Economics Bulletin

Volume 42, Issue 2

A note on Cournot equilibria under incomplete information

Alexey Shvedov
National Research University Higher School of Economics

Abstract

The usual assumptions that underlie the theory of Cournot Bayesian-Nash equilibrium under incomplete information are that the rivals' marginal costs are independently and identically distributed. Using a new mathematical method, this paper shows that the Cournot Bayesian-Nash equilibrium exists under much more general conditions. An expression of equilibrium solutions is presented.

Citation: Alexey Shvedov, (2022) "A note on Cournot equilibria under incomplete information", *Economics Bulletin*, Volume 42, Issue 2, pages 788-792
Contact: Alexey Shvedov - ashvedov@hse.ru.
Submitted: February 11, 2022. **Published:** June 30, 2022.

1. Introduction

The Cournot (1838) model is one of cornerstones of the modern theory of oligopoly. The Cournot competition under incomplete information was studied by many authors. Novshek and Sonnenschein (1982) and Vives (1984) investigated a duopoly model in which firms have private information about an uncertain demand. Gal-Or (1986) and Sakai (1986) analysed the role of information in a duopoly market. Palfrey (1985) considered an oligopoly model where identical firms have private information about an uncertain demand curve and an uncertain marginal cost of production. Cramton and Palfrey (1990) investigated cartel agreements when firms have private information about production costs and considered the Cournot competition. Lepore (2008) studied the Cournot competition as a two-stage game. Yan et al. (2016) compared Bertrand and Cournot competitions under random production quantities. Crespi et al. (2017) represented duopolistic markets as robust games and studied robust-optimization equilibrium. Chatterjee et al. (2019) investigated how the information about the rivals' cost can affect the research and development decision of a firm. Dynamic oligopoly with incomplete information was studied by several authors (see for example Bonatti et al. 2017). Lofaro (2002) computed the Cournot Bayesian–Nash equilibrium under incomplete information about rivals' costs when the inverse demand function and cost functions are linear; but the Lofaro's result is incorrect (see below in Section 3).

The paper is organized as follows. In Section 2, the Cournot Bayesian–Nash equilibrium under incomplete information about rivals' costs is computed. The paper considers only linear inverse demand functions and linear cost functions. This allows to find a closed form solution. But the case is not limited to application of well known mathematical methods. An expression for moments of random variables through quantile functions, which is found recently, is used. In Section 3, the case of equal expected marginal costs is analysed.

2. Cournot equilibria

Suppose that there are n firms in the industry. Each firm produces the quantity q_i of a homogeneous good and incurs linear production costs $c_i q_i$; $i = 1, \ldots, n$. Assume that the inverse demand is

$$P(q_1, \ldots, q_n) = a - b \sum_{i=1}^{n} q_i,$$

where a and b are positive real numbers. A firm's marginal cost c_i is known privately. Other firms know only that the marginal cost c_i is distributed with the expectation μ_i. The only assumption about the joint distribution of firms' costs is that the expectations μ_i exist. The inverse demand function and the form of the cost function is common knowledge.

Firm i's profit is

$$\pi(c_i, q_1, \dots, q_n) = \left(a - b \sum_{j=1}^{n} q_j \right) q_i - c_i q_i.$$

A strategy is a function γ_j such that $\gamma_j(c_j) = q_j$. So, the firm's expected profit is

$$\Pi(c_i, q_i) = \left(a - b \left(q_i + \sum_{j \neq i} E(\gamma_j(C_j)) \right) \right) q_i - c_i q_i, \tag{1}$$

where $E(C_j) = \mu_j$. Recall that the quantile function of the random variable C_j is $r_j(p) = \inf \Xi_p$, where

$$\Xi_p = \{\xi \in \mathbb{R} \colon Prob(C_j \leq \xi) > p\}, \qquad 0 < p < 1.$$

Assume that φ is a Borel function and expectation of the random variable $\varphi(C_j)$ exists. Shvedov (2016) proved that

$$E(\varphi(C_j)) = \int_0^1 \varphi(r_j(p)) dp. \tag{2}$$

A profile $\gamma_1, \dots, \gamma_n$ is called a Bayesian–Nash equilibrium if

$$\Pi(c_i, \gamma_i(c_i)) = \max_{q_i} \Pi(c_i, q_i)$$

for any c_i and any i.

For the sake of brevity we assume that $i = 1$ and drop notation specifying the firm 1. From (1) and (2) we obtain

$$\frac{d}{dq} \Pi(c, q) = -2bq + (a - c) - b \sum_{j=2}^{n} \int_0^1 \gamma(r_j(p)) dp.$$

The first-order condition becomes

$$-2bq + (a - c) = b \sum_{j=2}^{n} \int_0^1 \gamma(r_j(p)) dp.$$

The solution is a point of maximum because $\dfrac{d^2}{dq^2} \Pi(c, q) < 0$. Since γ is an equilibrium strategy we have

$$-2b\gamma(c) + (a - c) = b \sum_{j=2}^{n} \int_0^1 \gamma_j(r_j(p)) dp. \tag{3}$$

Assume that the function γ is continuously differentiable. Then from (3) we obtain

$$-2b\gamma'(c) - 1 = 0.$$

Hence,

$$\gamma(c) = -\frac{c}{2b} + \delta, \tag{4}$$

where δ is a value to be determined. Note that $\delta = \delta_1$. By substituting (4) in (3) we get

$$-2b\left(-\frac{c}{2b} + \delta\right) + (a - c) = b\sum_{j=2}^{n}\int_0^1 \left(-\frac{r_j(p)}{2b} + \delta_j\right) dp.$$

Using (2) with $\varphi(c) = c$, we obtain

$$-2b\delta + a = b\sum_{j=2}^{n}\delta_j - \frac{1}{2}\sum_{j=2}^{n}\mu_j.$$

Therefore

$$-b\delta_i + a = b\sum_{j=1}^{n}\delta_j - \frac{1}{2}\sum_{j\neq i}\mu_j, \qquad i = 1, \dots, n. \tag{5}$$

Denote $\Delta = \sum_{j=1}^{n}\delta_j$. Adding equations (5), we get

$$b(n+1)\Delta = a + \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}\mu_j = a + \frac{1}{2}(n-1)\sum_{i=1}^{n}\mu_i.$$

So,

$$\Delta = \frac{a}{b(n+1)} + \frac{n-1}{2b(n+1)}\sum_{i=1}^{n}\mu_i. \tag{6}$$

From (5) we obtain

$$\delta_i = \frac{a}{b} + \frac{1}{2b}\sum_{j\neq i}\mu_j - \Delta.$$

By substituting (6) we get

$$\delta_i = \frac{1}{b(n+1)}\left(a(n+1) + \frac{n+1}{2}\sum_{j\neq i}\mu_j - a - \frac{n-1}{2}\sum_{j=1}^{n}\mu_j\right) =$$

$$= \frac{1}{b(n+1)}\left(a + \sum_{j\neq i}\mu_j - \frac{n-1}{2}\mu_i\right).$$

Finally,

$$\gamma_i(c_i) = -\frac{c_i}{2b} + \frac{1}{b(n+1)}\left(a + \sum_{j\neq i}\mu_j - \frac{n-1}{2}\mu_i\right). \tag{7}$$

Denote $\lambda_i = \lim\limits_{p\to 1} r_i(p)$. Assume that $\gamma_i(\lambda_i) \geq 0$ for any i. In practice, formula (7) allows to distinguish between the average marginal cost μ_i and the current marginal cost c_i.

3. Equal expected marginal costs

Suppose that $\mu_1 = \ldots = \mu_n = \mu$. It follows from (7) that

$$\gamma(c) = -\frac{c}{2b} + \frac{1}{b(n+1)}\left(a + \frac{n-1}{2}\mu\right). \tag{8}$$

Note that $\gamma(\tilde{c}) = 0$ whenever

$$\tilde{c} = \frac{2}{n+1}\left(a + \frac{n-1}{2}\mu\right). \tag{9}$$

Example 1. Suppose that $r_i(p) \equiv \mu$, $i = 1, \ldots, n$. From (8) it follows that

$$\gamma(\mu) = -\frac{\mu}{2b} + \frac{1}{b(n+1)}\left(a + \frac{n-1}{2}\mu\right) = \frac{1}{b(n+1)}(a-\mu).$$

Example 2. Suppose that the marginal cost C_i is uniformly distributed on $[0, 2\mu]$ for any i. Then $\lambda_i = 2\mu$ for any i. Assume that $a < \dfrac{n+3}{2}$. Then $\tilde{c} < 2\mu$ and $\gamma(2\mu) < 0$. Formally, (8) is valid. However, $\left(a - b\sum_{j=1}^{n} q_j\right) q_i - c_i q_i$ is not firm i's profit when $q_i < 0$. It is reasonable to suppose that $\gamma(c) = 0$ for $c \geq \tilde{c}$. Lofaro (2002) assumes that the marginal costs are independently and uniformly distributed on [0, 1] and considers $a = 1$. Then, using (9), we get

$$\tilde{c} = \frac{n+3}{2(n+1)}.$$

In Lofaro (2002), another value of $\tilde{c}$, which is incorrect, is given. Consequently, Lofaro's Eq. (20) is wrong.

4. Conclusion

In this paper, a new expression of equilibrium solutions for Cournot competitions under incomplete information is presented. The assumptions of independence and identical distribution of marginal costs play no role. The inverse demand function and cost functions are linear. A similar approach may be used when the inverse demand function and cost functions are nonlinear; a generalization of (3) is obvious. However, in general, numerical methods should be used to solve the equation.

References

Bonatti, A., G. Cisternas, and J. Toikka (2017) "Dynamic oligopoly with incomplete information" *Review of Economic Studies* **84**, 503-546.

Chatterjee, R., S. Chattopadhyay, and T. Kabiraj (2019) "R&D in a duopoly under incomplete information" *International Journal of Economic Theory* **15**, 341-359.

Cournot, A. (1838) *Recherches sur les principles mathématiques de la théorie des richesses*, Hachette: Paris. (*Researches into the mathematical principles of the theory of wealth*, Kelley: New York, 1960.)

Cramton, P.C. and T.R. Palfrey (1990) "Cartel enforcement with uncertainty about costs" *International Economic Review* **31**, 17-47.

Crespi, G.P., D. Radi, and M. Rocca (2017) "Robust games: theory and application to a Cournot duopoly model" *Decisions in Economics and Finance* **40**, 177–198.

Gal-Or, E. (1986) "Information transmission – Cournot and Bertrand equilibria" *Review of Economic Studies* **53**, 85-92.

Lepore, J.J. (2008) "Cournot and Bertrand – Edgeworth competition when rivals' costs are unknown" *Economics Letters* **101**, 237-240.

Lofaro, A. (2002) "On the efficiency of Bertrand and Cournot competition under incomplete information" *European Journal of Political Economy* **18**, 561-578.

Novshek, W. and H. Sonnenschein (1982) "Fulfilled expectations Cournot duopoly with information acquisition and release" *Bell Journal of Economics* **13**, 214–218.

Palfrey, T.R. (1985) "Information aggregation and the Cournot competitive limit" *Review of Economic Studies* **52**, 69-83.

Sakai, Y. (1986) "Cournot and Bertrand equilibria under imperfect information" *Journal of Economics* **46**, 213-232.

Shvedov, A.S. (2016) "Quantile function of a fuzzy random variable and an expression for expectations" *Mathematical Notes* **100**, 477-481.

Vives, X. (1984) "Duopoly information equilibrium: Cournot and Bertrand" *Journal of Economic Theory* **34**, 71-94.

Yan, X., Y. Wang, and Z. Hong (2016) "Comparison of Bertrand and Cournot competitions under random yield" *International Journal of Production Research* **54**, 3256-3276.