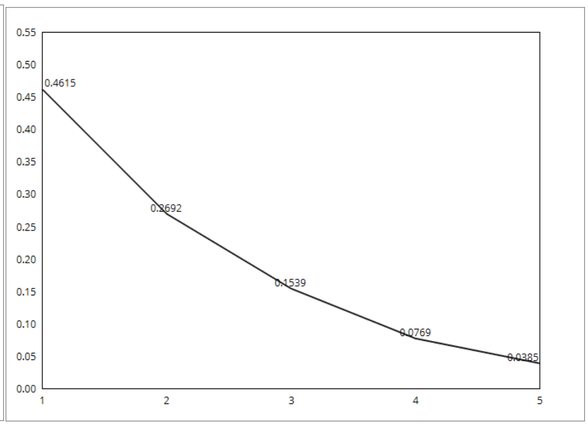


Figure 1(c): $\varpi^2(i)$

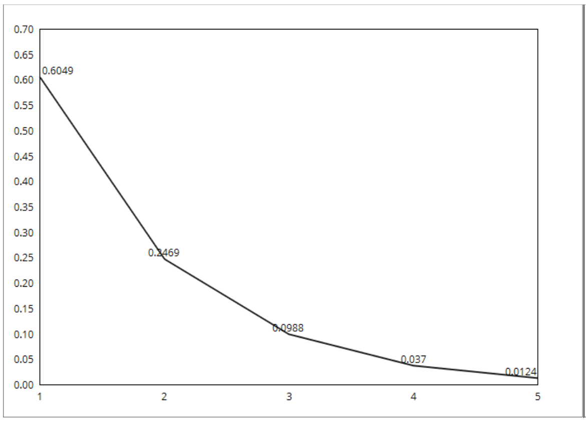
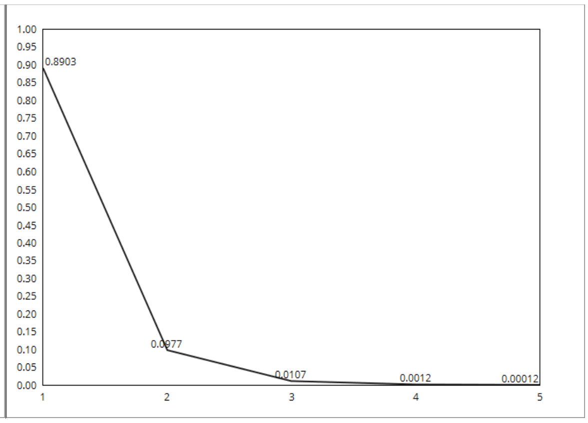


Figure 1(d): $\varpi^9(i)$



Note: The figures have been generated employing the ChartGo software: https://www.chartgo.com/modify.do .

In what follows, we consider in slightly greater detail the inequality measure I^1 which, for obvious reasons, will be called the Fibonacci measure of inequality.

4. The Fibonacci Inequality Coefficient

A preliminary remark: in what follows, we shall be employing certain basic and well-known facts about the Fibonacci and Pell number sequences. The reader unfamiliar with this literature is referred to the very helpful text by Koshy (2001).

Given an ordered n-vector of incomes $\mathbf{x} = (x_1,...x_i,...,x_n)$, the welfare function $W^1(.)$ can be written as:

$$W^1(\mathbf{x}) = \sum_{i=1}^{n} S^1(i)x_i = \sum_{i=1}^{n}\sum_{j=1}^{n+1-i} f^1(n+1-j)x_i\,. \tag{5}$$

Note that $f^1(n+1-j)$ is the $(n+1-j)th$ Fibonacci number $F(n+1-j)$, so $\sum_{j=1}^{n+1-i} f^1(n+1-j)$ is the sum of the first $(n+1-i)$ Fibonacci numbers: $F(1)+F(2)+...+F(n+1-i)$, which is just $F(n+3-i)-1$, a consequence that follows from the well known fact that the sum of the first p Fibonacci numbers $(F(1)+...+F(p))$ is $F(p+2)-1$. Equation (5) can therefore be written as

$$W^1(\mathbf{x}) = \sum_{i=1}^{n}[F(n+3-i)-1]x_i = \sum_{i=1}^{n} F(n+3-i)x_i - n\mu. \tag{6}$$

The equally distributed equivalent income—call it $\hat{\mu}^F$ --is given by:

$$\hat{\mu}^F = [\sum_{i=1}^{n} F(n+3-i)x_i - n\mu]/[\sum_{i=1}^{n} F(n+3-i)-n]. \tag{7}$$

But noting that

$$\sum_{i=1}^{n} F(n+3-i) = F(3)+...+F(n+2) = (F(1)+F(2)+...+F(n+2)) - (F(1)+F(2)) = F(n+4)-3,$$

we can re-write Equation (7) as:

$\hat{\mu}^F = [\sum_{i=1}^{n} F(n+3-i)x_i - n\mu]/[F(n+4)-(n+3)]$. The inequality index corresponding to the welfare function W^1 is now given by

$$I^1(\mathbf{x}) \equiv F(\mathbf{x}) = 1-\hat{\mu}^F/\mu = 1-[\sum_{i=1}^{n} F(n+3-i)x_i - n\mu]/[F(n+4)-(n+3)]\mu. \tag{8}$$

Yet another well-known fact about the Fibonacci sequence is that the ratio of two consecutive Fibonacci numbers $F(i)$ and $F(i-1)$ converges asymptotically on the 'golden ratio' $\varphi = [1+\sqrt{5}]/2$, which is the positive root of the quadratic equation $x^2 - x - 1 = 0$, and can be approximated by 1.618; further, in what is known as Binet's Formula, the *i*th term of the Fibonacci sequence, $F(i)$, can be shown to be equal to the quantity $[\varphi^i - (1-\varphi)^i]/\sqrt{5}$ which, for 'large' values of i can be approximated to $\varphi^i/\sqrt{5}$. Employing this approximation, and making the appropriate substitutions in Equation (8) yields the following asymptotic expression for the Fibonacci inequality measure:

$$F \approx 1 - \frac{\sum_{i=1}^{n} \varphi^{n+3-i}x_i - \sqrt{5}n\mu}{[\varphi^{n+4} - \sqrt{5}(n+3)]\mu}. \tag{9}$$

The expression for F in (9) is yet another example of the ubiquitous presence of the golden ratio in the affairs of the world!

By way of an addendum to this section, I state, without deriving, an expression for the inequality index corresponding to the welfare function W^k for $k = 2$, which I shall call the 'Pell Index', *P*. [A derivation of this result is available on request.]

$$P \approx 1 - \frac{\sqrt{2}(3\delta+1)\sum_i \delta^{n-i} x_i - 4n\mu}{[(3\delta+1)(\delta^n - 1) - 4n]\mu} \tag{10}$$

5. The Lorenz Curve and the Fibonacci Curve

The Lorenz curve is typically defined as the curve obtained by plotting, for each cumulated share *p* of the population arranged from poorest to richest, the corresponding income share $q(p)$ of the poorest *p*th fraction of the population. However, it can also be defined in terms of a simple transformation, as the curve obtained by plotting $p - q(p)$ against *p* for all $p \in [0,1]$. Given an ordered income *n*-vector $\mathbf{x} = (x_1,..x_i,...,x_n)$ with mean μ, the typical ordinate of the Lorenz curve is given by:

$$\mathcal{L}_i(\mathbf{x}; i/n) = (i/n) - (\sum_{j=1}^{i} x_j / n\mu), \forall i = 0,1,...,n, \text{ and } \mathcal{L}_0(\mathbf{x};0) \equiv 0. \tag{11}$$

Letting $F(i)$ stand for the *i*th Fibonacci number, we now define the *Fibonacci curve* as one whose typical ordinate is given by:

$$\mathcal{F}_i(\mathbf{x}; i/n) = F(n+1-i)\mathcal{L}_i(\mathbf{x}; i/n), \forall i = 0,1,...,n. \tag{12}$$

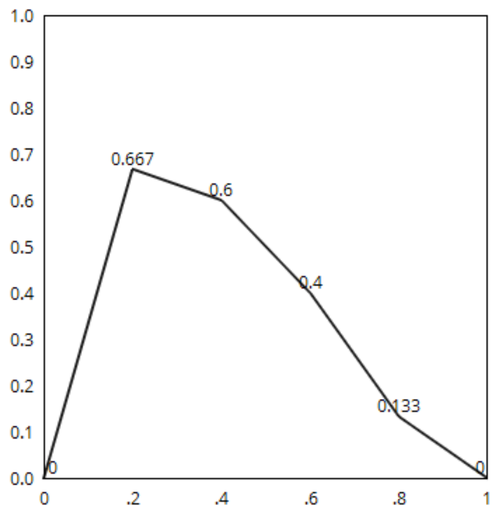
The coordinates of the Fibonacci curve are then the set of points $\{(0,0);(1/n, F(n)\mathcal{L}_1);...;(i/n, F(n+1-i)\mathcal{L}_i;...;(1,0)\}$.

By way of a simple numerical example, consider a situation in which $n = 5$ and $\mathbf{x} = (10,20,30,40,50)$. The Lorenz curve for this distribution is defined by the points (0,0); (.2,.133); (.4,.2); (.6,.4), (.8,.133); (1,0). From (11), (12) and the coordinates of the Lorenz curve for the distribution $\mathbf{x} = (10,20,30,40,50)$, and noting that $F(1) = F(2) = 1, F(3) = 2, F(4) = 3$ and $F(5) = 5$, the Fibonacci curve for the distribution **x** in our example can be seen to be given by the points (0,0); (.2,.6667); (.4,.6); (.6,.4); (.8,.133); (1,0). A typical Fibonacci curve would be an inverted U-curve, commencing at (0,0), initially rising, peaking, then declining, and terminating at (1,0). The Fibonacci curve for the distribution $\mathbf{x} = (10,20,30,40,50)$ can be drawn as a step function or as a piece-wise linear function, and the latter representation is provided in Figure 2. Notice that the area under the Fibonacci curve is just the value of the Fibonacci index of inequality *F*.

Finally, for all distributions **x**,**y**, **x** will be said to *Lorenz-dominate* **y**, written $\mathbf{x} \succ_L \mathbf{y}$, if and only if the Lorenz curve for **x** lies somewhere below the Lorenz curve for **y**, and nowhere above it. Similarly, for all distributions **x**,**y**, **x** will be said to *Fibonacci-dominate* **y**, written $\mathbf{x} \succ_F \mathbf{y}$, if and only if the Fibonacci curve for **x** lies somewhere below the Fibonacci curve for **y**, and nowhere above it. Since any point on a Fibonacci curve is just a positive multiple of the corresponding point on the Lorenz curve, it follows that for all distributions **x**,**y**,

$\mathbf{x} \succ_L \mathbf{y}$ will hold if and only if $\mathbf{x} \succ_F \mathbf{y}$ holds. Whenever these dominance relations hold for any **x** vis-à-vis any **y**, we can assert that there is unambiguously less inequality in **x** than in **y**.

Figure 2: A 'piece-wise' linear Fibonacci curve for the 5-distribution (10,20,30,40,50)



Note: The graph has been generated employing the GoChart software: https://www.chartgo.com/modify.do .

6. Some Properties of the Fibonacci Index

The commonly invoked axioms for inequality measures are well-enough known not to require elaborate treatment. The basic axioms are those of symmetry (the requirement that the measure does not depend on the personal identities of income-recipients); scale-invariance (the requirement that the measure be mean-independent); replication-invariance (the requirement that the measure be invariant with respect to population replications); and, most fundamentally, the Pigou-Dalton transfer axiom (the requirement that, other things equal, a rank-preserving progressive transfer of income should cause inequality to decline). The Gini coefficient of inequality satisfies all four of these axioms as does the Fibonacci index.

To see that F satisfies the symmetry axiom, note that it is constructed from an *ordered* income distribution; since the ordering according to income levels is independent of the personal identities of the income-recipients, the resulting index is also invariant with respect to any permutation of incomes across individuals.

If all incomes in a distribution **x** are uniformly scaled up or down by any positive scalar ρ, then one can see from Equation (9) that $F(\rho\mathbf{x}) \approx 1 - \dfrac{[\sum_{i=1}^{n} \varphi^{n+3-i} x_i - \sqrt{5} n\mu]\rho}{[\varphi^{n+4} - \sqrt{5}(n+3)]\rho\mu} = F(\mathbf{x})$, that is, F satisfies scale-invariance.

Further, as noted in the preceding section, a typical ordinate of the Fibonacci curve is a positive multiple of the corresponding ordinate of the Lorenz curve:

$\mathcal{F}_i(\mathbf{x};i/n) = F(n+1-i)\mathcal{L}_i(\mathbf{x};i/n), \forall i = 0,1,...,n.$ It is well known that the Lorenz curve remains unchanged with any *k*-fold replication of the underlying income distribution, and therefore this must be true for the Fibonacci curve as well; and since the Fibonacci index is just the area under the Fibonacci curve, F is a replication-invariant inequality measure.

That F satisfies the Pigou-Dalton transfer axiom is evident from the fact that the income-weights in the underlying welfare function $W^1(.)$ unlike G, decline with income.

Finally, F, unlike G, is transfer-sensitive, that is, it satisfies the property that, other things equal, the reduction in inequality following on a progressive rank-preserving transfer is greater the lower down the income distribution the transfer occurs. There are two ways of giving expression to this requirement, as discussed by Foster (1985), and captured in the following two axioms.

Tansfer-Sensitivity-1 (TS-1) requires that, other things equal, the reduction in inequality from a progressive transfer of a fixed amount of income between two persons *a fixed number of incomes* apart should be greater the poorer the pair of individuals involved in the transfer.

Tansfer-Sensitivity-2 (TS-2) requires the same outcome for pairs of individuals *a fixed income* apart.

For our purposes, I shall combine these two properties into a single property of Transfer-Sensitivity which is weaker than either of TS-1 or TS-2:

Transfer-Sensitivity (TS) requires that, other things equal, the reduction in inequality from a progressive transfer of a fixed amount of income between two persons who are both *a fixed number of incomes and a fixed income* apart should be greater the poorer the pair of individuals involved in the transfer.

That the Fibonacci index satisfies the TS axiom is evident from the fact that the income-weighting function of the underlying welfare function $W^1(.)$ is not only declining but also strictly convex. [A more elaborate demonstration of the transfer-sensitivity proposition is available from the author on request.]

The class of 'metallic' inequality indices $\{I^k\}$ becomes more and more distributionally sensitive as k increases; and in this regard, the $\{I^k\}$ series mimics the *S*-Gini series of Donaldson and Weymark (1980). The principal point of departure of *F* from *G* is that the former, unlike the latter, satisfies the property of Transfer-Sensitivity. It should be admitted here that the Fibonacci index, and other higher-order 'metallic' indices, are not the only 'rank-order-based' inequality measures that satisfy transfer-sensitivity: this is true also of measures such as the Bonferroni (1930) and De Vergottini (1940) indices. That is to say, a measure such as the Fibonacci index has properties shared with other measures: it is an *addition* to an existing stock, rather than a unique *replacement* of other extant measures.

7. Concluding Observations

This note has been concerned to provide an alternative extension to the by now standard extension of the Gini coefficient in terms of the single-parameter 'S-Ginis' of Donaldson and Weymark (1980), Kakwani (1980), Yitzhaki (1983), Chakravarty (1988) and others. (See also Chameni, 2006, on the class of 'α − Ginis'.) The Gini coefficient is constructed from the Borda rank-order weighting system in which the weight on the income of the *i*th poorest person in an n-person non-decreasingly ordered income vector is given by $r(i) \equiv (n+1-i)$. While the S-Gini generalization relies on transforming Gini's $r(i)$ income-weights via a power function, the route to generalization explored in the present note is via the 'metallic' number sequences of Fibonacci, Pell, and similar sequences generated by a generalized recursive relationship between each number in the sequence and its two immediately preceding numbers. In the specific case of the Fibonacci index, the Borda rank-order weight $r(i)$ is replaced by the Fibonacci number corresponding to $r(i)$. The effect of the generalization, as in the case of the S-Gini generalization, is to produce a family of inequality indices of increasing levels of distribution-sensitivity, ranging from the Gini-coefficient, which is not transfer-sensitive, to a Rawlsian measure which ranks distributions solely according to the income share of the poorest individual. A special, transfer-sensitive member of this family of indices, based on an income-weighting scheme inspired by the Fibonacci sequence, has been derived, and called the Fibonacci index.

Finally, it is as well to clarify that no claim is advanced as to any special virtues that may be possessed by the 'metallic sequence generalization' proposed in this note in relation to other available generalizations. Indeed, it is even possible that the Fibonacci and other 'higher-order' measures reflect degrees of inequality-aversion that may not appeal to all practitioners. What has been proposed, as stated earlier, is in the spirit of introducing a new member to a club, as eligible for entry, but by no means as deserving preferential treatment. In the end, this note is of interest, perhaps, primarily in the way of a curiosum featuring the 'golden ratio' (and other 'metallic' ratios) as playing a part in the measurement of inequality, as in so many other aspects of the worlds of both nature and artefact.

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