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Robust drawdown-based performance measures

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Abstract

In this paper, we propose a robust optimization framework for drawdown-based performance measures that substantially improves upon conventional portfolio choices. In particular, we motivate and develop a robust optimization method that is typically used with conventional robust statistical estimation techniques, directly and explicitly addressing the estimation errors in the portfolio optimization process of the drawdown-based performance measures. Empirical analyses validate the proposed methodologies and confirm that robust drawdown-based performance measures yield better out-of-sample performance than their classic versions.

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1. Introduction

Optimal portfolio allocation is one of the classic uncertain decision-making problems that involve parameters estimation and distributional properties of sets of investment opportunities. The most well-known performance measure for optimal portfolio selection is the Sharpe ratio (Sharpe, 1994), a reward-risk measure that optimizes the relationship between the mean and the standard deviation of the returns generated by a given portfolio. While the Sharpe ratio considers the portfolio's standard deviation (which is not a risk measure), it penalizes both negative and positive deviations from the mean symmetrically (Rachev et al., 2008). Moreover, it is theoretically justified under the Gaussian assumption of asset returns. Thus, it leads to incorrect investment decisions when the returns distribution presents heavy tails or skewness. Indeed, unrealistic distributional assumptions of asset returns combined with the implications of estimation error usually negatively impacts the overall performance. During the last decades, many alternatives to the Sharpe ratio have been proposed (Sortino & Price, 1994; Konno & Yamazaki, 1991; Biglova et al., 2004; Farinelli et al., 2008; Ortobelli et al., 2019). A common argument for proposing alternative reward-risk measures is the presence of asymmetry and fat tails in returns. Drawdown-based measures are a special class of these alternative reward-risk performance measures. While having the same reward measure as the Sharpe ratio, this special class of performance measures differs in terms of the measure used to quantify risk (Bacon, 2008; Auer & Schuhmacher, 2013).

In most practical situations, however, the statistical input parameters are unknown and approximated using available economic data. Thus, the reward-risk performance measures are likely sensitive to estimation errors and may provide poor out-of-sample performance (Chopra & Ziemba, 1993; DeMiguel & Nogales, 2009; Behr et al., 2012). In particular, the problem of parameter estimation increases as the number of assets increases. Several authors have accordingly developed alternative approaches to deal with this practical problem, including, for example, naïve diversification, shrinkage estimators, constraining portfolio weights, and robust optimization (see, e.g., Wang, 1998; Ledoit & Wolf, 2004; Pflug et al., 2012; Anderson & Cheng, 2016).

The aim of this paper, therefore, is to develop a robust portfolio framework for drawdown-based performance measures and minimize the negative impact of estimation errors on optimal portfolio weights. Two commonly accepted ways to robustify portfolio optimizations are to assume a certain structure for uncertainty sets or optimize portfolios under the worst-case analyses (Ben-Tal & Nemirovski, 1998; Goldfarb & Iyengar, 2003). Other methods involve robustification via the Bayes and Black-Litterman models or the adoption of robust statistical techniques (Schottle et al., 2010; Anderson & Chen, 2016). Existing empirical studies provided by several authors suggest that robust portfolios improve the out-of-sample performance and are more stable than their non-robust versions (see, e.g., DeMiguel & Nogales, 2009; Sehgal & Mehra, 2020; Kouaissah, 2021). The main contribution of this paper is to develop a robust portfolio optimization for drawdown-based performance measures that directly addresses the estimation errors in the optimization problem itself. In particular, two specific constraints are designed to achieve this purpose: one is based on robust estimators and one assumes that the true mean lies with a certain confidence. Thus, the proposed portfolio strategies combine the ability of two well-recognized robust methods (see, e.g., Fabozzi et al., 2007, DeMiguel & Nogales, 2009). The effectiveness of the proposed robust portfolio optimization is evaluated by applying it to the S&P 500 components. The empirical results confirm that the robust drawdown-based performance measures improve upon their classical non-robust versions in terms of out-of-sample performance.

The paper is structured in four sections. Section 2 introduces robust portfolio optimization for drawdown-based performance measures. Section 3 applies the proposed portfolio selection methodologies to the S&P 500 components. Finally, Section 4 summarizes our conclusions.

2. Methodology: robust drawdown-based performance measures

This section proposes robust portfolio formulations for drawdown-based performance measures. Consider n risky assets with a vector of gross returns[1] $z = [z_1, \ldots, z_n]'$ and define portfolio returns as $x'z$, where $x = [x_1, \ldots, x_n]'$ is the vectors of portfolio weights among n risky assets. Moreover, we assume no short sales are allowed (i.e., $x_i \geq 0$), thus the vector of portfolio weights x belongs to the simplex $C = \{x \in \mathbb{R}^n | \sum_{i=1}^n x_i = 1; x_i \geq 0; \forall i = 1, \ldots, n\}$. In this context, investors choose a portfolio that optimizes a given drawdown-based performance measure. In particular, when the reward v and drawdown measure q are both positive measures, the market portfolio is the solution to the following optimization problem:

$$\begin{aligned} \max_{x} \quad & \frac{v(x'z - z_b)}{q(x'z - z_b)} \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i = 1 \\ & x_i \geq 0;\ i = 1, \ldots, n \end{aligned} \tag{2.1}$$

where z_b denotes the returns of a given benchmark (that generally represents a risk-free asset) and $x'z = \sum_{i=1}^n x_i z_i$ computes the portfolio returns. Depending on the drawdown type measure used to quantify risk we can define various drawdown-based performance measures; for recent state-of-the-art surveys, readers should refer to Bacon (2008) and Auer and Schuhmacher (2013). The analytical and numerical assessments of drawdown magnitudes have been widely discussed in the literature (see, e.g., Schuhmacher & Eling, 2011; Auer, 2015; Goldberg & Mahmoud, 2017; and literature therein). This paper focuses on four well-known drawdown-based performance measures: the Calmar, Sterling, Martin, and Burke ratios. For instance, the Calmar ratio measures risk based on the maximum drawdown, while the Sterling ratio quantifies risk by the average of the K-most important drawdowns.

[1] We define the i-th gross return between time $t-1$ and time t as $z_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$ and the i-th log return as $ln(z_{i,t})$ where $P_{i,t}$ is the price of the i-th asset at time t.

Four well-known drawdown-based performance measures that are considered in the empirical analysis are formalized and reported in Table 1.

Table 1: Drawdown-based performance measures: reward and drawdown measures

Performance measure	Reference	Reward measure	Drawdown measure
Calmar ratio (CR)	Young (1990)	$E(x'z - z_b)$	$\max_{t=1,\dots,T} dd_t(x'z)$
Sterling ratio (SR)	Bacon (2008)	$E(x'z - z_b)$	$\lvert\frac{1}{K}\sum_{k=1}^{K} ldd_k(x'z)\rvert$
Burke ratio (BR)	Burke (1994)	$E(x'z' - z_b)$	$\sqrt{\sum_{k=1}^{K} lld_k^2(x'z)}$
Martin ratio (MR)	Bacon (2008)	$E(x'z - z_b)$	$\sqrt{\frac{1}{T}\sum_{t=1}^{T} DDP_t^2(x'z)}$

where $E(x'z)$ is the portfolio's expected return, $dd_t(x'z) = \max_{s=1,\dots,T} w_s(x'z) - w_t(x'z)$, $x_s(x'z) = \sum_{s=1}^{t} x'z_s - z_{t,b}$, $t = 1,\dots,T$, ldd_k is the kth-largest drawdown, ldd_k^2 is the square of the kth-largest drawdown, DDP_t, $t = 1,\dots,T$ are drawdowns from a previous peak and are defined as the cumulated uncompounded excess returns since the previous peak, which can be defined as $DDP_t = \max_{1\le i\le t}(-(z_i - z_b))$. Following performance measurement literature, this paper considers $K = 5$ to calculate the Sterling, Martin, and Burke ratios (see, e.g., Auer & Schuhmacher, 2013; Kouaissah & Hocine, 2021; and references therein).

A large body of literature has confirmed the negative impact of estimation errors on the optimal portfolios (see, e.g., Chopra & Ziemba, 1993; DeMiguel & Nogales, 2009). Our contribution here consists of proposing a simple robust portfolio optimization for drawdown-based type performance measures that render them less prone to estimation errors. In particular, two specific constraints are designed: one is based on robust estimators and one assumes that the true mean lies with a certain confidence. Thus, we integrate into a single portfolio optimization two types of robustification one based on robust estimators (DeMiguel & Nogales, 2009) and one based on a robust methodology proposed by Ceria and Stubbs (2006) (see also Fabozzi et al., 2007). In general, there is a consensus that robust portfolios have better stability properties than their traditional version and usually provide better out-of-sample performance (Anderson & Cheng, 2016; Sehgal & Mehra, 2020). In doing so, we follow a "standard" robust optimization approach proposed by Ceria and Stubbs (2006) (see also Fabozzi et al., 2007), where the vector of true mean $\mu = [\mu_1, \dots, \mu_n]'$ is assumed to be normally distributed. Then, given an estimate of the mean $\bar{\mu}$ and a covariance matrix Σ, it can be been shown that the true mean lies inside the confidence region:

$$(\mu - \bar{\mu})'\Sigma^{-1}(\mu - \bar{\mu}) \le \kappa^2 \tag{2.2}$$

with probability 100η percent, where $\kappa^2 = \chi_d^2(1-\eta)$ and χ_d^2 is the inverse cumulative distribution function of the chi-squared distribution with d degrees of freedom. Further, inspired by robust estimators' literature (see, e.g., DeMiguel & Nogales, 2009), we consider the following portfolio constraint:

$$\frac{1}{T}\sum_{t=1}^{T} \rho\left(\frac{x'z_t - m}{s}\right) \ge S \tag{2.3}$$

where z_t is the vector of asset returns at time t, T is the sample size, ρ is a convex symmetric loss function, S is the expectation of this loss function evaluated at a standard normal random variable, m is the M-estimator of portfolio return and s is the S-estimator for risk as suggested in DeMiguel and Nogales (2009). For more details about the robust statistics, readers should refer to Maronna et al. (2006) and Huber and Ronchetti (2009). Under these conditions, the portfolio optimization problem (2.1) can be reformulated as robust portfolio optimization. In particular, any optimal robust portfolio can be obtained a solution to the following optimization problem:

$$\begin{aligned} \min_{x} \quad & \frac{q(x'z - z_b)}{v(x'z - z_b)} \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i = 1 \\ & x_i \geq 0;\ i = 1, \ldots, n \\ & \frac{1}{T}\sum_{t=1}^{T} \rho\left(\frac{x'z_t - m}{s}\right) \geq S \\ & \varphi(x) \geq m \end{aligned} \tag{2.4}$$

where $\varphi(x) = x'\bar{\mu} - \kappa\sqrt{x'\Sigma x}$ is the corrected mean and represents an alternative reward measure. In practical applications, $\kappa = 0$ yields the non-robust performance measures, whilst, for the robust versions, we use a significance level of $\eta = 0.95$. This extension allows for the establishment of a simple robust optimization that assumes the true mean lies with a certain confidence and incorporates certain robust estimators known as M- and S-estimators, which have better properties than the classical mean absolute deviation thereby the proposed portfolios are constructed using robust methodologies that are commonly justified from previous studies (Fabozzi et al., 2007; DeMiguel & Nogales, 2009).

3. Empirical Results

This section presents ex-ante empirical analyses and discusses the behavior of robust and non-robust portfolio selection problems on a real data set. For both problems, we consider all active components in the S&P 500 index available in Thomson Reuters DataStream between January 2010 and December 2020 and as a risk-free asset the U.S. 3-Month Treasury Bill. We use a moving average window of 500 trading days (about two years) of observations to compute each optimal portfolio, and we recalibrate the portfolio either monthly or every three months to explore the sensitivity of optimal portfolios to recalibration time (Ortobelli et al., 2019). Further, to guarantee minimal diversification, we invest no more than 20% in a single asset (i.e., $x_i \geq 0.2$; $\forall i = 1, \ldots, n$) (see, e.g., Statman, 2004; Ortobelli et al., 2017). Thus, investing an initial wealth of $W_0 = 1$ on December 1, 2011, we evaluate the out-of-sample performance by optimizing four different drawdown-based performance measures either using robust portfolio optimization (2.4) or non-robust portfolio optimization (2.1).

In particular, at each kth optimization, three steps are performed to compute the ex-ante final wealth:

Step 1: Approximate the expected returns using Fama and French standard five-factor model (Fama & French, 2015), where its data is freely available on the French Data Library website.

Step 2: Determine the optimal portfolio x_M that optimizes a given drawdown-based performance measure, i.e. the solution of optimization (2.4) for robust portfolios or the solution of optimization (2.1) for non-robust versions. In practice, optimization problems (2.1) and (2.4) may present more local optima, as argued by Ortobellli et al. (2019). Therefore, to optimize these performance measures in an acceptable computational time, a heuristic algorithm proposed by Angelelli and Ortobelli (2009) is used as a starting point. Furthermore, the obtained solution is further improved by applying the Matlab heuristic function pattern-search to approximate the global optimum (Ortobelli et al., 2017).

Step 3: Compute the ex-ante final wealth taking into account 20 basis points as proportional transaction costs as suggested in several studies (see, e.g., Ortobelli et al., 2019; Kouaissah et al., 2020).

Steps 1–3 are repeated until all observations are available for every drawdown-based performance measure. Further, we consider the behavior of the S&P 500 index, the equally weighted portfolio (EWP) (i.e. $1/n$ in each component), and the Sharpe ratio during the examined period as the classic benchmarks of the market (Sharpe, 1994; Pflug et al., 2012). The results of this empirical analysis are described in Tables 2–3 and Figures 1–2. Tables 2–3 report five different statistics (mean, standard deviation (SD), value at risk (VaR5%), conditional value at risk (CVaR5%), and final wealth) and two performance measures (Sharpe (mean/SD) and STARR (mean/CVaR5%) ratios) of the ex-ante log-returns of all optimized portfolios. In particular, for simplicity, we denote the robust version of a given drawdown-based performance measure (PM) as R–PM.

Table 2: Statistics of the ex-ante returns obtained by optimizing drawdown-based performance measures with and without robust formulations: monthly recalibration.

PM	Mean	SD	VaR5%	CVaR5%	Final W	Sharpe	STARR
S&P 500	0.0448%	1.093%	1.843%	2.739%	2.8916	4.098%	1.636%
Sharpe	0.0491%	1.168%	1.971%	2.634%	3.1980	4.200%	1.863%
EWP	0.0402%	1.109%	1.864%	2.766%	2.5881	3.621%	1.451%
Robust portfolios							
R–CR	0.0732%	1.255%	2.138%	2.985%	5.6544	5.828%	2.450%
R–SR	0.0589%	1.249%	2.113%	2.934%	4.0344	4.714%	2.007%
R–BR	0.0602%	1.201%	2.036%	2.883%	4.1626	5.013%	2.088%
R–MR	0.0628%	1.205%	2.044%	2.916%	4.4257	5.212%	2.154%
Non-robust versions							
CR	0.0488%	1.214%	2.046%	2.878%	3.1757	4.017%	1.695%
SR	0.0487%	1.159%	1.955%	2.671%	3.1673	4.199%	1.822%
BR	0.0508%	1.169%	1.973%	2.666%	3.3309	4.346%	1.905%
MR	0.0515%	1.188 %	2.007%	2.731 %	3.3817	4.327%	1.883%

Table 3: Statistics of the ex-ante returns obtained by optimizing drawdown-based performance measures with and without robust formulations: three months recalibration.

PM	Mean	SD	VaR5%	CVaR5%	Final W	Sharpe	STARR
S&P 500	0.0448%	1.093%	1.843%	2.739%	2.8916	4.098%	1.636%
Sharpe	0.0367%	1.100%	1.847%	2.745%	2.3872	3.338%	1.338%
EWP	0.0402%	1.109%	1.864%	2.766%	2.5881	3.621%	1.451%
Robust portfolios							
R–CR	0.0579%	1.208%	2.045%	2.941%	3.9384	4.791%	1.968%
R–SR	0.0601%	1.241%	2.102%	2.961%	4.1549	4.844%	2.030%
R–BR	0.0685%	1.156%	1.971%	2.676%	5.0677	5.924%	2.560%
R–MR	0.669%	1.218%	2.070%	2.914%	4.8737	5.491%	2.295%
Non-robust versions							
CR	0.0539%	1.259%	2.125%	3.081%	3.5801	4.276%	1.748%
SR	0.0532%	1.115%	1.888%	2.650%	3.525	4.768%	2.007%
BR	0.0625%	1.152%	1.957%	2.745%	4.3902	5.422%	2.275%
MR	0.0505%	1.130 %	1.909%	2.660 %	3.3053	4.467%	1.897%

From Tables 2–3, we can observe that robust strategies perform much better than their respective versions. In particular, robust strategies present the highest mean return, final wealth, SR, and STARR ratios compared to their respective non-robust strategies, as well as comparable risks (SD, VaR5%, and CVaR5%); thus, robust formulations improve their relatively good out-of-sample performance. These results confirm the superiority of robust portfolio optimization as stressed by several studies (see, e.g., Goldfarb & Iyengar, 2003; DeMiguel & Nogales, 2009). Moreover, the S&P 500, Sharpe, and EWP benchmarks present comparable results to non-robust strategies but lower risk-adjusted returns and final wealth than those based on robust strategies. The empirical investigation also highlights the sensitivity of portfolio strategies to recalibration times, which remains an open question in the financial economics literature. For further confirmation and stress obtained results, Figures 1–2 depict the sample paths of the ex-ante wealth obtained by optimizing two performance measures (MR and BR) that respectively recalibrate the portfolio on a monthly basis and every three months.

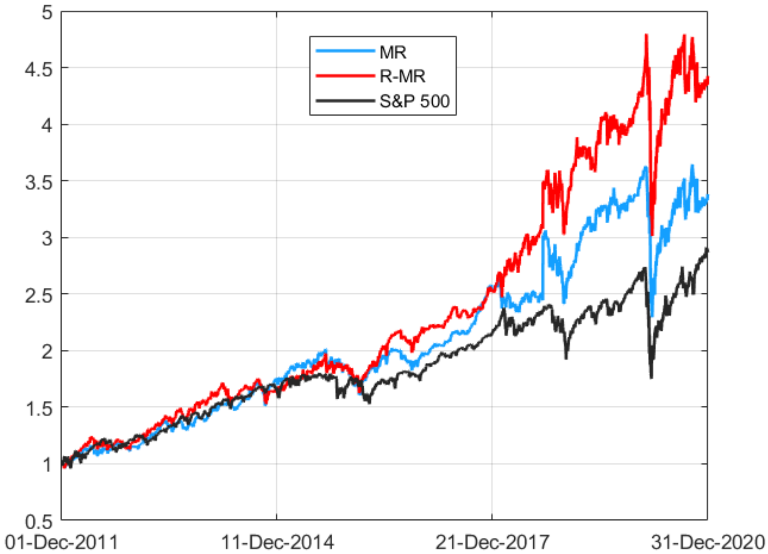

Fig. 1 Ex-ante wealth obtained by maximizing the MR with and without robust approach, compared with the S&P 500 benchmark.

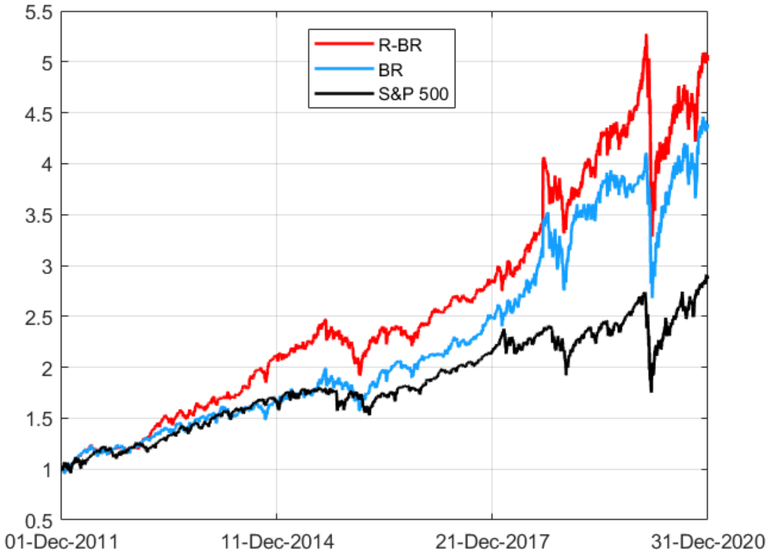

Fig. 2 Ex-ante wealth obtained by optimizing the BR with and robust approach, compared with the S&P 500 benchmark.

According to Figures 1–2, the portfolio strategies based on robust drawdown-based performance measures outperform their respective non-robust versions. Thus, these results confirm and strongly support the assertion that portfolio managers and investors must account for the negative effects of estimation errors through robust methodologies. Furthermore, all examined portfolios show better results than the S&P 500 benchmark. These observations confirm the importance of the proposed robust optimization for drawdown-based performance measures. Performing various types of sensitivity analyses always supports the usefulness of the proposed methodologies

4. Conclusions

The problem of portfolio choice under uncertainty involves unknown parameters that have to be estimated using available economic data. Therefore, in this paper, we develop a robust portfolio optimization for drawdown-based performance measures. In particular, we robustify the drawdown-based performance measures to address the estimation errors directly and explicitly in the portfolio optimization problem itself. To this end, two specific constraints are designed to assume that the true mean lies with a certain confidence and to lessen estimation errors through robust estimation techniques. Empirical analyses on the S&P 500 components confirm that the robust portfolio selection models of drawdown-based performance measures yield the best out-of-sample performance and outperform their respective non-robust versions. Thus, this study supports the importance of robust optimization techniques for portfolio selection problems. A promising direction for future research would be to apply robust and non-parametric statistical methods to portfolio selection problems. In this respect, it has been shown that errors with a heavy-tailed distribution can significantly affect the approximated returns. For this reason, following Ortobelli et al. (2019), we consider a non-parametric regression analysis that relaxes the assumptions of linearity in the financial data set used to estimate the returns and is suitable even for non-Gaussian distributions.

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