Economics Bulletin

Volume 42, Issue 2

The expenditure approach to income and substitution effects

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Abstract

Visible substitutions between goods as a result of a price change have to occur insofar as the price elasticity of demand deviates from unity. A price-elastic demand leads to a larger expenditure share after a price decrease. This implies that the aggregate expenditure share for all other goods, including money in one's cash balance, has to decrease. A visible substitution takes place as the demand for other goods is reduced. Similarly, a price-inelastic demand for a specific good leads to a larger expenditure share after a price increase. The aggregate expenditure share of all other goods has to decrease and the consumer reduces demand for at least one other good. By following the expenditure approach to income and substitution effects it is shown that the conventional analysis of deadweight loss from taxation is misleading. The deadweight loss is underestimated when demand is inelastic and overestimated when it is elastic.

I would like to thank Professor Joseph T. Salerno for having challenged me to think more carefully about income and substitution effects. I also thank two anonymous referees for very helpful comments.
Citation: Karl-Friedrich Israel, (2022) "The expenditure approach to income and substitution effects", *Economics Bulletin*, Volume 42, Issue 2, pages 431-446
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Submitted: March 19, 2021. **Published:** June 30, 2022.

1 Introduction

Economists engage in partial analyses to divide the complexities of real-world phenomena into manageable chunks. We often focus on specific markets in isolation to get a better handle on a given research question. Problems arise insofar as we forget secondary effects on other markets that have repercussions on our conclusions.

The theoretical concepts we use in economic analysis can be more or less conducive to reminding us of these additional effects. In some situations they are misleading. The separation of income and substitution effects in microeconomic price theory is one such example. The term "substitution effect" suggests that it relates to the connectedness of different markets. One good is substituted for another one. However, the substitution effect of standard microeconomics occurs also in situations where no genuine substitution between goods takes place.

Take the example of the standard model of consumer choice between two goods, x_1 and x_2, with a Cobb-Douglas utility function, a budget of y and unit prices of p_1 and p_2. The primal optimization problem,

$$\max_{x_1,x_2} \; x_1^{\alpha} x_2^{1-\alpha} \; \text{ given } \; p_1 x_1 + p_2 x_2 \leq y,$$

gives rise to the Marshallian demand functions:

$$x_1(p_1, y) = \alpha \frac{y}{p_1} \; \text{ and } \; x_2(p_2, y) = (1-\alpha)\frac{y}{p_2}.$$

The expenditure shares for both goods, α and $(1-\alpha)$, remain the same, regardless of their unit prices. Any price change for one good has absolutely no effect on the quantity demanded of the other. And yet standard microeconomic analysis would identify a substitution effect attached to any given price change. In other words, in spite of the fact that the cross-price elasticity between the goods is zero, and that the goods are unrelated in that sense, i.e. neither substitutable nor complementary, there still is a substitution effect.[1] This is not to say that the standard substitution effect is of no analytical interest, but merely suggests that this effect remains below the surface and that it does not necessarily tell us anything about the repercussions that the developments on one market may have on other markets. A different conceptualization is needed to bring the actual substitutions or trade-offs between goods to the foreground.

This paper suggests one such conceptualization by focusing on price elasticity and the corresponding changes in expenditure shares. The purpose of this paper is not to criticize the standard versions of the substitution and income effects, but rather to complement them and to provide an alternative perspective that might be analytically more convenient in situations where secondary effects on other markets are important. An application of this alternative approach to the analysis of deadweight loss from taxation is provided in the last section.

2 Substitution and income effects in standard microeconomic analysis

The intuition behind decomposing the effects of price changes into income and substitution effects is straightforward. Any price variation implies changes in real income or wealth as well as a shift in relative prices. A good becomes more or less expensive relative to other goods. Consumption decisions depend on both real income and relative prices. With income and substitution effects economists try to disentangle the different effects that arise from these two causes.

There are two well-known versions of these effects, the Hicks and the Slutsky decomposition. The Hicksian demand function tries to get rid of the income effects of price changes by holding the level of utility constant along the demand curve. The dual optimization problem in consumer choice theory, that gives rise to the Hicksian demand function, minimizes the

[1] A numerical example is given in the appendix.

expenditure, given a certain level of utility:

$$\min_{x_1,x_2,...,x_n} \sum_{i=1}^{n} p_i x_i \text{ subject to } u(x_1,x_2...,x_n) = \bar{u}.$$

Any change in the quantity demanded that arises from a price change in the corresponding demand functions, $x_i^H(p_1,p_2,...,p_n,\bar{u}),\ \forall i \in [1:n]$, is interpreted as a substitution effect, since the level of utility and by implication real income have not changed.

In contrast, Marshallian demand functions, derived from the maximization of utility, given a certain nominal income, incorporate both substitution and income effects:

$$\max_{x_1,x_2,...,x_n} u(x_1,x_2...,x_n) \text{ subject to } \sum_{i=1}^{n} p_i x_i = \bar{y}.$$

The corresponding demand functions, $x_i^M(p_1,p_2,...,p_n,\bar{y}),\ \forall i \in [1:n]$, are said to be uncompensated for changes in real income. For monotonic and convex preferences, Marshallian and Hicksian demand are identical when the fixed level of utility in the dual optimization problem corresponds to the maximum level of utility reached in the primal optimization problem at the given nominal income and unit prices.

Starting from such an identical optimal consumption bundle, the difference between the Marshallian and Hicksian demand for a specific good after a change in its unit price from p_i to $p_i^{'}$, i.e., $x_i^M(p_1,....,p_i^{'},...,p_n,\bar{y}) - x_i^H(p_1,....,p_i^{'},...,p_n,\bar{u})$, provides the corresponding income effect to the Hicksian substitution effect which is given by the change in Hicksian demand.

In contrast, the Slutsky decomposition holds real income constant by adjusting the available nominal income such that the optimal bundle prior to the price change just remains affordable at the new price and the adjusted nominal income. Hence, the adjusted nominal income is $\bar{y}^{'} = \bar{y} + (p_i^{'} - p_i)x_i^M(p_1,...,p_i,...,p_n,\bar{y})$. It is increased when the unit price p_i increases $(p_i^{'} > p_i)$, and reduced when the unit price falls $(p_i^{'} < p_i)$, in order to compensate the implicit change of real income or purchasing power.

The difference in Marshallian demand at the new price and the actual income, and Marshallian demand at the new price and the adjusted income i.e., $x_i^M(p_1, ..., p_i^{'}, ..., p_n, \bar{y}) - x_i^M(p_1, ..., p_i^{'}, ..., p_n, \bar{y}^{'})$, gives the Slutsky income effect. The difference between Marshallian demand at the new price and the adjusted income, and Marshallian demand at the initial price and the actual income, i.e., $x_i^M(p_1, ..., p_i^{'}, ..., p_n, \bar{y}^{'}) - x_i^M(p_1, ..., p_i, ..., p_n, \bar{y})$, reflects the income-compensated change in demand, and hence the corresponding Slutsky substitution effect. Both effects thus add up to the total change in demand for the good whose price has changed.

As shown in the appendix the Hicksian and Slutsky decompositions into income and substitution effects are not identical. They tackle the same problem from a slightly different angle. While the Hicksian decomposition holds the level of utility constant, the Slutsky decomposition holds the buyer's purchasing power constant by adjusting nominal income. Both decompositions have in common that they identify substitution effects, even when the expenditure shares on all goods remain constant after the price change. What is labeled a substitution effect in standard microeconomic analysis does not actually refer to genuine substitutions or trade-offs between goods from a buyer's point of view.

3 The expenditure approach as an alternative perspective

There is another way to look at the demand side of markets. The purchasing power of any buyer is of course constrained and buying means deciding how much to spend on a given good and how much to withhold for other goods. A buying decision, given the budget and unit prices, thus involves determining the expenditure shares on various goods. Increasing one expenditure share implies lowering another one. It could also mean to lower one's cash balance, which can be interpreted as the "expenditure" of one's income on money.

Genuine substitutions between goods emerge in so far as one expenditure share is increased at the expense of another one (Israel, 2018, 2020). When the expenditure share on a given good remains constant after a price change, that is, when demand for the good is unit-elastic, the change in the quantity demanded does not necessitate a substitution between goods. Any increase in the quantity demanded of the good in response to a decrease of its unit price, for example, can be financed entirely out of the implied increase in real income due to the price change. There is no sacrifice necessary in terms of other goods.

The income effect of a price change

From that point of view, the income effect of a price change corresponds to the change in the quantity demanded of the good that would emerge if the expenditure share is held constant. Hence, unit elasticity of demand serves as the benchmark to determine the income effect.

Let $d(p)$ denote the quantity of a good demanded at the unit price p. For two prices, p and p', the corresponding quantities demanded are $q = d(p)$ and $q' = d(p')$. The price change $\Delta p = p' - p$ leads to a quantity change of $\Delta q = q' - q$. The change in the quantity demanded is separated into an income and a substitution effect: $\Delta q = \Delta^I q + \Delta^S q$.

The income effect is the quantity change that would emerge with unit price elasticity of demand or, in other words, with a constant expenditure share for the good in question, that is, under the condition: $pq = p'q'$. Under this condition, we obtain

$$q' = \frac{p}{p'} q$$

$$\Leftrightarrow q' - q = \left(\frac{p}{p'} - 1\right) q = -\frac{\Delta p}{p'} q.$$

The income effect so understood is defined as:

$$\Delta^I q = -\frac{\Delta p}{p'} q.$$

This income effect always has the opposite sign of the price change. It is negative for a price increase ($\Delta p > 0$), and it is positive for a price decrease ($\Delta p < 0$). Hence, this alternative conception of the income effect does not serve as a tool of identifying inferiority of goods or Giffen behavior. In fact, it leaves the main work to do for the substitution effect.

The substitution effect of a price change

In so far as the price elasticity of demand deviates from unity, there is an additional substitution effect with respect to other goods and services. Total expenditure can increase or decrease as a result of any price change. The substitution effect can thus take any sign regardless of the direction of the price change. It can be determined on the basis of the variation in total expenditure: $p'q' - pq$. The quantity change that emerges due to the adjustment in total expenditure is:

$$\Delta^S q = \frac{p'q' - pq}{p'}$$

$$\Leftrightarrow \Delta^S q = q' - \frac{p}{p'}q = q' - q + q - \frac{p}{p'}q = \Delta q + \left(1 - \frac{p}{p'}\right) q = \Delta q - \left(-\frac{\Delta p}{p'}q\right).$$

Hence, the substitution effect so understood is the difference between the total quantity change and the income effect:

$$\Delta^S q = \Delta q - \Delta^I q.$$

The substitution effect takes on a positive value, when the expenditure share increases after the price change. This is the case for a price-inelastic demand in combination with a price increase, or a price-elastic demand in combination with a price decrease. The substitution effect turns negative, when the expenditure share decreases after the price change. This is the case for a price-inelastic demand in combination with a price decrease, or a price-elastic demand in combination with a price increase. At unit price elasticity of demand the substitution effect is 0 and the expenditure share remains constant.

A discussion of different scenarios

An exogenous decline in the unit price of a good leads to an increase in real income or purchasing power of every money unit with respect to that good. The question is how a buyer wishes to expend the additional income. When it is entirely spent on the good whose price has decreased, the quantity demanded increases proportionally to the price change and the substitution effect as defined above is 0:

$$\Delta q = \Delta^I q = -\frac{\Delta p}{p'} q \;\text{ and }\; \Delta^S q = 0.$$

When demand is price-elastic, the consumer wishes to increase the quantity demanded further and the substitution effect takes on a positive value. The expenditure share for the good whose price has fallen increases and the expenditure share of at least one other good has to decrease, that is, the demand schedule of at least one other good shifts to the left in the corresponding price quantity diagram. A genuine substitution takes place. When we consider all other goods in the aggregate, then the aggregate demand function must shift to the left to the extent that the expenditure share for the good whose price has changed has increased. This mechanism is shown in the upper panels of Figure 1.

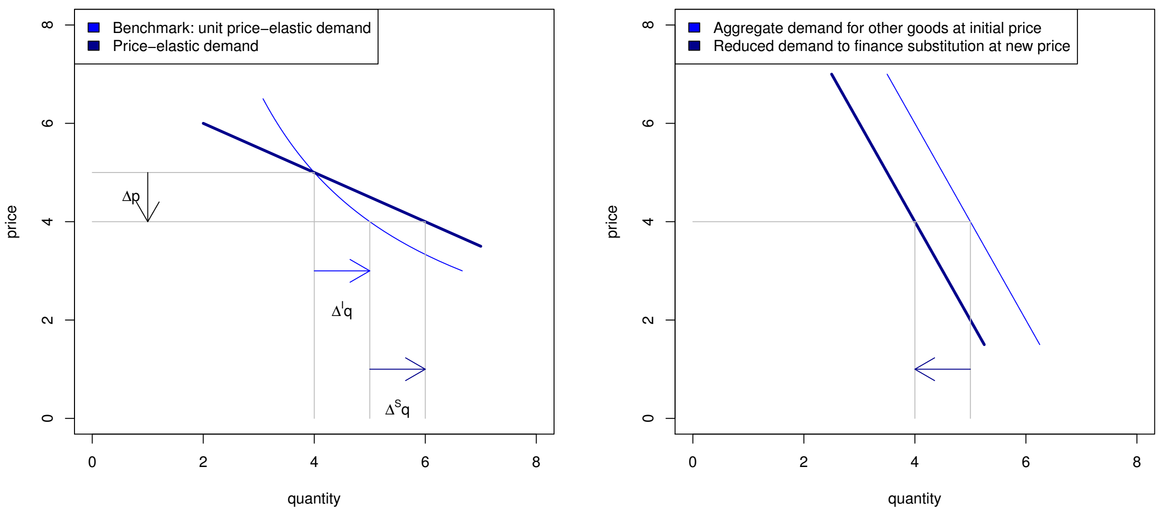
When demand is price-inelastic, the expenditure share is reduced at the lower unit price, which means that the expenditure share of at least one other good increases. The positive income effect of the price decrease is combined with a negative substitution effect, which allows the demand for other goods to increase. In a sense, a part of the positive income effect spills over to other goods. This is illustrated in the bottom panels of Figure 1.

The corresponding cases of price-elastic and price-inelastic demand when the unit price increases are shown in Figure 2. The income effect as defined above is always negative, when a good becomes more expensive, but the substitution effect can take on both signs depending on

Figure 1: Income ($\Delta^I q$) and substitution ($\Delta^S q$) effects after a price decrease

Price-elastic demand

leads to a reduction of the expenditure for other goods



Price-inelastic demand

leads to an increase of the expenditure for other goods

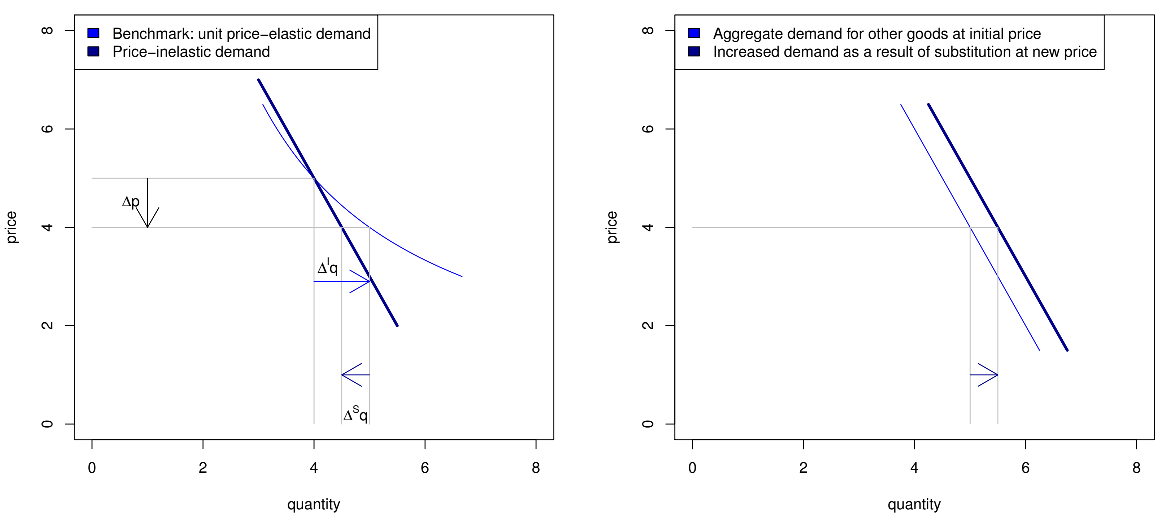
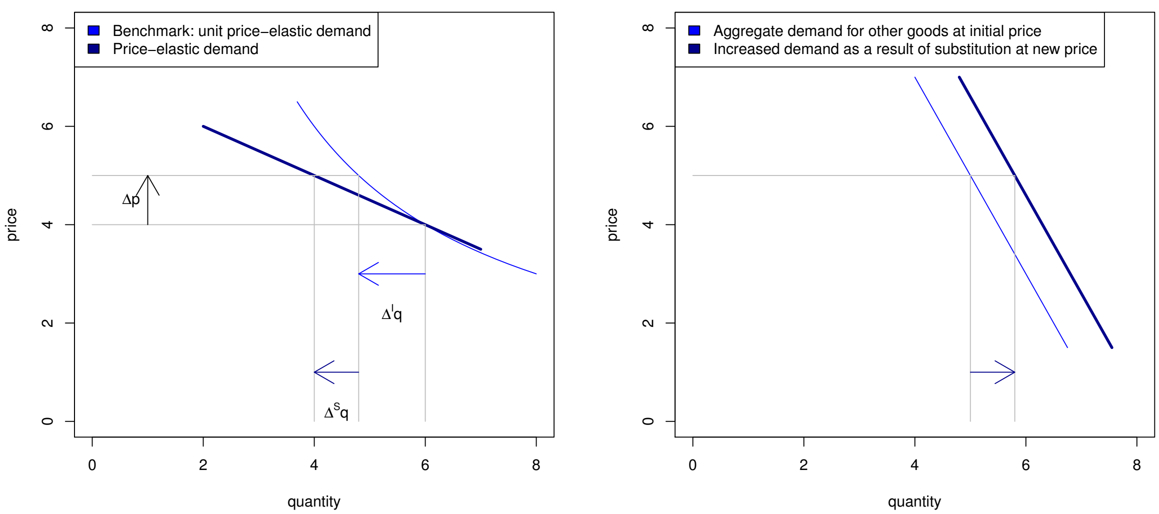


Figure 2: Income ($\Delta^I q$) and substitution ($\Delta^S q$) effects after a price increase

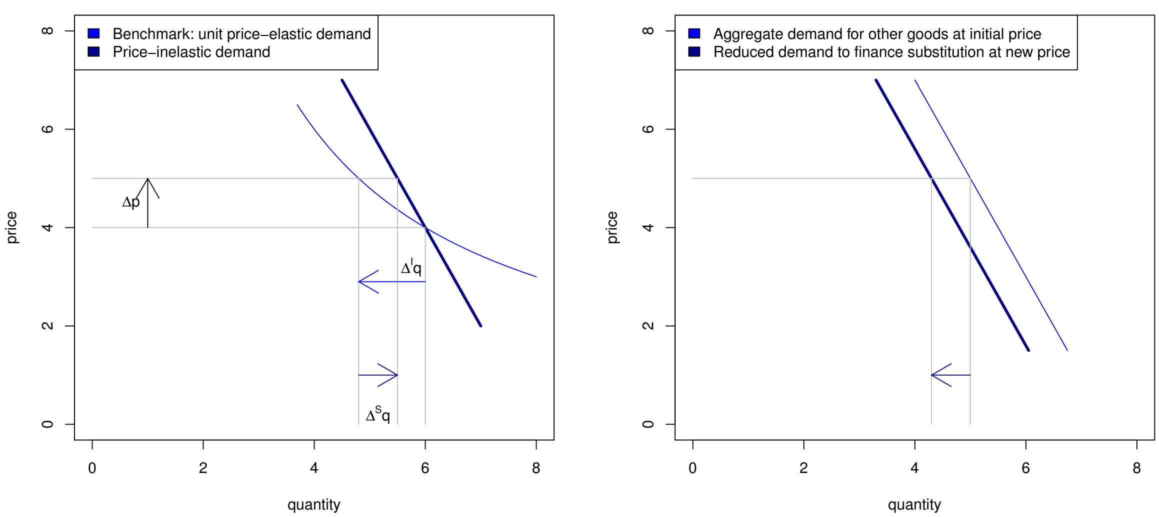
Price-elastic demand

leads to an increase of the expenditure for other goods



Price-inelastic demand

leads to a decrease of the expenditure for other goods



the buyer's subjective preferences. It is positive, when demand is inelastic (upper panels). It is negative, when demand is elastic (bottom panels).

In the special case of Giffen goods, income and substitution effects have opposite signs and the substitution effect outweighs the income effect. This is similar to the standard decompositions. In contrast, however, following the expenditure approach it is the substitution effect that does the work of bringing about Giffen behavior and not the reverse income effect that emerges when goods are inferior. Standard microeconomic theory holds that any Giffen good is also an inferior good. It has to be so inferior that the reverse income effect outweighs the substitution effect that is always positive when the price decreases or negative when it increases (Varian, 1992, ch. 8).

Table 1: Summary of different cases

Price	$\Delta^I q$	$\Delta^S q$	Δq	expenditure share	price elasticity
increases	negative	negative	negative	decreases	elastic
increases	negative	0	negative	unchanged	unit
increases	negative	positive	negative	increases	inelastic
increases	negative	positive	positive	increases	positive (Giffen good)
decreases	positive	positive	positive	increases	elastic
decreases	positive	0	positive	unchanged	unit
decreases	positive	negative	positive	decreases	inelastic
decreases	positive	negative	negative	decreases	positive (Giffen good)

4 An application of the expenditure approach

The expenditure approach to substitution and income effects can be applied to show that the standard analysis of Harberger triangles (Hines, 1999) for the evaluation of deadweight loss from market distortions, such as an excise tax on a good, is misleading (Fegley et al., 2021). This analysis holds that the the overall welfare loss from an excise tax is lower, the more price-inelastic the demand function. It ignores changes in expenditure shares and the corresponding substitution effects between goods.

An excise tax increases the demand price that buyers have to pay along the demand function. When demand is price-elastic the expenditure share decreases. This implies that the expenditure on at least one other market increases. The high deadweight loss observed on the taxed market is thus partly compensated by an increase in consumer and producer surplus on other markets. There is a negative substitution effect at work for the taxed good that reinforces the negative income effect. Demand and expenditure for at least one other good increase. The upper panels of Figure 3 illustrate this case.

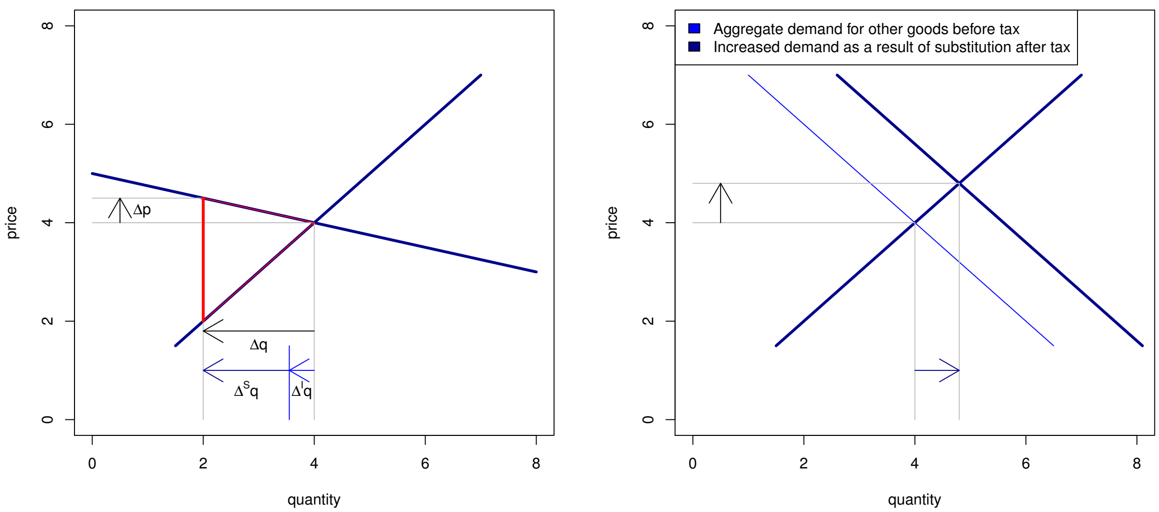
When demand is price-inelastic, a positive substitution effect acts as a counterbalance to the negative income effect. This can only be the case at the expense of demand for some other good. As the expenditure for the taxed good increases, demand for at least one other good decreases. Hence, the relatively small deadweight loss in the taxed market is accompanied by a reduction of consumer and producer surplus in other markets.

This shows that the basic neoclassical analysis of deadweight loss from taxation is misleading. It ignores secondary effects of taxes on other markets. It overestimates the deadweight loss when demand is price-elastic, and it underestimates the deadweight loss when demand is price-inelastic on the market where the tax is imposed. The expenditure approach to income and substitution effects helps to bring the connectedness of markets to the foreground and thus sheds new light on the overall effects of excise taxes in a straightforward and simple fashion. It underlines the importance of more complex measures of deadweight loss (Zabalza, 1982), be it on the basis of equivalent variation (Kay, 1980) or compensating variation (Diamond & McFadden, 1974).

Figure 3: The deadweight loss of an excise tax

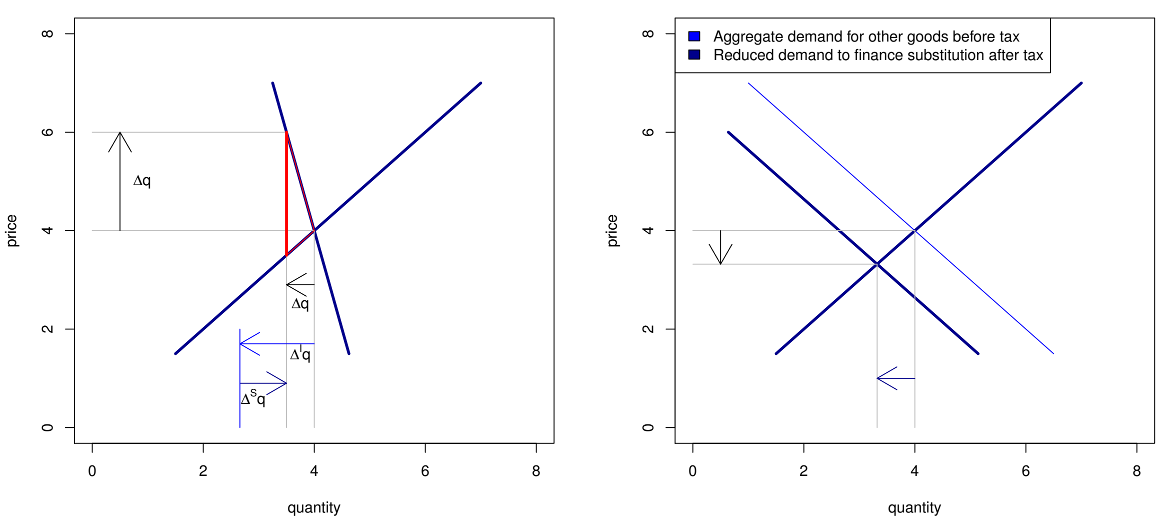
Price-elastic demand

leads to large deadweight loss on the taxed market but increased surplus on other markets



Price-inelastic demand

leads to small deadweight loss on the taxed market but reduces surplus on other markets



5 Conclusion

This new conceptualization of income and substitution effects does not put into question the importance of the standard microeconomic decompositions and their various applications. It rather complements them. It aims at providing an alternative perspective that has one important advantage. It impels the analyst to zoom out of a partial equilibrium perspective and to take into consideration secondary effects on other markets. A substitution effect as defined here refers to a genuine substitution or trade-off between goods from the perspective of a buyer. It highlights the connection between markets and helps to follow the chain of cause and effect that emerges in one market and propagates onto others.

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Appendix

This appendix provides a numerical example to illustrate the difference between the standard decompositions into income and substitution effects and the alternative expenditure approach presented in the paper. We assume a consumer, who has the choice between two goods, x_1 and x_2. The unit prices of the two goods are p_1 and p_2 and the consumer's available income is y. The consumer's preferences are described by a simple Cobb-Douglas utility function $u(x_1, x_2) = \sqrt{x_1 x_2}$. The primal optimization problem gives rise to the Marshallian demand functions:

$$x_1^M(y, p_1) = \frac{y}{2p_1} \text{ and } x_2^M(y, p_2) = \frac{y}{2p_2}.$$

For a fixed level of utility, $\bar{u}$, the dual optimization problem gives rise to the Hicksian demand functions:

$$x_1^H(\bar{u}, p_1, p_2) = \sqrt{\frac{p_2}{p_1}}\bar{u} \text{ and } x_2^H(\bar{u}, p_1, p_2) = \sqrt{\frac{p_1}{p_2}}\bar{u}.$$

For $y = \$32$, and $p_1 = p_2 = \$4$, the consumer buys 4 units of each good according to the Marshallian demand functions. Assume a price change for x_1 from $p_1 = \$4$ to $p_1 = \$1$. The consumer now buys 16 units of the first good. The quantity of the second good does not change. Hence, we have a quantity change for the first good of $\Delta x_1 = 12$, which according to the expenditure approach is a pure income effect, since no substitution or trade-off with respect to the second good is required.

However, standard microeconomic analysis decomposes this quantity change in the following two ways.

First, the price reduction implies a gain in real income that corresponds to $3 saved per unit bought of the first good. If the consumer decides to buy the bundle that is optimal at the initial price, i.e. $(x_1 = 4; x_2 = 4)$, a total amount of $12 is saved at the new price. In other words, the initial bundle at the new price costs $12 less than at the initial price. The income is thus adjusted from $y = \$32$ to $y = \$20$, and the Slutsky income effect is

$$\Delta^I x_1 = x_1^M(y = 32, p_1 = 1) - x_1^M(y = 20, p_1 = 1) = 16 - 10 = 6.$$

The corresponding substitution effect is

$$\Delta^S x_1 = x_1^M(y = 20, p_1 = 1) - x_1^M(y = 32, p_1 = 4) = 10 - 4 = 6,$$

so that the income effect and the substitution effect add up to the total quantity change:

$$\Delta x_1 = \Delta^I x_1 + \Delta^S x_1 = 6 + 6 = 12.$$

Second, the optimal bundle at the initial price corresponds to a utility level of $u(x_1 = 4, x_2 = 4) = \sqrt{16} = 4$. The Hicksian demand at the new price, given this utility level, is $x_1^H(\bar{u} = 4, p_1 = 1, p_2 = 4) = 8$. Hence, the HIcks substitution effect is

$$\Delta^S x_1 = x_1^H(\bar{u} = 4, p_1 = 1, p_2 = 4) - x_1^H(\bar{u} = 4, p_1 = 4, p_2 = 4) = 8 - 4 = 4,$$

and the corresponding income effect is

$$\Delta^I x_1 = x_1^M(y = 32, p_1 = 1) - x_1^H(\bar{u} = 4, p_1 = 1, p_2 = 4) = 16 - 8 = 8,$$

such that, once again, the two effects add up to the total effect:

$$\Delta x_1 = \Delta^I x_1 + \Delta^S x_1 = 8 + 4 = 12.$$

Table 2: Three decompositions into income and substitution effects

	Income effect $\Delta^I x_1$	Substitution effect $\Delta^S x_1$	Total effect Δx_1
Slutsky decomposition	6	6	12
Hicks decomposition	8	4	12
Expenditure approach	12	0	12

For a price change from $p_1 = \$4$ to $p_1 = \$1$, given $p_2 = \$4$, $y = \$32$ and $u(x_1, x_2) = \sqrt{x_1 x_2}$, we thus have the three decompositions summarized in Table 2. As no genuine substitution between the two goods is required, the increase in the quantity demanded of the first good is interpreted as a pure income effect according to the expenditure approach outlined in the paper.