Economics Bulletin

Volume 42, Issue 2

Optimal two-part tariff licensing in a Stackelberg duopoly

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Abstract

This article studies the patent licensing of a cost-reducing innovation by an incumbent innovator in a Stackelberg duopoly. We show that two-part ad valorem profit royalty licensing, which is verified to be equal to pure ad valorem profit royalties, is superior to both two-part per-unit royalty and two-part ad valorem revenue royalty licensing offers for the patentee if the innovation is non-drastic and relatively small. Then, both consumer and social welfare are lower under ad valorem profit royalty licensing than under the other two-part royalty licensing offer, as well as under no licensing.

I would like to acknowledge two anonymous referees for valuable comments and suggestions. The author is supported by the Japan Society for the Promotion of Science KAKENHI 20K01619.
Citation: Masashi Umezawa, (2022) "Optimal two-part tariff licensing in a Stackelberg duopoly", *Economics Bulletin*, Volume 42, Issue 2, pages 629-642
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Submitted: July 23, 2021. **Published:** June 30, 2022.

1 Introduction

The licensing of patented technology has played a crucial role in the advancement of technology in recent years. There are many forms of licensing, including fixed fees and/or royalties; thus, patentees must consider which licensing scheme is superior for their patented technology. We investigate the optimal two-part tariff licensing of a cost-reducing innovation by one incumbent producer in an industry to its rival firm in a leadership structure and shed light on the nature of the firms' profitability and the welfare effects of licensing.[1]

In this paper, three types of two-part tariff licensing are examined: two-part per-unit royalty licensing, two-part ad valorem profit royalty licensing, and two-part ad valorem revenue royalty licensing. Two-part tariff licensing is widely observed in practice. For example, in his study of US firms, Rostoker (1984) reports that 46% of licensing contracts use a down payment plus a running royalty, 39% use royalties alone, and 13% use a fixed fee alone. Moreover, in their study of French data, Bousquet et al. (1998) find that 78% of licensing contracts include royalties, and among these 96% are ad valorem royalties, while only 4% are per-unit royalties. Thus, ad valorem royalties have a significant impact on licensing schemes.

Given the wide diffusion of ad valorem royalty contracts in practice, there has been increasing interest in the ad valorem royalties in the literature. Among others, for the case of the incumbent innovator in a Cournot duopoly, San Martín and Saracho (2010) show that a patentee always prefers pure ad valorem royalties to pure per-unit royalties. Other studies on ad valorem royalty contracts in Cournot competition include Niu (2013, 2017), Heywood et al. (2014), San Martín and Saracho (2015), Colombo and Filippini (2016)[2], Hsu et al. (2019), and San Martín and Saracho (2021).

This paper extends the analysis of ad valorem royalties to the case of a Stackelberg duopoly. The analysis of this paper is particularly meaningful for industries that have a dominant innovating firm. For example, privatized firms that were previously run by the government tend to be the leaders as well as the dominant firm in each industry (e.g., gas, electricity, and phone) and new entrants are the followers. When a dominant firm develops a superior technology, it can share the technology innovation with its rivals through a patent license. It takes time for the licensee to introduce the new technology, which causes the licensee to also become a follower after sharing their innovation.

Some papers analyzed the licensing mechanisms of a cost-reducing technology by an internal patentee in a Stackelberg structure. Mukherjee (2001) examines the possibility of technology transfer using fixed fees. Wang and Yang (2004) study optimal licensing under both fixed fees and pure per-unit royalties. Kabiraj (2005) focuses on fixed-fee contracts and pure per-unit royalty contracts, and examines which contract is chosen and how welfare is affected (see also Cao and Kabiraj 2018). In a similar model, Filippini (2005) examines two-part per-unit royalty licensing. Notably, he shows that the constraint that the royalty rate cannot exceed the cost reduction is redundant. That is, two-part per-unit royalty licensing performs better for patentees under the unconstrained solution for the royalty,

[1]This article studies a process innovation (see, e.g., Kamien and Tauman 1986, Katz and Shapiro 1986, Gallini and Winter 1985, Wang 1998, and Sen and Tauman 2007). Other authors consider product innovation (see, e.g., Kitagawa et al. 2014, 2020 and San Martín and Saracho 2016).

[2]They point out the difference between ad valorem profit royalties and ad valorem revenue royalties. As they mention, these schemes are identical when the production costs after the innovation are zero.

compared with the constrained solution in Wang and Yang (2004) and Kabiraj (2005). Li and Yanagawa (2011) extend the analysis of Kabiraj (2005) to the case of differentiated products. Antelo and Bru (2020) study the superiority of pure ad valorem royalty licensing over pure per-unit royalty licensing for the case of innovation such that all firms are active under no licensing. However, none of these contributions consider two-part ad valorem royalty licensing. This paper attempts to provide some useful implications of adopting two-part ad valorem royalties considering the firms' profitability and welfare effects in comparison with the results of Filippini (2005).

Our findings are summarized as follows. First, if the innovation is non-drastic, the patentee prefers licensing the innovation to no licensing, regardless of which of the three licensing schemes is used. Second, a two-part ad valorem profit royalty contract is superior to both two-part per-unit royalty and two-part ad valorem revenue royalty contracts for the patentee if the innovation is non-drastic and relatively small. Patent licensing by means of ad valorem profit royalties allows the patentee to be much less aggressive in increasing the joint profit, which increases the profitability of technology sharing for the patentee. Finally, both consumer and social welfare are lower under two-part ad valorem profit royalty licensing than under the other two-part royalty licensing offer, as well as under no licensing, for a small innovation.

The rest of the paper is organized as follows. In Section 2, we formally present the Stackelberg model with no licensing. In Section 3, we examine optimal two-part tariff licensing and discuss the results including the welfare analysis. Finally, in Section 4, we present our conclusions.

2 The Model

We consider a Stackelberg duopoly industry that produces a homogeneous product, where firms 1 and 2 are the leader and follower, respectively. Let q_i be the quantity produced by firm $i = 1, 2$. The inverse demand function is given by $p(Q) = a - Q$, where p is the price of the product, and $Q = q_1 + q_2$ is the industry output. Under the old technology, each firm produces with a constant marginal cost of production c, where $0 < c < a$. There are no fixed costs of production. We assume that an innovator has a patent on a new technology that reduces the marginal cost of production from c to $c - \varepsilon$, where $0 < \varepsilon \leq c$. Moreover, we assume that the leader (firm 1) is the innovator.

2.1 Case of No Licensing

Let c_i be the constant unit cost of production by firm $i = 1, 2$. The follower chooses q_2 to maximize its profit $\Pi_2 = (a - q_1 - q_2)q_2 - c_2 q_2$. Assuming interior solutions, the first-order condition implies that the follower's best response function is given by $q_2 = (a - c_2 - q_1)/2$. Given the follower's best response, the leader chooses q_1 to maximize its profit $\Pi_1 = (a - q_1 - q_2)q_1 - c_1 q_1$. Then, the resulting Stackelberg equilibrium quantities are $q_1 = (a - 2c_1 + c_2)/2$ and $q_2 = (a - 3c_2 + 2c_1)/4$, and the profits are $\Pi_1 = (a - 2c_1 + c_2)^2/8$ and $\Pi_2 = (a - 3c_2 + 2c_1)^2/16$. As is well known, a cost-reducing innovation is drastic if the monopoly price under the new technology is equal to or less than the preinnovation marginal cost (see Arrow 1962). In the context studied here, $c_1 = c - \varepsilon$ and $c_2 = c$. Hence, the innovation is *drastic* if $\varepsilon \geq a-c$, and it is *non-drastic* if $\varepsilon < a-c$. If a firm's innovation is drastic, then the firm holds a pure monopoly and the rival firm produces zero output. However, we note that a situation can occur in which the innovator produces a positive output and the noninnovator produces zero output, even if the innovation is non-drastic. It follows that firm 2 sets $q_2 = 0$ for $\varepsilon \geq (a-c)/2$. As the follower's best response is $q_2 = (a-c-q_1)/2$, the leader produces only output $q_1 = a-c$ so that the follower produces zero output if $(a-c)/2 \leq \varepsilon < a-c$, where the leader behaves as a restricted monopoly.

Based on the above-mentioned model setup, we consider the Stackelberg equilibrium when licensing of the new technology does not occur. Thus, the equilibrium quantities and the firms' profits are as follows:

$$\left(q_1^N, q_2^N, Q^N, \Pi_1^N, \Pi_2^N\right) = \begin{cases} \left(\frac{a-c+2\varepsilon}{2}, \frac{a-c-2\varepsilon}{4}, \frac{3a-3c+2\varepsilon}{4}, \frac{(a-c+2\varepsilon)^2}{8}, \frac{(a-c-2\varepsilon)^2}{16}\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ (a-c, 0, a-c, \varepsilon(a-c), 0) & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c \\ \left(\frac{a-c+\varepsilon}{2}, 0, \frac{a-c+\varepsilon}{2}, \frac{(a-c+\varepsilon)^2}{4}, 0\right) & \text{if } \varepsilon \geq a-c, \end{cases} \tag{1}$$

where the superscript N denotes the case of no licensing.

When the innovation is drastic (i.e., $\varepsilon \geq a-c$), the innovator (the leader) obtains the monopoly profit of the market without licensing. As it is known that the innovator does not license such an innovation, we focus only on the non-drastic innovation in the following.[3]

3 Patent Licensing

A licensing game consists of three stages. In the first stage, the patentee (firm 1) decides whether to offer a licensing contract or not. When the patentee offers a contract, it chooses the type of contract from a two-part per-unit royalty contract, a two-part ad valorem profit royalty contract, and a two-part ad valorem revenue royalty contract (two-part contract U, V, and R for short, respectively). We assume that the patentee does not offer a contract if its payoff when it does offer such a contract is equal to that when it does not offer it. In the second stage, the potential licensee (firm 2) decides whether to accept the contract from the patentee. Assume that the licensee accepts the licensing contract if and only if its profit with licensing is higher than or equal to its profit without licensing. In the last stage, both firms engage in a Stackelberg quantity competition game. We look for the subgame perfect Nash equilibrium by backward induction. All proofs are given in the Appendix.

3.1 Two-Part Per-Unit Royalty Licensing Mechanism

Under a two-part contract U, the patentee charges a nonnegative fixed fee F plus a nonnegative royalty r per unit of the licensee's output. The results from Filippini's (2005) study of this model are summarized below. Assuming interior solutions, the equilibrium production quantities in the last stage for each firm are $q_1^u = (a-c+\varepsilon)/2$ and $q_2^u = (a-c+\varepsilon-2r)/4$. It is obvious that $q_2^u \geq 0$ if and only if $r \leq (a-c+\varepsilon)/2 \equiv \bar{r}$. In the first stage, the patentee chooses the contract that maximizes the sum of the profit from its own production and the

[3]In the models of San Martín and Saracho (2015) and Colombo and Filippini (2016), the patentee transfers a drastic innovation under a two-part ad valorem royalty contract. It does not occur in our model.

licensing revenue. Let π_2^U be firm 2's profit without the fixed fee to be paid when it accepts a contract, i.e., $\pi_2^U = (a - q_1 - q_2)q_2 - (c - \varepsilon)q_2 - rq_2$. Then, the patentee solves

$$\begin{aligned}&\max_{r,F}\ (a - q_1 - q_2)q_1 - (c - \varepsilon)q_1 + rq_2 + F\\ &\text{subject to } 0 \le r \le \bar{r}, 0 \le F \le \pi_2^U - \Pi_2^N, \text{ and } (q_1, q_2) = (q_1^u, q_2^u).\end{aligned}$$

Thus, we have the following lemma, where Π_i^U denotes firm i's total profit under a two-part contract U (for details, see Filippini 2005).

Lemma 1 *The optimal two-part contract U involves the royalty rate and the fixed fee given by*

$$(r^U, F^U) = \begin{cases} \left(\frac{3\varepsilon}{2}, 0\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ (\bar{r}, 0) & \text{if } \frac{a-c}{2} \le \varepsilon < a - c. \end{cases}$$

Then, the Stackelberg equilibrium quantities and the profits are

$$(q_1^U, q_2^U, Q^U) = \begin{cases} \left(\frac{a-c+\varepsilon}{2}, \frac{a-c-2\varepsilon}{4}, \frac{3(a-c)}{4}\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ \left(\frac{a-c+\varepsilon}{2}, 0, \frac{a-c+\varepsilon}{2}\right) & \text{if } \frac{a-c}{2} \le \varepsilon < a - c \end{cases} \quad \textit{and}$$

$$(\Pi_1^U, \Pi_2^U) = \begin{cases} \left(\frac{(a-c)^2+8(a-c)\varepsilon-2\varepsilon^2}{8}, \frac{(a-c-2\varepsilon)^2}{16}\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ \left(\frac{(a-c+\varepsilon)^2}{4}, 0\right) & \text{if } \frac{a-c}{2} \le \varepsilon < a - c. \end{cases}$$

The patentee prefers a two-part licensing U to no licensing if the innovation is non-drastic.

3.2 Two-Part Ad Valorem Profit Royalty Licensing Mechanism

Under a two-part contract V, the patentee charges a nonnegative fixed fee F plus a nonnegative ad valorem profit royalty $d \in [0, 1]$. In the last stage, the licensee first solves

$$\max_{q_2}\ (1 - d)(a - q_1 - q_2 - (c - \varepsilon))q_2 - F$$

subject to $q_2 \ge 0$. Assuming interior solutions, the first-order condition yields firm 2's reaction function $q_2^V(q_1) = (a - c + \varepsilon - q_1)/2$.

Next, the patentee solves

$$\max_{q_1}\ (a - q_1 - q_2 - (c - \varepsilon))q_1 + d(a - q_1 - q_2 - (c - \varepsilon))q_2 + F$$

subject to $q_1 \ge 0$ and $q_2 = q_2^V(q_1)$. Assuming interior solutions, the equilibrium quantities in the last stage for each firm are $q_1(d) = \frac{(1-d)(a-c+\varepsilon)}{2-d}$ and $q_2(d) = \frac{a-c+\varepsilon}{4-2d}$. Note that $q_2(d) \ge 0$ for any $d \in [0, 1]$. Let Π_i^V denote firm i's total profit under a two-part contract V. Then,

$$\Pi_1^V = \frac{(a - c + \varepsilon)^2}{4(2 - d)} + F,\ \Pi_2^V = \frac{(1 - d)(a - c + \varepsilon)^2}{4(2 - d)^2} - F. \tag{2}$$

Let π_2^V be firm 2's profit without the fixed fee to be paid when it accepts a contract. That is, $\pi_2^V = \frac{(1-d)(a-c+\varepsilon)^2}{4(2-d)^2}$. Then, firm 1's problem is

$$\max_{d,F} \Pi_1^V$$

subject to $0 \le d \le 1$ and $0 \le F \le \pi_2^V - \Pi_2^N$. The solution of this optimization problem establishes the following lemma.

Lemma 2 *The optimal two-part contract V involves the royalty rate and the fixed fee given by*

$$\left(d^V, F^V\right) = \begin{cases} \left(\bar{d}, 0\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ (1, 0) & \text{if } \frac{a-c}{2} \leq \varepsilon < a - c, \end{cases}$$

where $\bar{d} = 2\{-3\varepsilon(2a - 2c - \varepsilon) + (a - c + \varepsilon)\sqrt{3\varepsilon(2a - 2c - \varepsilon)}\}/(a - c - 2\varepsilon)^2$*. Then, the Stackelberg equilibrium quantities and the profits are*

$$\left(q_1^V, q_2^V\right) = \begin{cases} \left(\frac{a-c+\varepsilon-\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{2}, \frac{a-c+\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{4}\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ \left(0, \frac{a-c+\varepsilon}{2}\right) & \text{if } \frac{a-c}{2} \leq \varepsilon < a - c, \end{cases}$$

$$\left(\Pi_1^V, \Pi_2^V\right) = \begin{cases} \left(\frac{(a-c+\varepsilon)^2+(a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{8}, \frac{(a-c-2\varepsilon)^2}{16}\right) & \text{if } \varepsilon < \frac{a-c}{2} \\ \left(\frac{(a-c+\varepsilon)^2}{4}, 0\right) & \text{if } \frac{a-c}{2} \leq \varepsilon < a - c. \end{cases}$$

The patentee prefers two-part licensing V to no licensing if the innovation is non-drastic.

Under this licensing, it is optimal for the patentee to set a fixed fee $F = (1-d)\{p(Q)-(c-\varepsilon)\}q_2-\Pi_2^N$, which makes the licensee indifferent between accepting and rejecting the licensing offer. Then, the profit of the patentee is rewritten as $\Pi_1 = \{p(Q)-(c-\varepsilon)\}q_1+\{p(Q)-(c-\varepsilon)\}q_2-\Pi_2^N = \{p(Q)-(c-\varepsilon)\}Q-\Pi_2^N = \pi_M(Q)-\Pi_2^N$, where $\pi_M(Q)$ is the monopolist's profit at quantity Q under marginal cost $c-\varepsilon$. Let $Q(d)$ be the equilibrium total quantity under ad valorem profit royalty rate d. Then, $\Pi_1(Q(d)) = \pi_M(Q(d))-\Pi_2^N$. Note that $Q(d)$ is more than the monopoly quantity Q_M. As $\pi_M(Q)$ is decreasing for $Q > Q_M$ and $Q(d)$ is decreasing in d, $\Pi_1(Q(d))$ is increasing in d. Therefore, it is optimal for the patentee to choose the maximum possible d, which makes the licensee indifferent between accepting and rejecting the license. Then, the royalty rate d satisfies the condition $(1 - d)\{p(Q) - (c - \varepsilon)\}q_2(d) = \Pi_2^N$, which means that the fixed fee must be zero. When $\Pi_2^N = 0$ (i.e., $(a - c)/2 \leq \varepsilon < a - c$), the condition gives $d = 1$. When $\Pi_2^N > 0$ (i.e., $\varepsilon < (a - c)/2$), there is a unique $\bar{d}$ $(0 < \bar{d} < 1)$ satisfying the condition above by the monotonicity of the expression on the left-hand side with respect to d.

3.3 Two-Part Ad Valorem Revenue Licensing Mechanism

Under a two-part contract R, the patentee charges a nonnegative fixed fee F plus a nonnegative ad valorem revenue royalty $h \in [0, 1]$. In the last stage, the licensee first solves

$$\max_{q_2} \; (1 - h)(a - q_1 - q_2)q_2 - (c - \varepsilon)q_2 - F$$

subject to $q_2 \geq 0$. Assuming interior solutions, the first-order condition yields firm 2's reaction function $q_2^R(q_1) = \{a - c + \varepsilon - ah - (1 - h)q_1\}/\{2(1 - h)\}$.

Next, the patentee solves

$$\max_{q_1} \; (a - q_1 - q_2 - (c - \varepsilon))q_1 + h(a - q_1 - q_2)q_2 + F$$

subject to $q_1 \geq 0$ and $q_2 = q_2^R(q_1)$. Assuming interior solutions, the equilibrium quantities in the last stage for each firm are $q_1 = \frac{a(1-h)^2+(2h-1)(c-\varepsilon)}{(2-h)(1-h)}$ and $q_2 = \frac{a-c+\varepsilon-(a+c-\varepsilon)h}{2h^2-6h+4}$. It is obvious that $q_2 \geq 0$ if and only if $h \leq (a-c+\varepsilon)/(a+c-\varepsilon) \equiv \bar{h}$. Notice that $\bar{h} \leq 1$. Let Π_i^R denote firm i's total profit under a two-part contract R. Then,

$$\Pi_1^R = \frac{-4a(c-\varepsilon)h^2 - (a-5c+5\varepsilon)(a-c+\varepsilon)h + (a-c+\varepsilon)^2}{4(2-h)(1-h)} + F \tag{3}$$

and $\Pi_2^R = \frac{\{a(h-1)+(h+1)(c-\varepsilon)\}^2}{4(2-h)^2(1-h)} - F$. Let π_2^R be firm 2's profit without the fixed fee to be paid when it accepts a contract. That is, $\pi_2^R = \frac{\{a(h-1)+(h+1)(c-\varepsilon)\}^2}{4(2-h)^2(1-h)}$. Then, firm 1's problem is

$$\max_{h,F} \ \Pi_1^R$$

subject to $0 \leq h \leq \bar{h}$ and $0 \leq F \leq \pi_2^R - \Pi_2^N$. The solution of this optimization problem establishes the following lemma.

Lemma 3 *The optimal two-part contract R involves the royalty rate and the fixed fee given by*

$$\left(h^R, F^R\right) = \begin{cases} (h^*, 0) & \text{if } \varepsilon < \frac{a-c}{2} \\ (\bar{h}, 0) & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c, \end{cases}$$

where h^* is the solution of $\pi_2^R = \Pi_2^N (= \frac{(a-c-2\varepsilon)^2}{16})$ such that $h^* < \bar{h}$. The patentee prefers two-part licensing R to no licensing if the innovation is non-drastic.

We find h^* by numerical examples in the Appendix, as in Colombo and Filippini (2016), because we have a third-degree equation in h that is not easy to solve analytically.

Under this licensing, it is optimal for the patentee to set fixed fee $F = (1-h)p(Q)q_2 - (c-\varepsilon)q_2 - \Pi_2^N$, where the licensee is indifferent between accepting and rejecting the licensing offer. Then, the profit of the patentee is rewritten by $\Pi_1 = \{p(Q) - (c-\varepsilon)\}q_1 + p(Q)q_2 - (c-\varepsilon)q_2 - \Pi_2^N = \{p(Q)-(c-\varepsilon)\}Q - \Pi_2^N = \pi_M(Q) - \Pi_2^N$. Let $Q(h)$ be the equilibrium total quantity under ad valorem revenue royalty rate h. Then, $\Pi_1(Q(h)) = \pi_M(Q(h)) - \Pi_2^N$. The quantity $Q(h)$ is more than the monopoly quantity Q_M. As in subsection 3.2, $\Pi_1(Q(h))$ is increasing in h, so it is optimal for the patentee to choose the maximum possible h. The royalty rate h is the one that makes the licensee indifferent between accepting and rejecting the license. Let $q_2(h)$ be the equilibrium quantity of the licensee at h. Then, the maximum possible h satisfies the condition $\{(1-h)p(Q) - (c-\varepsilon)\}q_2(h) = \Pi_2^N$, which means that the fixed fee must be zero. When $\Pi_2^N = 0$ (i.e., $(a-c)/2 \leq \varepsilon < a-c$), a royalty rate $\bar{h}$ satisfying the condition above can be uniquely found. When $\Pi_2^N > 0$ (i.e., $\varepsilon < (a-c)/2$), by the monotonicity of the expression on the left-hand side with respect to h, there is a unique h^* $(0 < h^* < 1)$ satisfying the condition above such that $h^* < \bar{h}$.

3.4 Comparing the Two-Part Tariff Contracts

We now derive the optimal two-part tariff licensing offer for the patentee among the three licensing offers.

Proposition 1 *Suppose that the innovation is non-drastic. If the innovation is small (i.e., $\varepsilon < (a-c)/2$), the patentee prefers two-part licensing V (equal to pure licensing V) to two-part licensing U, as well as to two-part licensing R. Otherwise, the patentee is indifferent among two-part licensing U, two-part licensing V, and two-part licensing R.*

The effect of ad valorem profit royalties on the quantities produced by each firm is given below. The effect on the patentee is negative, because $\partial q_1(d)/\partial d = -(a-c+\varepsilon)/(2-d)^2 < 0$. The effect on the licensee is positive because $\partial q_2(d)/\partial d = (a-c+\varepsilon)/\{2(2-d)^2\} > 0$. Then, we have $|\partial q_1(d)/\partial d| > |\partial q_2(d)/\partial d|$. Therefore, the effect of ad valorem profit royalties on the patentee's output is larger than that on the licensee's output. Eventually, $Q^V < Q^U < Q^N$ is easily confirmed when $\varepsilon < a - c$ (see proof of Proposition 2). That is, because under a contract V, the patentee extracts a portion of the licensee's profit, it is concerned with the licensee's profit. This fully reduces the patentee's incentive to compete. The reduced competition raises the joint profit of the two firms, which increases the profitability of technology sharing for the patentee. This causes the patentee to choose a contract V rather than a contract U even if the latter contract could give the patentee an advantage in terms of marginal cost.

Consider the cases of pure royalty licensing, instead of two-part tariff licensing. Because the fixed fees are not used in Lemmas 1, 2, and 3, we can confirm that the result of Proposition 1 is still unchanged.

Moreover, we evaluate the welfare effects of licensing. Consumer welfare is measured by consumer surplus (CS), and social welfare (SW) is measured by the sum of CS and the firms' profits. Let CS^ℓ and W^ℓ denote CS and SW under each licensing contract ℓ, respectively.

Proposition 2 *Suppose that the innovation is non-drastic. If the innovation is small (i.e., $\varepsilon < (a-c)/2$), we have $CS^V < CS^R < CS^U < CS^N$. Otherwise, we have $CS^V = CS^R = CS^U < CS^N$.*

Proposition 3 *Suppose that the innovation is non-drastic. If the innovation is sufficiently small (i.e., $\varepsilon < (a-c)/7$), we have $W^V < W^R < W^N < W^U$. For the intermediate size of innovation (i.e., $(a-c)/7 < \varepsilon < (a-c)/2$), we have $W^V < W^R < W^U < W^N$. Otherwise, we have $W^V = W^R = W^U < W^N$.*

4 Conclusion

We show that two-part ad valorem profit royalty licensing (equal to the pure licensing) is profitable compared with the other two-part royalties if the innovation is small, in a Stackelberg duopoly model, where the leader is the patentee. Then, both CS and SW are, however, lower under two-part ad valorem profit royalty licensing than under the other licensing offers, as well as under no licensing.

Our model assumes that firm 1, which is the leader prior to the innovation, is the innovator (patentee). However, it is also worth considering alternative situations in which firm 2 (the follower) is now the innovator. This raises the possibility of some Stackelberg game scenarios, depending on which firm is the leader in each stage, both before and after licensing the innovation. This can be open to debate considering the actual market. However, it may be natural to assume that a more cost-efficient firm becomes a leader in a market. Under this assumption, firm 2 becomes a leader after the innovation when it does not license the innovation. Moreover, firm 2 can play the leader role also after licensing if it takes time for firm 1 to adopt the new technology. This means that our analysis can also be applied to this case. Then, firm 2 also chooses the ad valorem profit royalty licensing scheme if the innovation is small.

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Appendix

Proof of Lemma 2

The solution of the optimization problem implies that the optimal contract (d, F) satisfies $F = \pi_2^V - \Pi_2^N \equiv F(d)$. Then, from (1), we have

$$F(d) = \begin{cases} \frac{(1-d)(a-c+\varepsilon)^2}{4(2-d)^2} - \frac{(a-c-2\varepsilon)^2}{16} & \text{if } \varepsilon < \frac{a-c}{2} \\ \frac{(1-d)(a-c+\varepsilon)^2}{4(2-d)^2} & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c. \end{cases} \tag{4}$$

In both cases of (4), $\partial F(d)/\partial d = d(a-c+\varepsilon)^2/\{4(-2+d)^3\} \leq 0$ when $0 \leq d \leq 1$. Therefore, $F(d)$ is always maximized when $d = 0$. Moreover, we can confirm that $F(0) > 0$.

First, consider the case of $\varepsilon < (a-c)/2$. We have $F(1) = -(a-c-2\varepsilon)^2/16 < 0$. Thus, there is $\bar{d}$ such that $F(\bar{d}) = 0$ and $0 < \bar{d} < 1$. A simple calculation reveals that

$$\bar{d} = \frac{2\left\{-3\varepsilon(2a-2c-\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}\right\}}{(a-c-2\varepsilon)^2}.$$

Hence, the inequality constraint $F(d) \geq 0$ implies that $0 \leq d \leq \bar{d}$. It follows from (2) and (4) that

$$\Pi_1^V = \frac{(a-c+\varepsilon)^2}{4(2-d)} + \frac{(1-d)(a-c+\varepsilon)^2}{4(2-d)^2} - \frac{(a-c-2\varepsilon)^2}{16}.$$

Therefore, $\partial \Pi_1^V / \partial d = (1-d)(a-c+\varepsilon)^2/\{2(2-d)^3\}$, which implies that $\partial \Pi_1^V/\partial d = 0$ if $d = 1$ and that $\partial \Pi_1^V/\partial d > 0$ if $0 < d < 1$. Moreover, when $d = 0$,

$$\Pi_1^V = \frac{2(a-c+\varepsilon)^2 + (a-c+\varepsilon)^2 - (a-c-2\varepsilon)^2}{16} > 0,$$

because $a-c+\varepsilon > a-c-2\varepsilon > 0$. Thus, the solution of firm 1's optimization problem is $d = \bar{d}$. Then, $F = 0$. Thus, we have $q_1 = \{a-c+\varepsilon - \sqrt{3\varepsilon(2a-2c-\varepsilon)}\}/2$, $q_2 = \{a-c+\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)}\}/4$, and $Q = \{3(a-c+\varepsilon) - \sqrt{3\varepsilon(2a-2c-\varepsilon)}\}/4$. Then, $\Pi_1^V = \{(a-c+\varepsilon)^2 + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}\}/8$ and $\Pi_2^V = \Pi_2^N$. Moreover, we have $\Pi_1^V - \Pi_1^N = \{-\varepsilon(2a-2c+3\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}\}/8$. Let $\alpha \equiv \varepsilon(2a-2c+3\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)} > 0$. Multiplying both sides of the above-mentioned equation by 8α, we have

$$\begin{aligned} 8\alpha(\Pi_1^V - \Pi_1^N) &= \varepsilon\left\{6(a-c)^3 + 5(a-c)^2\varepsilon - 12(a-c)\varepsilon^2 - 12\varepsilon^3\right\} \\ &= \varepsilon\left[(a-c-2\varepsilon)\{6(a-c)^2 + 17\varepsilon(a-c) + 22\varepsilon^2\} + 32\varepsilon^3\right] > 0, \end{aligned}$$

because $\varepsilon < (a-c)/2$. Thus, $\Pi_1^V > \Pi_1^N$.

Next, consider the case of $(a-c)/2 \leq \varepsilon < a-c$. We have $F(1) = 0$. Thus, $F(d) \geq 0$ implies that $0 \leq d \leq 1$. Then, from (2) and (4),

$$\Pi_1^V = \frac{(a-c+\varepsilon)^2}{4(2-d)} + \frac{(1-d)(a-c+\varepsilon)^2}{4(2-d)^2}.$$

As with the case of $\varepsilon < (a-c)/2$, it is confirmed that $\partial \Pi_1^V/\partial d = 0$ if $d = 1$ and that $\partial \Pi_1^V/\partial d > 0$ if $0 < d < 1$. When $d = 0$, $\Pi_1^V = (a-c+\varepsilon)^2/16 > 0$. Thus, the solution of firm 1's optimization problem is $d = 1$. Then, $F = 0$. Thus, we have $q_1 = 0$, $q_2 = (a-c+\varepsilon)/2$, and $Q = (a-c+\varepsilon)/2$. Then, $\Pi_1^V = (a-c+\varepsilon)^2/4$ and $\Pi_2^V = \Pi_2^N = 0$. Thus, $\Pi_1^V - \Pi_1^N = (a-c+\varepsilon)^2/4 - \varepsilon(a-c) = (a-c-\varepsilon)^2/4 > 0$ for $(a-c)/2 \leq \varepsilon < a-c$.

Proof of Lemma 3

The solution of the optimization problem implies that the optimal contract (h, F) satisfies $F = \pi_2^R - \Pi_2^N \equiv F(h)$. Then, from (1), we have

$$F(h) = \begin{cases} \frac{\{a(h-1)+(h+1)(c-\varepsilon)\}^2}{4(2-h)^2(1-h)} - \frac{(a-c-2\varepsilon)^2}{16} & \text{if } \varepsilon < \frac{a-c}{2} \\ \frac{\{a(h-1)+(h+1)(c-\varepsilon)\}^2}{4(2-h)^2(1-h)} & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c. \end{cases} \quad (5)$$

In both cases of $F(h)$, $\partial F(h)/\partial h = \frac{a^2(h-1)^2h + 2a(h-1)^2(h+4)(c-\varepsilon) + (h+1)(h(h+5)-8)(c-\varepsilon)^2}{4(h-2)^3(h-1)^2} = 0$ generates three solutions h_1, h_2, h_3 such that $h_1 \leq h_2 \leq 1 \leq h_3$ and $h_2 = \bar{h}$, because the numerator of $\partial F(h)/\partial h$ is $\{(a+c-\varepsilon)h - (a-c+\varepsilon)\}\{(a+c-\varepsilon)h^2 + (-a+5c-5\varepsilon)h - 8(c-\varepsilon)\}$. When $h = 0$, we have

$$F(0) = \begin{cases} \frac{3\varepsilon(2a-2c-\varepsilon)}{16} > 0 & \text{if } \varepsilon < \frac{a-c}{2} \\ \frac{(a-c+\varepsilon)^2}{16} > 0 & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c, \end{cases}$$

because $2a - 2c - \varepsilon > 3(a-c)/2 > 0$ if $\varepsilon < (a-c)/2$. Moreover,

$$F(\bar{h}) = \begin{cases} -\frac{(a-c-2\varepsilon)^2}{16} < 0 & \text{if } \varepsilon < \frac{a-c}{2} \\ 0 & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c. \end{cases}$$

This implies that when $\varepsilon < (a-c)/2$, there exists h^* such that $F(h^*) = 0$ and $0 \leq h^* \leq \bar{h}$. Thus, the constraints $F \geq 0$ and $0 \leq h \leq \bar{h}$ generate a constraint $0 \leq h \leq h^*$ if $\varepsilon < (a-c)/2$ and a constraint $0 \leq h \leq \bar{h}$ if $(a-c)/2 \leq \varepsilon < a-c$. By (5), (3) becomes

$$\Pi_1^R = \begin{cases} -\frac{\{2ah-3(a-c+\varepsilon)\}\{a+(2h-1)(c-\varepsilon)\}}{4(h-2)^2} - \frac{(a-c-2\varepsilon)^2}{16} & \text{if } \varepsilon < \frac{a-c}{2} \\ -\frac{\{2ah-3(a-c+\varepsilon)\}\{a+(2h-1)(c-\varepsilon)\}}{4(h-2)^2} & \text{if } \frac{a-c}{2} \leq \varepsilon < a-c. \end{cases}$$

In both cases of Π_1^R, $\partial\Pi_1^R/\partial h = \frac{(a+3c-3\varepsilon)\{a(h-1)+(h+1)(c-\varepsilon)\}}{2(h-2)^3} = 0$ generates $h = (a-c+\varepsilon)/(a+c-\varepsilon)(=\bar{h})$. Thus, the optimal contract is $(h^*, F(h^*))$, where $F(h^*) = 0$, for $\varepsilon < (a-c)/2$, and $(\bar{h}, F(\bar{h}))$, where $F(\bar{h}) = 0$, for $(a-c)/2 \leq \varepsilon < a-c$.

When $\varepsilon < (a-c)/2$, we find the optimal profit Π_1^R by numerical examples and confirm that $\Pi_1^R > \Pi_1^N$ in the proof of Proposition 1. When $(a-c)/2 \leq \varepsilon < a-c$, it is easily verified that $\Pi_1^R = (a-c+\varepsilon)^2/4 > \varepsilon(a-c) = \Pi_1^N$.

Proof of Proposition 1

I. Comparison between two-part ad valorem profit and two-part per-unit royalty licensing

First, for $\varepsilon < (a-c)/2$, we have

$$\Pi_1^V - \Pi_1^U = \frac{1}{8}\left\{-3\varepsilon(2a-2c-\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}\right\}.$$

Let $\beta \equiv 3\varepsilon(2a-2c-\varepsilon)+(a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}$. Then, $\beta > 0$, because $2a-2c-\varepsilon > 0$ when $\varepsilon < (a-c)/2$. Thus, $8\beta(\Pi_1^V - \Pi_1^U) = 3\varepsilon(2a-2c-\varepsilon)(a-c-2\varepsilon)^2 > 0$. Hence, $\Pi_1^V > \Pi_1^U$.

Next, for $(a-c)/2 \leq \varepsilon < a-c$, $\Pi_1^V = \Pi_1^U = (a-c+\varepsilon)^2/4$.

II. Comparison among three kinds of two-part tariff licensing offers including two-part ad valorem revenue royalty licensing

First, consider the case of $\varepsilon < (a-c)/2$. Figure 1 illustrates the profits of the patentee under three types of contracts as a function of the magnitude of innovation when $a = 10$ and $c = 2$. Furthermore, for other parameters a and c, we have similar figures, although they are not provided here. Moreover, Table I gives numerical examples for $a = 10$. Note that $a > c$ and $c \geq \varepsilon$. It is confirmed from Figure 1 and Table I that $\Pi_1^V > \Pi_1^R > \Pi_1^U > \Pi_1^N$.

Next, consider the case of $(a-c)/2 \leq \varepsilon < a-c$. From the proof of Lemma 3, $\Pi_1^R = (a-c+\varepsilon)^2/4$. Therefore, $\Pi_1^R = \Pi_1^V = \Pi_1^U$.

Proof of Proposition 2

Note that CS increases with the industry output in markets with homogeneous goods. For the case of $\varepsilon < (a-c)/2$, we have $Q^N - Q^U = \varepsilon/2 > 0$. Moreover, $Q^U - Q^V = \frac{-3\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{4} = \frac{(-3\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)})(3\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)})}{4(3\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)})} = \frac{6\varepsilon(a-c-2\varepsilon)}{4(3\varepsilon+\sqrt{3\varepsilon(2a-2c-\varepsilon)})} > 0$. Therefore,

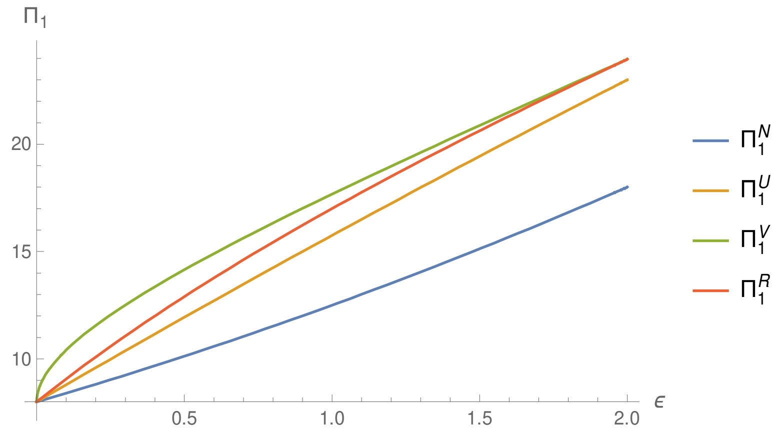

Figure 1: Patentee's profits with small innovation ($a = 10$, $c = 2$)

Table I: Numerical example ($a = 10$)

c	ε	Π_1^N	Π_1^U	Π_1^V	Π_1^R (h^*)
0.001	0.0001	12.498	12.4985	12.5946	12.5902 (0.0146756)
	0.00035	12.4993	12.501	12.6795	12.6763 (0.028061)
	0.0006	12.5005	12.5035	12.7361	12.7342 (0.0369271)
	0.00085	12.5018	12.506	12.7819	12.7812 (0.0440493)
0.01	0.0015	12.4825	12.49	12.8532	12.8144 (0.0520462)
	0.004	12.495	12.515	13.0966	13.0689 (0.0889428)
	0.0065	12.5075	12.5399	13.2711	13.2549 (0.114919)
	0.009	12.52	12.5649	13.4153	13.4107 (0.136086)
0.1	0.01	12.3008	12.3502	13.2305	12.9148 (0.0925325)
	0.035	12.4251	12.5974	14.1271	13.879 (0.2122)
	0.06	12.55	12.8444	14.747	14.5938 (0.292641)
	0.085	12.6756	13.0909	15.261	15.204 (0.357106)
1	0.05	10.3512	10.5744	12.0941	10.9233 (0.0704762)
	0.35	11.7612	13.2444	15.9592	14.8934 (0.382986)
	0.65	13.2613	15.8694	18.6566	18.1242 (0.608495)
	0.95	14.8513	18.4494	21.0453	20.974 (0.801317)
5	0.01	3.15005	3.17498	3.48035	3.17962 (0.00299644)
	0.81	5.47805	7.01098	7.65151	7.1701 (0.222858)
	1.61	8.44605	10.527	10.7213	10.6087 (0.412812)
	2.41	12.0541	13.723	13.725	13.7242 (0.582009)
9	0.02	0.1352	0.1449	0.173996	0.145092 (0.00332956)
	0.17	0.22445	0.287775	0.312401	0.288539 (0.0280647)
	0.32	0.3362	0.4194	0.427344	0.419824 (0.052395)
	0.47	0.47045	0.539775	0.54	0.539792 (0.0763376)
9.9	0.001	0.0013005	0.00134975	0.0015836	0.00134984 (0.000151507)
	0.016	0.002178	0.002786	0.00304469	0.0027867 (0.00242228)
	0.031	0.0032805	0.00410975	0.00419802	0.00411017 (0.00468962)
	0.046	0.004608	0.005321	0.005325	0.00532103 (0.00695354)

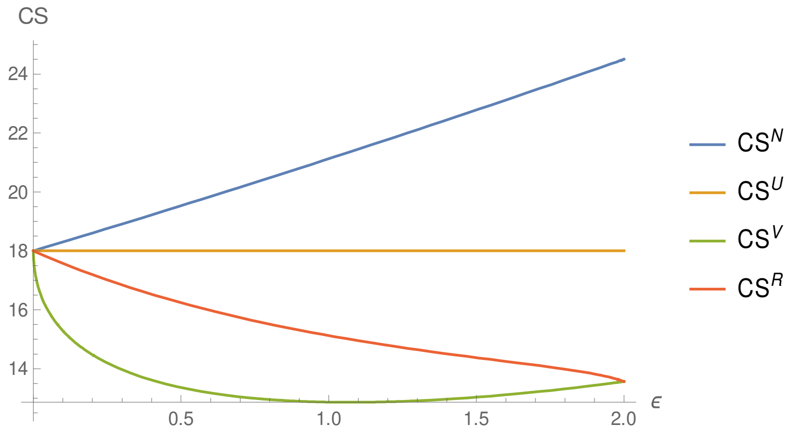

Figure 2: Consumer surplus with small innovation ($a = 10$, $c = 2$)

$Q^V < Q^U < Q^N$, which leads to $CS^V < CS^U < CS^N$. As for the comparison of two-part ad valorem revenue royalty licensing with the other licensing offers, see Figure 2.

For the case of $(a-c)/2 \leq \varepsilon < a-c$, it is obvious that $Q^V = Q^U = (a-c+\varepsilon)/2 < Q^N(= a-c)$. Moreover, we have $q_1^R = (a-c+\varepsilon)/2$ and $q_2^R = 0$, so $Q^R = Q^V = Q^U$, which leads to $CS^R = CS^V = CS^U < CS^N$.

Proof of Proposition 3

We can obtain $W^N = \frac{15}{32}(a-c)^2 + \frac{5}{8}(a-c)\varepsilon + \frac{7}{8}\varepsilon^2$ if $\varepsilon < (a-c)/2$, and $W^N = \frac{1}{2}(a-c)(a-c+2\varepsilon)$ if $(a-c)/2 \leq \varepsilon < a-c$.

First, compare W^V with W^N for the case of $\varepsilon < (a-c)/2$. We have

$$W^N - W^V = \frac{-\varepsilon(2a-2c-5\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{16}.$$

When $2(a-c)/5 \leq \varepsilon < (a-c)/2$, $-\varepsilon(2a-2c-5\varepsilon) \geq 0$, which implies that $W^N - W^V > 0$. Consider now the case of $\varepsilon < 2(a-c)/5$. Let $\gamma_1 \equiv \varepsilon(2a-2c-5\varepsilon) + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}$. Then, $\gamma_1 > 0$. We have $16\gamma_1(W^N - W^V) = \varepsilon[\{2(a-c)-5\varepsilon\}\{3(a-c)^2 + 10(a-c)\varepsilon + 6\varepsilon^2\} + 58(a-c)\varepsilon^2 + 2\varepsilon^3] > 0$. Therefore, $W^N - W^V > 0$.

Second, compare W^V with W^U for the case of $\varepsilon < (a-c)/2$. We have

$$W^U - W^V = \frac{-9\varepsilon^2 + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}}{16}.$$

Let $\gamma_2 \equiv 9\varepsilon^2 + (a-c+\varepsilon)\sqrt{3\varepsilon(2a-2c-\varepsilon)}$. Then, $\gamma_2 > 0$. We have $16\gamma_2(W^N - W^V) = 9\varepsilon^2(a-c)^2 + 6\varepsilon(a-c)^3 - 84\varepsilon^4 = \{(a-c)-2\varepsilon\}\{6(a-c)^2 + 21(a-c)\varepsilon + 42\varepsilon^2\}\varepsilon > 0$. Therefore, $W^U - W^V > 0$.

Third, compare W^U with W^N for the case of $\varepsilon < (a-c)/2$. We have $W^U - W^N = (a-c-7\varepsilon)\varepsilon/8$. Thus, this proposition is confirmed.

Fourth, for the comparison of two-part ad valorem revenue royalty licensing with the other licensing offers in the case of $\varepsilon < (a-c)/2$, see Figure 3.

Finally, consider the case of $(a-c)/2 \leq \varepsilon < a-c$. We have $W^U = W^V = W^R = 3(a-c+\varepsilon)^2/8$. Thus, we have $W^N - W^R = (a-c-\varepsilon)(a-c+3\varepsilon)/8 > 0$.

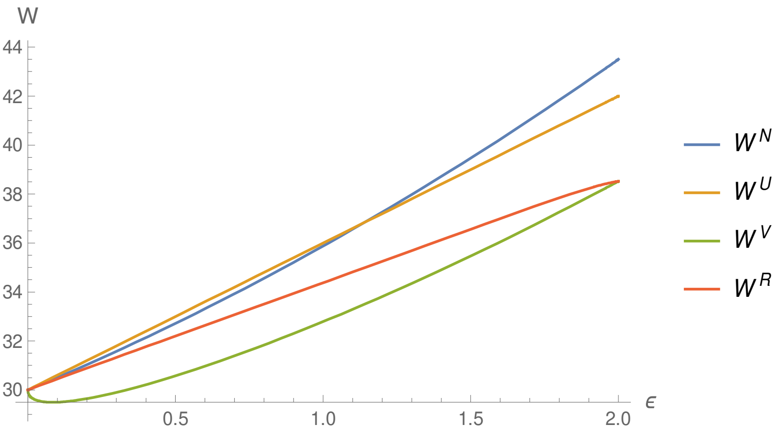

Figure 3: Social welfare with small innovation ($a = 10$, $c = 2$)