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The relationship between mandatory retirement and patterns of human capital accumulation

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Abstract

This study examines the relationship between mandatory retirement, patterns of human capital accumulation, and economic growth. A key feature of the model is that the agents of the working generation not only educate their children, but also have the opportunity to educate themselves for their labor supply in old age. We show that they educate themselves if a mandatory retirement age is sufficiently high. However, the extension of the retirement age in this case is neutral for the growth rate.

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1 Introduction

Almost all developed countries have witnessed rapid population aging, which is triggered by declines in fertility and mortality rates. To address this, in these countries, the extension of the mandatory retirement age is in progress or planned. This study examines the relationship between mandatory retirement, patterns of human capital accumulation, and the long-run rate of economic growth.

Since population aging is a widely observed phenomenon and is believed to be a trigger of serious threats (e.g., slow economic growth, sustainability of social security systems, and so on), many existing studies have paid attention to the effects of increases in the retirement age. Some studies have examined how increasing the mandatory retirement age affects economic growth through changes in savings and human capital accumulation. Zhang and Zhang (2009) show that the extension of mandatory retirement promotes the long-run growth rate. However, Kunze (2014) and Chen and Miyazaki (2020) theoretically find an inverted U-shaped relationship between retirement age and long-run growth rate.[1]

In contrast to the above studies, a feature of this study is that the agents of the working generation not only educate their children, but can also educate themselves in preparation for their labor supply in old age. The above existing studies only assume either one of the two. Zhang and Zhang (2009) assume that the agents' human capital when young is endogenously determined by their parents' decisions, but assume that when they are old, it is exogenous.[2] Thus, they overlook the opportunity of recurrent education for old-age workers. Kunze (2014) and Chen and Miyazaki (2020) introduced this opportunity into the model. However, they omit the agents' educational decisions for their children. Instead, they assume that the young agents in a period are unconditionally endowed with the same amount of human capital as the old generation in that period. Although there is little doubt about such an externality, at the same time, it is taken for granted that people decide the length of time to educate their children.

We introduce both kinds of human capital accumulation into an overlapping generations model with Uzawa–Lucas endogenous growth [Uzawa (1965) and Lucas (1988)]. We obtain the following two main results. First, the retirement age is a key determinant of the patterns of human capital accumulation. In this model, the agents always invest in human capital for their children. In contrast, they invest in human capital for themselves only if the mandatory retirement age is sufficiently high. This is because the marginal benefit of the agents' recurrent education becomes large as its working period of old age becomes long. Second, it crucially depends on this pattern whether an increase in the mandatory retirement age promotes the long-run growth rate. If the agents only educate their children, which occurs when the mandatory

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[1] Chen and Miyazaki (2020) also consider the labor-leisure choice of old workers, whereas Kunze (2014) simply assumes that old workers inelastically supply their labor.

[2] More specifically, they assume that the proportion of human capital when young is carried over into the older period.

retirement age is low, the increase in the retirement age enhances the time of parental teaching. In this case, the retirement age extension raises the long-run growth rate. If the agents also receive recurrent education, which occurs when the mandatory retirement age is high, the increase in the retirement age enhances the time of such recurrent education. Although this increases the relative supply of the old workers to the young, the long-run growth rate is neutral. In sum, once we explicitly consider the two aspects of education, the results become dramatically different from that of existing studies.

The remainder of this paper is organized as follows. Section 2 describes the setup of the model. Section 3 characterizes the equilibrium and investigates the relationship among mandatory retirement, patterns of human capital accumulation, and the long-run growth rate. Section 4 concludes the paper. Proofs and details of the derivations of key equations are provided in the Appendix.

2 Model

Firms and households

A single final good, used for consumption and investment in physical capital, is competitively produced from physical capital and human capital by a Cobb-Douglas technology: $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$ $(A > 0, \alpha \in (0,1))$, where Y_t is output, K_t is the demand for capital, and L_t is the demand for human capital. Profit maximization under perfect competition leads to

$$r_t = \alpha A x_t^{\alpha-1},\ w_t = (1-\alpha)A x_t^{\alpha}, \tag{1}$$

where $x_t \equiv K_t/L_t$, r_t is the rental rate of capital, and w_t is the wage rate. We assume that the depreciation rate of physical capital is one, which means that r_t corresponds to the gross interest rate.

For simplicity, there is no population growth. Each agent lives for three periods, namely, childhood, young, and old periods. In the childhood period, an agent receives education from his/her parents. Let h_{yt} denote his/her human capital in the young period. In the young period, he/she uses this human capital in the following three ways: working, the education of his/her child, and his/her own education in preparation for work at an older age. Let e_t and m_t denote the fractions of human capital used for the education of the child and him/herself, respectively. The budget constraint in the young period is

$$w_t(1-e_t-m_t)h_{yt} = c_{yt} + s_t, \tag{2}$$

where c_{yt} and s_t are consumption when young and savings, respectively. Then, his/her child's human capital (denoted by h_{yt+1}) and his/her human capital when old (h_{ot+1}) are, respectively, given by

$$h_{yt+1} = \gamma_e e_t h_{yt},\ \gamma_e > 0, \tag{3}$$

$$h_{ot+1} = \lambda h_{y_t} + \gamma_m m_t h_{yt} = (\lambda + \gamma_m m_t)h_{yt},\ \gamma_m > 0, \lambda \in [0,1]. \tag{4}$$

In the above equation, $\gamma_j (j = e, m)$ is the efficiency of human capital accumulation and $1 - \lambda$ is the depreciation rate of human capital.

In the old period, each agent potentially has one unit of time, but can only work $\theta \in [0, 1]$ units. θ is the fraction of the period that on old household is required to work. Following Kunze (2014), we interpret θ as the retirement age. The budget constraint when old is

$$r_{t+1} s_t + w_{t+1} \theta h_{ot+1} = c_{ot+1}. \tag{5}$$

The utility function of the agent born in period $t - 1$ is assumed to be

$$U = \ln c_{yt} + \beta \left(\ln c_{ot+1} + \zeta \ln h_{yt+1} \right),$$

where $\beta \in (0, 1)$ is the subjective discount factor. The term $\ln h_{yt+1}$ is the altruistic utility from his/her child's human capital and $\zeta > 0$ is the weight attached to this utility.

The agent chooses s_t, e_t, and m_t to maximize the utility function subject to (2)–(5). The first order conditions are given by

$$\frac{1}{w_t(1 - e_t - m_t)h_{yt} - s_t} = \frac{\beta r_{t+1}}{r_{t+1} s_t + w_{t+1} h_{ot+1} \theta}, \tag{6}$$

$$\frac{w_t h_{yt}}{w_t(1 - e_t - m_t)h_{yt} - s_t} = \frac{\beta \zeta}{e_t}, \tag{7}$$

$$\frac{w_t}{w_t(1 - e_t - m_t)h_{yt} - s_t} \geq \frac{\beta w_{t+1} \gamma_m \theta}{r_{t+1} s_t + w_{t+1} h_{ot+1} \theta}. \tag{8}$$

Noting that h_{ot+1} is given by (4), we can rewrite (6) as

$$s_t = \frac{h_{yt}}{1 + \beta} \left[\beta w_t (1 - e_t - m_t) - \frac{w_{t+1}}{r_{t+1}} \theta (\lambda + \gamma_m m_t) \right]. \tag{9}$$

Equations (6) and (8) imply the following inequality:

$$r_{t+1} \geq \frac{w_{t+1} \gamma_m \theta}{w_t}. \tag{10}$$

The equality of (10) holds when $m_t > 0$. If this is the case, (10) is the no-arbitrage condition between physical capital investment and human capital investment for himself.

Market-clearing conditions

The market-clearing conditions of physical and human capital are respectively given by

$$K_{t+1} = s_t,$$
$$L_{t+1} = (1 - e_{t+1} - m_{t+1}) h_{yt+1} + \theta h_{ot+1}.$$

From these conditions with (3) and (4), we obtain

$$x_{t+1} = \frac{s_t}{[(1 - e_{t+1} - m_{t+1}) \gamma_e e_t + \theta(\lambda + \gamma_m m_t)] h_{yt}}. \tag{11}$$

Note that the final good market automatically clears owing to Walras' law.

3 Growth effect of the retirement age extension

Case of the equilibrium with $m_t > 0$

First, we focus on the equilibrium in which $m_t > 0$, that is, young agents receive education for old-age work. In this case, (10) holds with equality, and (9) is rewritten as

$$s_t = \frac{w_t h_{yt}}{(1+\beta)} \left[\beta(1-e_t) - (1+\beta)m_t - \frac{\lambda}{\gamma_m} \right]. \tag{12}$$

Substituting this result into (7), we find that the agent's time used for his/her child is constant over time:

$$e_t = e^* \equiv \frac{\beta\zeta}{1+\beta+\beta\zeta} \left(1 + \frac{\lambda}{\gamma_m}\right).$$

For $e^* < 1$ to be ensured, $\gamma_m > \frac{\beta\zeta\lambda}{1+\beta}$ must be satisfied. The no-arbitrage condition means that savings and human capital accumulation are indifferent for the agents as a way to increase their old-age income. In fact, using (10) with equality, we can express the lifetime income I_t as

$$\begin{aligned} I_t &\equiv w_t(1-e_t-m_t)h_{yt} + \frac{w_{t+1}}{r_{t+1}}\theta(\lambda+\gamma_m m_t)h_{yt} \\ &= w_t h_{yt}\left[(1-e_t) + \lambda/\gamma_m\right], \end{aligned}$$

which clearly shows the independence of I_t from w_{t+1}, m_t, and θ. Accordingly, the agents' expenditure on educating their children ($w_t e_t h_{yt}$) does not depend on them.

From (1) and (10) with equality, we obtain

$$x_{t+1} = \frac{\alpha A}{\theta\gamma_m} x_t^{\alpha}. \tag{13}$$

Let $u_t \equiv 1 - e_t - m_t$ denote the young agent's working time. Substituting (12) into (11) and using (13), we obtain the autonomous dynamic equation of u_t as follows:

$$u_{t+1} = \frac{\theta}{\alpha\gamma_e e^*} \left\{ \gamma_m u_t - \frac{1+\beta\alpha}{1+\beta}[\lambda + \gamma_m(1-e^*)] \right\}. \tag{14}$$

From its definition, u_t is a jump variable.[3] Therefore, it must be the case that $\theta > \alpha\gamma_e e^*/\gamma_m$. Then, u_0 jumps to its stationary value, u^*, and u_t remains its value, where u^* is given by

$$u^* = \frac{1+\beta\alpha}{1+\beta} \frac{\theta[\lambda + \gamma_m(1-e^*)]}{\theta\gamma_m - \alpha\gamma_e e^*},$$

which is positive as long as $\theta > \alpha\gamma_e e^*/\gamma_m$ is assumed. m_t is determined as $m^* \equiv 1 - e^* - u^*$. Since we assume that this is positive at first, we must derive the condition of $m^* > 0$ (i.e, $u^* < 1 - e^*$). Given the assumption $\theta > \alpha\gamma_e e^*/\gamma_m$, the condition $m^* > 0$ is rewritten as

$$(1+\beta)\alpha\gamma_e e^*(1-e^*) < \theta\Delta, \tag{15}$$

[3] To see why, recall the definition of u_t, $u_t \equiv 1 - e_t - m_t$ and the fact that e_0 and m_0 are endogenously determined by the young agents in period 0.

where

$$\Delta \equiv [\beta(1-\alpha)\gamma_m(1-e^*) - (1+\beta\alpha)\lambda].$$

If $\Delta \leq 0$, the above inequality never holds and m_t is always zero. To avoid such a trivial case, we consider the situation in which $\Delta > 0$. Using the definition of e^*, we can rewrite this inequality as follows:

Assumption 1. $\gamma_m > \widetilde{\gamma}_m \equiv \dfrac{\beta\zeta\lambda}{1+\beta} + \dfrac{(1+\beta\alpha)(1+\beta+\beta\zeta)\lambda}{(1-\alpha)\beta(1+\beta)}.$

Since $\widetilde{\gamma}_m > \frac{\beta\zeta\lambda}{1+\beta}$, the condition $e^* < 1$ is automatically satisfied as long as it is based on Assumption 1. In sum, $m^* > 0$ holds if

$$\theta > \widetilde{\theta}(\gamma_m) \equiv \frac{(1+\beta)\alpha\gamma_e e^*(1-e^*)}{\Delta}.$$

Note that $\widetilde{\theta}(\gamma_m) > \alpha\gamma_e e^*/\gamma_m$ as long as $\gamma_m > \widetilde{\gamma}_m$ (see the Appendix). Then, we show the following lemma:

Lemma 1. *Suppose that Assumption 1 holds. Then, e_t and m_t are given by their stationary values e^* and m^* for all periods if $\theta > \widetilde{\theta}(\gamma_m)$.*

We characterize the balanced growth path (BGP) in this case. Let $H_t \equiv h_{ot} + h_{yt}$ denote the aggregate level of human capital in the economy. From (3) and (4) with $e_t = e^*$ and $m_t = m^*$, we obtain its growth rate on the BGP as

$$\frac{H_{t+1}}{H_t} = \frac{\lambda + \gamma_m m^* + \gamma_e e^*}{h_{ot}/h_{yt} + 1}.$$

Since h_{ot}/h_{yt} is given by $(\lambda + \gamma_m m^*)/(\gamma_e e^*)$, the growth rate of aggregate human capital is given by $g^* \equiv \gamma_e e^* - 1$.

The labor supply L_t is given by

$$L_t = \chi^* H_t, \quad \chi^* \equiv \frac{(1-e^*-m^*)\gamma_e e^* + \theta(\lambda + \gamma_m m^*)}{\lambda + \gamma_m m^* + \gamma_e e^*} > 0.$$

Let $k_t \equiv K_t/H_t$ denote the ratio of physical capital to human capital. Using (13) and the definition of x_t, the dynamic equation of k_t is given by

$$k_{t+1} = \frac{\alpha A}{\theta\gamma_m}\chi^{*1-\alpha}k_t^{\alpha}.$$

Given the initial condition $k_0 > 0$, k_t converges to its steady state, $k^* \equiv \chi^*[\alpha A/(\gamma_m\theta)]^{1/(1-\alpha)}$. Thus, in the long run, physical capital, human capital, and GDP grow at a rate of g^*.

We now examine the effect of a retirement age extension, captured by an increase in θ, on the growth rate. An increase in θ unambiguously decreases u^*. Then, m^* increases. However, the long-run growth rate is determined solely by e^* and is independent of θ. Then, we can show the following proposition:

Proposition 1. *Suppose that Assumption 1 and $\theta > \widetilde{\theta}(\gamma_m)$ hold. An increase in θ promotes the agents' education for their labor supply at an old age, but this is neutral for the long-run growth rate.*

Kunze (2014) and Chen and Miyazaki (2020) assume that young agents in a period are endowed with the same amount of human capital as the old agents in that period. Then, in their model, an increase in retirement age affects both young and old agents' human capital and, hence, the long-run growth rate. Proposition 1 states that once we explicitly consider agents' decisions about educating their children, the results become dramatically different.

Case of the equilibrium with $m_t = 0$

If $\theta \leq \widetilde{\theta}(\gamma_m)$, $m_t > 0$ never holds in the equilibrium. In this case, (8) and (10) hold with strict inequality and hence we can not use (13). From (7), (9), and (11) with $m_t = 0$, we obtain

$$(1+\beta\zeta)e_t = \beta\zeta\left(1 - \frac{1}{(1-\alpha)Ax_t^{\alpha}}\frac{s_t}{h_{yt}}\right), \tag{16}$$

$$\frac{s_t}{h_{yt}} = \frac{1-\alpha}{(1+\beta)\alpha}\left[\beta\alpha Ax_t^{\alpha}(1-e_t) - \theta\lambda x_{t+1}\right], \tag{17}$$

$$x_{t+1} = \frac{1}{(1-e_{t+1})\gamma_e e_t + \theta\lambda}\frac{s_t}{h_{yt}}. \tag{18}$$

From (16)–(18), we obtain the dynamic equation of e_t (the derivation is given in the Appendix):

$$F(e_t) = G(e_t, e_{t+1}), \tag{19}$$

where functions F and G are defined as

$$F(e_t) \equiv \frac{(1+\beta)\left[\left(1+\frac{1}{\beta\zeta}\right)e_t - 1\right]}{\beta(1-e_t)},$$

$$G(e_t, e_{t+1}) \equiv \frac{1-\alpha}{\left[\frac{\gamma_e}{\theta\lambda}e_t(1-e_{t+1}) + 1\right](1+\beta)\alpha + 1 - \alpha} - 1.$$

We examine the existence and uniqueness of the steady state that solves $F(e) = G(e, e)$. We can show the following lemma:

Lemma 2. *Suppose that Assumption 1 and $\theta \leq \widetilde{\theta}(\gamma_m)$ hold. Let $e^{**} \in (0,1)$ denote the steady state that solves $F(e) = G(e,e)$. If $e^{**} < 1/2$, e_0 jumps to e^{**}, and e_t remains e^{**}.*

Proof. See the Appendix. □

Therefore, we focus on the case of $e^{**} < 1/2$, which is automatically satisfied if $\zeta \leq 1/\beta$.[4]

[4] To see why, note that $G(e,e) < 0$ for all $e \in (0,1)$. This means $F(e^{**}) < 0$. Therefore,

$$\left(1+\frac{1}{\beta\zeta}\right)e^{**} - 1 < 0,$$

which means $e^{**} < \frac{\beta\zeta}{1+\beta\zeta}$. Thus, if $\zeta \leq 1/\beta$, $e^{**} < 1/2$. We would like to thank an anonymous referee for pointing out this.

The growth rate is given by $g^{**} = \gamma_e e^{**} - 1$. An increase in θ increases the location of $G(e, e)$, which implies that e^{**} increases.

Proposition 2. *Suppose that Assumption 1 and $\theta \leq \widetilde{\theta}(\gamma_m)$ hold. Then, an increase in θ promotes young agents' human capital accumulation for their children and then boosts the long-run growth rate.*

Thus, in this case, we obtain a result similar to that of Zhang and Zhang (2009). However, as we have already shown, the increase in θ eventually moves the economy to an equilibrium with $m_t > 0$, which implies that the growth-enhancing effect eventually disappears.

Throughout this study, we do not introduce the aspect of population aging. In the Appendix, we introduce the survival probability from young to old ages and show that we qualitatively obtain the same results even in such a case.

4 Conclusion

We examined the relationship between mandatory retirement and patterns of human capital accumulation in a simple overlapping generations model with Uzawa–Lucas endogenous growth. In order to obtain clear-cut results, we did not assume externalities in human capital accumulation, because there are various ways to introduce such externalities and the results may depend on the method. Thus, it is a promising extension to introduce several types of externalities in human capital accumulation and examine the robustness of this study's implications. In addition, we assumed a logarithmic utility in common with existing studies. Therefore, it is important to relax this assumption and examine how this assumption affects the results in this study, although to accomplish this task requires numerical analysis or requires simplifying assumptions elsewhere.[5] Finally, following Fanti ang Gori (2012) and Chen (2018), we treated fertility as exogenous. Therefore, to introduce this aspect and examine the effects of postponement of retirement age in such a framework is an interesting extension. Nonetheless, the results obtained in this study provide a benchmark.

5 Appendix

Proof of $\widetilde{\theta}(\gamma_m) > \alpha\gamma_e e^*/\gamma_m$

We can arrange $\widetilde{\theta}(\gamma_m) - \alpha\gamma_e e^*/\gamma_m$ as

$$\widetilde{\theta}(\gamma_m) - \alpha\gamma_e e^*/\gamma_m = \frac{\alpha\gamma_e e^*}{\gamma_m}\left[\frac{(1+\beta)(1-e^*)\gamma_m}{\Delta} - 1\right]. \tag{20}$$

Since $\Delta = \beta(1-\alpha)(1-e^*)\gamma_m - (1+\beta\alpha)\lambda < (1+\beta)(1-e^*)\gamma_m$, we can verify that $\widetilde{\theta}(\gamma_m) - \alpha\gamma_e e^*/\gamma_m > 0$.

[5]For example, Chen and Miyazaki (2020) also consider the case of the CRRA utility function in the final section, whereas in that case, they abstract an agent's intertemporal consumption-saving decisions.

Derivation of (19)

Substituting (17) into (16) to eliminate s_t/h_{yt}, we obtain

$$\begin{aligned}(1+\beta\zeta)e_t &= \beta\zeta\left[1-\frac{1}{(1-\alpha)Ax_t^\alpha}\frac{1-\alpha}{(1+\beta)\alpha}\left[\beta\alpha Ax_t^\alpha(1-e_t)-\theta\lambda x_{t+1}\right]\right] \\ &= \beta\zeta\left[1-\frac{\beta}{1+\beta}(1-e_t)+\frac{\theta\lambda}{1+\beta}\frac{x_{t+1}}{\alpha Ax_t^\alpha}\right] \\ &= \frac{\beta\zeta}{1+\beta}\left(1+\beta-\beta(1-e_t)+\theta\lambda\frac{x_{t+1}}{\alpha Ax_t^\alpha}\right). \qquad (21)\end{aligned}$$

Substituting (17) into (18) to eliminate s_t/h_{yt}, we obtain

$$\begin{aligned}x_{t+1} &= \frac{1}{(1-e_{t+1})\gamma_e e_t+\theta\lambda}\frac{1-\alpha}{(1+\beta)\alpha}\left[\beta\alpha Ax_t^\alpha(1-e_t)-\theta\lambda x_{t+1}\right] \\ \Leftrightarrow &\left\{[(1-e_{t+1})\gamma_e e_t+\theta\lambda](1+\beta)\alpha+(1-\alpha)\theta\lambda\right\}x_{t+1}=(1-\alpha)\beta\alpha Ax_t^\alpha(1-e_t) \\ \Leftrightarrow &\frac{x_{t+1}}{\alpha Ax_t^\alpha}=\frac{(1-\alpha)\beta(1-e_t)}{[(1-e_{t+1})\gamma_e e_t+\theta\lambda](1+\beta)\alpha+(1-\alpha)\theta\lambda}. \qquad (22)\end{aligned}$$

Substituting (22) into (21) to eliminate $x_{t+1}/(\alpha Ax_t^\alpha)$ and arranging the terms, we obtain

$$\frac{(1+\beta)\left[\left(1+\frac{1}{\beta\zeta}\right)e_t-1\right]}{\beta(1-e_t)}=\frac{\theta\lambda(1-\alpha)}{[(1-e_{t+1})\gamma_e e_t+\theta\lambda](1+\beta)\alpha+(1-\alpha)\theta\lambda}-1,$$

which is (19).

Proof of Lemma 2

We can easily verify that $F(0)=-\frac{1+\beta}{\beta}<G(0,0)=-\frac{\alpha(1+\beta)}{1+\beta\alpha}$ and $F(1)=\infty>G(1,1)=-\frac{\alpha(1+\beta)}{1+\beta\alpha}$. Since $F(e)$ is increasing and convex, and $G(e,e)$ is U-shaped, these intersect at least once within the interval $(0,1)$.

The linear approximation of (19) in the neighborhood of $(e_t,e_{t+1})=(e^{**},e^{**})$ is given by

$$e_{t+1}-e^{**}=\frac{1}{G_2^{**}}\left(F_1^{**}-G_1^{**}\right)(e_t-e^{**}),$$

where

$$\begin{aligned}F_1^{**}&\equiv F'(e^{**})=\frac{1+\beta}{\beta\zeta(1-e^{**})^2}>0, \\ G_1^{**}&\equiv\left.\frac{\partial G}{\partial e_t}\right|_{e_t=e_{t+1}=e^{**}}=-\frac{\theta\lambda(1-\alpha)\beta(1-e^{**})\gamma_e(1+\beta)\alpha}{\{[(1-e^{**})\gamma_e e^{**}+\theta\lambda](1+\beta)\alpha+(1-\alpha)\theta\lambda\}^2}<0 \\ G_2^{**}&\equiv\left.\frac{\partial G}{\partial e_{t+1}}\right|_{e_t=e_{t+1}=e^{**}}=-G_1^{**}\frac{e^{**}}{1-e^{**}}>0.\end{aligned}$$

Since e_t is a jump variable, its equilibrium is unique and determinate if $\left(F_1^{**}-G_1^{**}\right)/G_2^{**}>1$. We can arrange this condition as

$$\begin{aligned}\frac{1}{G_2^{**}}\left(F_1^{**}-G_1^{**}\right)>1 &\Leftrightarrow\frac{F_1^{**}}{G_2^{**}}>1+\frac{G_1^{**}}{G_2^{**}} \\ &\Leftrightarrow\frac{F_1^{**}}{G_2^{**}}>1+\frac{e^{**}-1}{e^{**}}.\end{aligned}$$

Thus, $e^{**} < 1/2$ is a sufficient condition to satisfy the above inequality.

Introduction of mortality probability

We modify the model such that survival from the young to old periods is uncertain. Let $p \in (0,1)$ denote the probability of survival. Then, the utility function is given by

$$U_{yt} = \ln c_{yt} + p\beta \left(\ln c_{ot+1} + \zeta \ln h_{yt+1} \right).$$

The agent's budget constraint in the old period is now given by

$$\frac{r_{t+1}}{p} s_t + w_{t+1}\theta h_{ot+1} = c_{ot+1},$$

which comes from the existence of an actuarially fair insurance company, as in Yaari (1965). The other constraints remain unchanged from those in the main body. The first-order-conditions are

$$\begin{aligned}
s_t: &\quad \frac{1}{w_t(1-e_t-m_t)h_{yt}-s_t} = \frac{\beta r_{t+1}}{\frac{r_{t+1}s_t}{p} + w_{t+1}h_{ot+1}\theta}, \\
e_t: &\quad \frac{w_t h_{yt}}{w_t(1-e_t-m_t)h_{yt}-s_t} = \frac{p\beta\zeta}{e_t}, \\
m_t: &\quad \frac{w_t}{w_t(1-e_t-m_t)h_{yt}-s_t} \geq \frac{p\beta w_{t+1}\gamma_m \theta}{\frac{r_{t+1}s_t}{p} + w_{t+1}h_{ot+1}\theta}.
\end{aligned}$$

Let $\hat{\beta} \equiv p\beta$ and $\hat{\theta} \equiv p\theta$. Then, we can easily verify that the above three conditions are rewritten as

$$\frac{1}{w_t(1-e_t-m_t)h_{yt}-s_t} = \frac{\hat{\beta} r_{t+1}}{r_{t+1}s_t + w_{t+1}h_{ot+1}\hat{\theta}}, \tag{23}$$

$$\frac{w_t h_{yt}}{w_t(1-e_t-m_t)h_{yt}-s_t} = \frac{\hat{\beta}\zeta}{e_t}, \tag{24}$$

$$\frac{w_t}{w_t(1-e_t-m_t)h_{yt}-s_t} \geq \frac{\hat{\beta} w_{t+1}\gamma_m \hat{\theta}}{r_{t+1}s_t + w_{t+1}h_{ot+1}\hat{\theta}}. \tag{25}$$

Thus, conditions (23)–(25) are essentially the same as (6)–(8) if we replace β and θ with $\hat{\beta}$ and $\hat{\theta}$. Firms' behavior (equation (1)) and the capital market equilibrium ($K_{t+1} = s_t$) do not change. The labor market equilibrium is replaced by

$$L_{t+1} = (1 - e_{t+1} - m_{t+1})h_{yt+1} + \hat{\theta} h_{ot+1}.$$

Thus, we obtain the same qualitative results as the main body, even if we introduce the mortality probability.

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