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Measurement of competitive balance in professional team sports using the adjusted entropy

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Abstract

Competitive balance is an important concept for professional team sports and relates with the balance among the sporting capabilities of teams. Among the different approaches introduced in the literature, indices from the industrial organization theory have been used to measure competitive balance. The attention of this study in on the Entropy Index applied in sports in the form of Relative Entropy (R). The application in English Premier League indicates that, in cases with variant number of teams N, R index leads to misleading interpretation of competitive balance. It is shown that the introduced Adjusted Entropy (AH) remains invariant to changes in N and solves the deficiencies of R.

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1. Introduction

Competitive balance, which is defined as the balance among the sporting capabilities of teams (Michie and Oughton 2004), it is an important concept for professional team sports. The importance of the concept derives from the fact that it creates an uncertainty of outcome, which instigates the interest of fans leading to an increased interest for sport events (El-Hodiri and Quirk, 1971, and Rottenberg 1956). For this reason, there is an interest for competitive balance either for an analysis over seasons or for the effect on fans' behavior (Fort and Maxcy 2003).

However, based on the empirical studies testing for the '*Uncertainty of Outcome Hypothesis'* (*UOH*), the relationship between competitive balance and fans' interest is a matter of debate. Although there are some studies that present a clear positive effect of competitive balance on demand (Humphreys 2002; Lee 2004; Manasis, Ntzoufras, and Reade 2022), there is also a number of studies showing that this effect is either weak or even contradictory (Coates, Humphreys, and Zhou 2014; Pawlowski 2013). The latter has triggered further research studying the subjective evaluation of competitive balance (Pawlowski & Budzniski 2014) and the related concept of competitive intensity (Scelles 2017). Based on a more recent study by Humphreys and Pérez (2019), the *UOH* is also not supported while the presence of dominant teams and the potential of 'historic' upsets emerge as possible determinants of demand.

The above discussion might be related with the measurement issue of competitive balance due to the multidimensionality aspect of the concept. A great diversity of different approaches has been introduced in the literature with a view to better quantifying competitive balance. As Zimbalist (2002) notices, "there are almost as many ways to measure competitive balance as there are to quantify money supply". The current study focuses on the seasonal dimension of competitive balance which relates with teams' performances in the course of a particular season.

Since competitive balance is essentially concerned with inequality of teams' performances, using in this context indices measuring the inequality of income distribution or market power is not surprising. The area of industrial organization theory offers a wide range of indices measuring the relative industry competitiveness. If we consider a professional sport league as an industrial sector, such concentration indices explain the distribution of teams' success in the league. Industrial economists investigate the concentration of output. In the professional sport setting, in which the performance can be measured by the winning percentage or winning share, it has been appeared relative indices such as the *Herfindahl-Hirschman Index* (Depken 1999) and *Gini coefficient* (Schmidt and Berri 2001, 2002).

The attention of our effort is on the *Entropy Index* (*H*), which originates from information theory and industrial organization, applied in sports by Horowitz (1997) as *Relative Entropy* (*R*). The main justification using such measures is their feature in capturing the degree of inequality within in a league. Essentially, the entropy captures the inequality by quantifying the uncertainty of the game's outcome in sporting events (Horowitz 2018). In this context, the more equitable the distribution of games' outcomes or teams' performances, the more balanced would be the competition with the league (Borooah and Mangan 2012). Also, a Generalized Entropy approach has been introduced by Borooah and Mangan (2012) to measure inequality both within and between groups of teams in a league.

Although *R* is one of the first proposed competitive balance indices, it has been rarely used in the literature (Humphreys and Watanabe 2012) or at least not very often (Brandes and Franck 2006).

One of the reasons for the absence of the *R* index in the analysis of competitive balance in professional team sports might be caused by the improper definition of the index's lower bound. In particular, we argue that the lower bound of *R* is not well documented when applied to leagues with different size. The size variation could be observed both across seasons (because of contraction and/or expansion) and across domestic leagues. This variation affects the index's range, which is noticed by Evans (2014) as a deficiency for a proper comparison of competitive balance.

This paper attempts a modification of *H* to offer a proper application when the number of teams varies. The work of Owen and Ryan and Weatherston (2007) by offering the *HHI** as a normalization of the *Herfindahl-Hirschman index* as well as the related work of Utt and Fort (2002) with the *AGINI* as an adjustment of the *Gini coefficient*, signifies the contribution of the proper modification of *H* to the sports economics literature.

In the following, we discuss the application of *H* in measuring competitive balance in professional team sports and the sensitivity of the index's bounds to the variation in the number of teams in the league. In Section 3, it is presented the proposed adjustment of the index which is then applied to the English Premier League (EPL) to illustrate its main qualities. Section 4 concludes with the main remarks.

2. The measurement of competitive balance with *H*

The Entropy index (*H*) has its origin in information theory as a measure of uncertainty (Shannon 1948). The meaning of entropy also relates with the information, or surprise inherent to a variable's possible outcome. Based on Shannon (1948), an analogous definition of entropy is the expected value of the self-information of a variable. The index has been further explored by Theil (1967) to measure economics inequality in industrial organization. According to Simko (2021), *H* is likely the most preferred index of diversity in ecology.

The index has been introduced to sports economics by Horowitz (1997). When applied to sports, *H* measures the probability of occurrence of the winning share of a team *i* in a league and it is given by:

$$H = -\sum_{i=1}^{N} s_i log_2 s_i, \tag{1}$$

where *N* stands for number of teams in the league, and s_i stands for the winning share of the *i*th team. The s_i is defined as the number of wins by a team during a season as a proportion of the total number of wins in the championship. Note that a draw is counted as a half of a win. The upper bound of the index (H_M) equals log_2N and corresponds to the maximum entropy. This is the special case of a perfectly balanced league with equal winning shares (s_i=1/*N*) for each of the *N* teams. A decrease in *H* indicates a decrease in competitive balance.

The H_M is positively related to *N*. This can be shown by derivating H_M with respect to *N*.

$$\frac{\vartheta H_M}{\vartheta N} = \frac{\vartheta log_2 N}{\vartheta N} = \frac{1}{N log 2} \tag{2}$$

The second derivative of H_M in equation (3), shows that H_M increases at a decreasing function of *N*.

$$\frac{\vartheta \frac{1}{Nlog2}}{\vartheta N} = -\frac{1}{N^2 log2} \tag{3}$$

The variation of H_M is presented in Table I and graphically illustrated in Figure 1. More specifically, the percentage difference in H_M is as high as 28.7% when comparing leagues with size 22 and 10 respectively.

After calculating H_M values, Horowitz (1997) proposes the *Relative Entropy* (R) defined by the equation below:

$$R = \frac{H}{H_M} = \frac{-\sum_{i=1}^{N} s_i \, log_2 \, s_i}{log_2 \, N} \tag{4}$$

Essentially, R captures the degree of uncertainty for a team to win a particular game relative to maximum possible uncertainty. Given that R controls for H_M, the upper bound of R is one. Thus, as R decreases, so does the level of competitive balance within the league. Alternatively, if the index approaches the upper bound of unity, the disparity among the teams decreases and the league becomes more balanced.

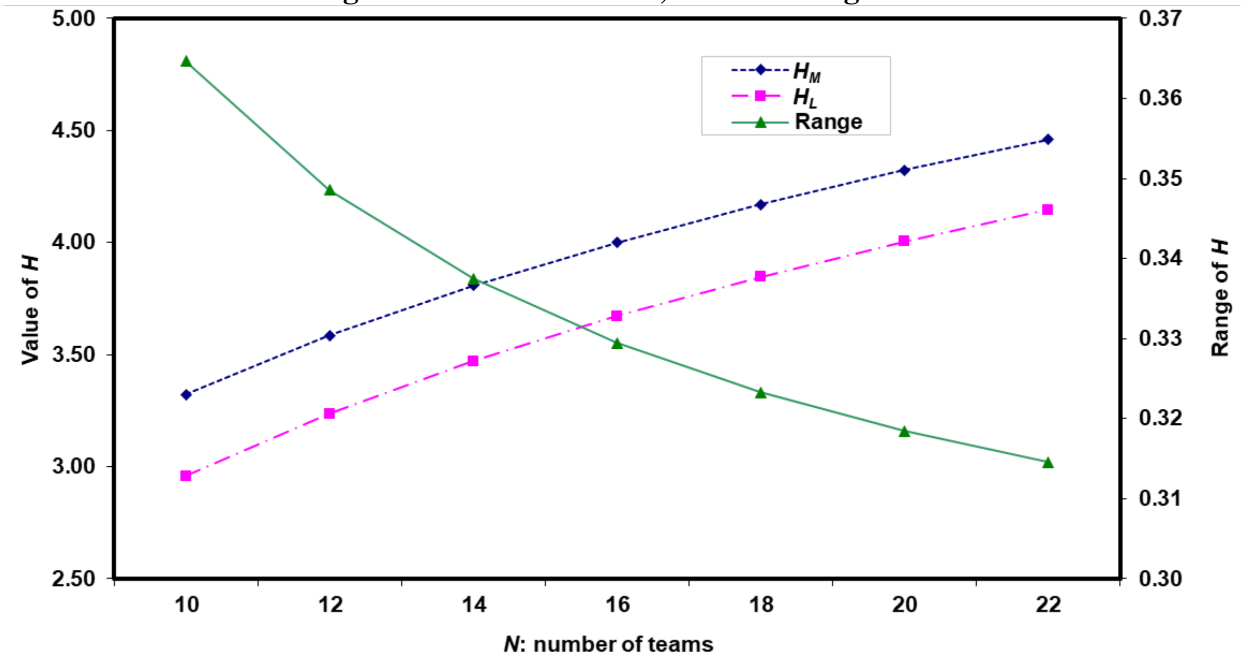
However, R is bounded below by a minimum value that depends on the number of teams in the league (Humphreys and Watanabe 2012). This is true since, when H is applied to sports, varying N not only affects the upper but also the lower bound (H_L). H_L corresponds to the minimum entropy, the extreme case of a completely unbalanced league defined as the case in which the first team wins all games, the second team wins all games against lower teams, and so down to the last team with no wins (Fort and Quirk 1997, and Horowitz 1997, and Utt and Fort 2002, and Owen and Ryan and Weatherston 2007).

Table I: Variation of H_L, H_M and Range of H

N	H_L	*D%*	*CD%*	H_M	*D%*	*CD%*	*Range*	*D%*	*CD%*
10	2.957			3.322			0.365		
12	3.236	0.086	0.086	3.585	0.073	0.073	0.349	0.046	0.046
14	3.470	0.067	0.154	3.807	0.058	0.132	0.338	0.033	0.079
16	3.671	0.055	0.208	4.000	0.048	0.180	0.329	0.024	0.103
18	3.847	0.046	0.254	4.170	0.041	0.221	0.323	0.019	0.122
20	4.003	0.039	0.293	4.322	0.035	0.256	0.318	0.015	0.138
22	4.145	0.034	0.327	4.459	0.031	0.287	0.315	0.012	0.150

D%: Percentage Difference, *CD%:* Cumulative Percentage Difference

Figure 1: Variation of H_L, H_M and Range of H



In such a hypothetical league with a balanced schedule, a team with rank i will win $2(N\text{-}i)$ out of the total $N(N\text{-}1)$ games provided that each team plays with the other twice (as in the case of soccer). For this extreme case, H_L is defined by:

$$H_L = \sum_{i=1}^{N} \frac{2(N-i)}{N(N-1)} log_2 \frac{2(N-i)}{N(N-1)} \tag{5}$$

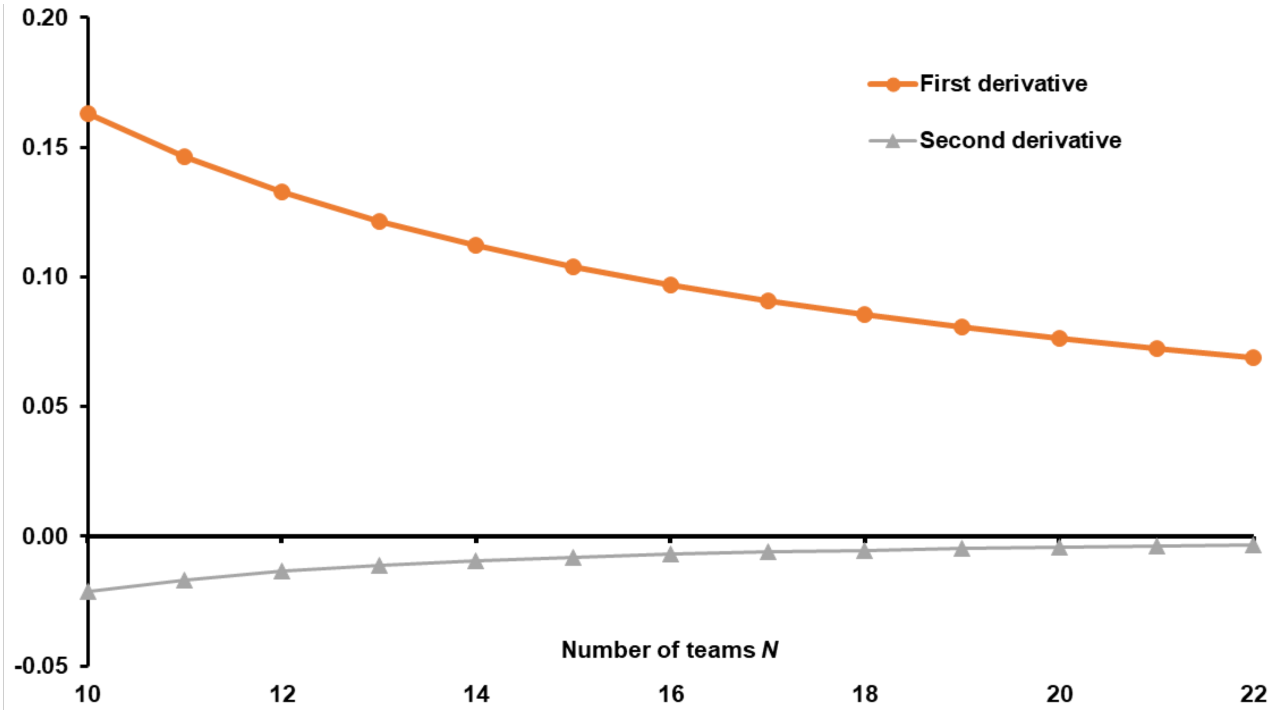
Based on the results in Figure 2 (from Matlab), the first and second derivatives of H_L with respect to N signify that H_L increases at the decreasing rate with variation in N. As presented in Table I and illustrated in Figure 1, the percentage increase of H_L (for realistic values of N) may rise to 32.7%. This difference is numerically considerable; and thus, it should be taken under consideration in any analysis of competitive balance using the R index.

Since for well-defined index both bounds should be well documented, R index as defined in equation (4), cannot be employed for a reliable comparison of competitive balance amongst seasons or countries with different N. As Evans (2014) notes, R is not an appropriate index to compare leagues with different size since the range of the index is dependent on the number of teams. The variation of the range of H is also presented in Table I and graphically depicted in Figure 1.

3. The *Adjusted Entropy* (*AH*)

Based on the above discussion, both bounds (lower H_L and upper H_M) of H are affected by the variation in N. Following the procedure offered by Manasis *et al.* (2011) and Owen, Ryan and Weatherston (2007), a two-step adjustment of *Entropy index* (H) is attempted:

Figure 2: First and second derivatives of H_L with respect to N



a) For a reliable calculation of the index, a point of reference is created. Hence, H_M is chosen as a benchmark from which H is subtracted. By choosing H_M as a benchmark, the boundaries match those of other conventional indices: *HHI** (Owen and Ryan and Weatherston 2007), *NAMSI* (Goosens 2006) and *AGINI* (Utt and Fort 2002).
b) The value of the index, which is re-located to zero, must be controlled for the variability in both bounds. Intuitively, this can be accomplished by dividing with the feasible range of the index.

The ratio of the above two conditions provide the *Adjusted Entropy* (*AH*), which is given by:

$$AH = \frac{H_M - H}{H_M - H_L} = \frac{log_2 N - H}{log_2 N - \sum_{i=1}^{N} \frac{2(N-i)}{N(N-1)} log_2 \frac{2(N-i)}{N(N-1)}} \quad (6)$$

The value of *AH* ranges from zero to one. Those two extremes correspond to cases of perfect competitive balance and complete imbalance respectively. The major advantage of this index is that is not affected by variations in *N* and can be easily interpreted and contrasted against other related indices.

The implementation of the discussed indices in EPL can illustrate the effect of league size variation in the analysis of competitive balance. During the 60 football seasons investigated (1959/60-2018/19), the number of teams *N* in EPL ranges from 20 to 22 as shown in Table II. Note that, in European soccer, *N* varies across seasons and countries usually from 10 (e.g., Switzerland and

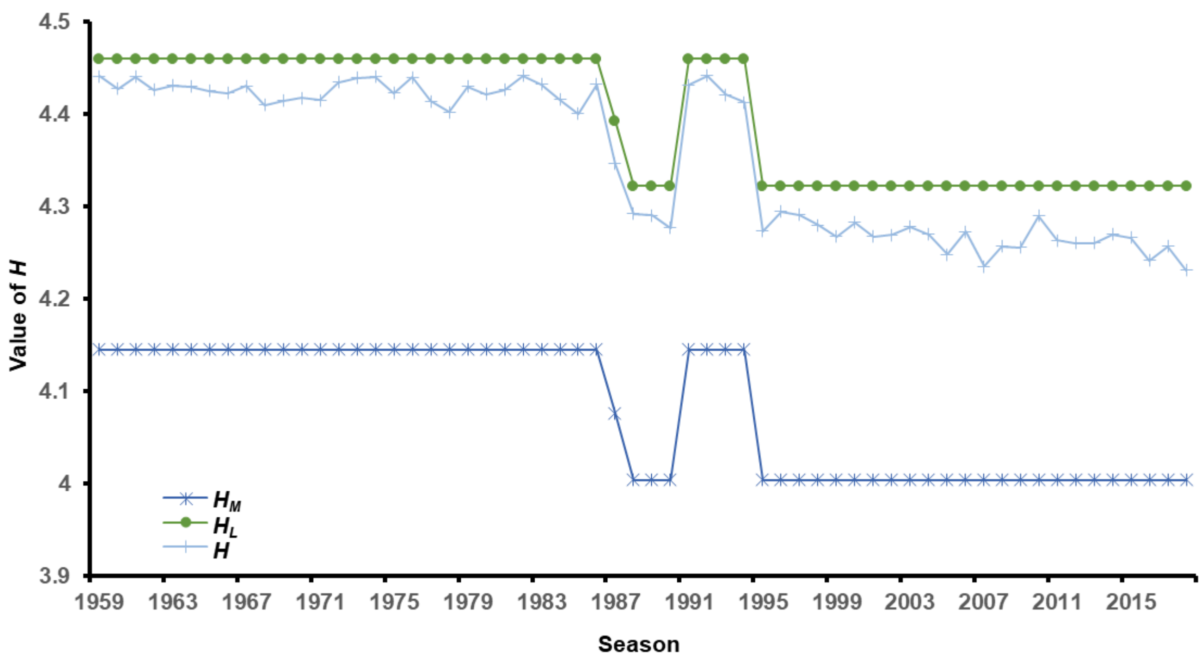
Albania) to 22 (only in EPL in the past). As in the case of EPL, the current league size in France, Italy and Spain is 20.

Table II. The number of teams (N) in EPL

	League size		
Seasons	20	21	22
1959/60 – 1986/87			•
1987/88		•	
1988/89 – 1990/91	•		
1991/92 – 1994/95			•
1995/98 – 2018/19	•		

Figure 3 shows that the calculated values of H in EPL are much closer to the upper bound H_M, which stands for the case of a perfectly balanced league. Based on equation (4), as Goosens (2006) also points out, a problem for R is that its value is always very close to unity when there are many teams in the league. Additionally, as observed in Figure 3, the pattern variation in H is similar with this of H_M and H_L. The above verify that both H_M and H_L should be taken into consideration when measuring competitive balance over time.

Figure 3: H, H_L and H_M values in EPL from 1959/60 – 2018/19



When comparing EPL seasons with different N, there are quite a few cases of percentage differences in which R index displays the same sign with AH leading to misleading perception of competitive balance behavior. Note that all these cases concern seasons with different N (as shown in Table II) some of which are presented in Table III. For instance, from 1985/86 to 2011/12 season, competitive balance as measured with AH and R indices, improves 1.92% and worsens

0.033% respectively. More impressive are the comparison results for seasons 1990/91 and 1994/95, in which competitive balance worsens 4.08% using *AH* but improves 0.04% according to *R*.

The behavior of the two indices can be also examined with trend analysis of competitive balance over the seasons. The examination of a deterministic trend using the regression analysis is allowed following the unit root test results. More specifically, using the augmented Dickey-Fuller (ADF) test for nonstationarity, the null hypothesis for integration one ($I(1)$) is rejected for both indices. According to test results in Table IV, trend coefficient for *AH* is much larger than this for *R*. Overall, *R* underestimates the worsening of competitive balance through seasons in EPL. Both percentage difference and trend results are robust to the proportion of points collected by a team to the total number of points in the league. After the adoption of the current point system (starting season 1981/92, three points for a win, one for a draw and zero for a loss), the calculated share of points account for the ratio of the total number of draws to the total number of wins in the league.

Table III. Seasons' comparison for *AH* and *R*

Seasons	*D%*	
	AH	*R*
1969/70 vs. 1990/91	-0.79%	-0.04%
1977/78 vs. 1990/91	-1.30%	-0.03%
1987/88 vs. 1990/91	-1.99%	-0.02%
1987/88 vs. 1994/95	2.01%	0.01%
1990/91 vs. 1994/95	4.08%	0.03%
1984/85 vs. 2003/04	-0.19%	-0.04%
1985/86 vs. 2011/12	-1.92%	-0.03%

D%: Percentage Difference

Table IV. Test for unit root and trend in *AH* and *R*

	AH	*R*
ADF[1]	-4.29*	-1.82*
Trend[2]	0.0017*	-0.0002*
	(0.0004)	(0.00002)

[1]ADF test statistic for the null hypothesis of a unit root based on a regression with a trend a constant term.
[2]The trend coefficient in regression with a constant and trend term.
*Denote statistical significance at the 1% level of significance based on asymptotic critical values from MacKinnon (1991).
Numbers in parentheses are standard errors.

4. Conclusion

The main issue addressed in this article was to develop an adjustment of the *Entropy Index* (*H*) for a reliable measurement of competitive balance in leagues with different number of teams (*N*). The *H* is a concentration index derived from the industrial organization to capture the degree of inequality in professional team sports in the form of the *Relative Entropy* (*R*). We argue that the application of *R* to team sports is problematic since the index does not account for the lower bound (H_L) which is a function of the variation in *N*. More specifically, it is shown that H_L increases at a decreasing rate as *N* increases. Such an increase may rise to 32.7% for realistic values of *N*. This deficiency can lead to misleading results when dealing with changes in *N* over time or across leagues. This weakness of *R*, one for the first proposed indices in the literature, might be the reason for its infrequent use in the sport setting as noticed by Humphreys and Watanabe (2012) and Brandes and Franck (2006).

As a solution to a proper calculation of competitive balance, the *Adjusted Entropy* (*AH*) is introduced. For the development of *AH*, both a point of reference and a relocation to zero are created for the *H*. This method takes also into consideration the H_L. The justification for the inclusion of the H_L in the calculation of *AH* is that in team sports, in contrast to a broad monopolized industry, a dominant team cannot win all league games (e.g., games that the team does not participate). Both bounds of the *AH* remain invariant to changes in *N* and the index can solve the deficiencies of *R*.

The application in the English Premier League (EPL) shows that, based on trend results, *R* underestimates the degree of deterioration of competitive balance across seasons. An indication of inaccurate perception of competitive balance behavior is also derived from the examination of percentage differences between seasons with different *N*. There are observed quite a few cases in which *R* display opposite change direction as compared with *AH*.

Following that, we argue that new proposed *AH* index can capture more effectively the degree of inequality within leagues with variant *N* and it can be easily interpreted and contrasted against other related indices when measuring the seasonal dimension of competitive balance. This is of greatest importance for studies focusing on either the analysis of competitive balance behavior across seasons or the examination of the *UOH*.

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