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R&D rivalry with the interaction of output subsidies in an extensive endogenous timing game

Jiaqi Chen
Chonnam National University

Sang-ho Lee
Chonnam National University

Abstract

This study investigates strategic interplay between government's output subsidies and firms' R&D rivalry in an extensive endogenous timing game. We find that research spillovers are crucial in determining multiple equilibria of the game, which yields different welfare consequences. We show that a simultaneous-move game appears at equilibrium if the spillovers rate is extremely low, but it is always socially undesirable. We also show that the government plays as a leader or a follower at equilibrium, while it could cause welfare loss unless spillovers rate is either high or sufficiently low. Our findings suggest that the appropriate role of the government in providing output subsidies should be based on the rate of research spillovers when the firms strategically choose their endogenous timings of R&D activities.

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Contact: Jiaqi Chen - chenjiaqi@naver.com, Sang-ho Lee - sangho@jnu.ac.kr.
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1. Introduction

A number of studies have examined the strategic interactions between output subsidies and firms' R&D activities, and revealed that research spillovers are critical to assess the welfare effect of government intervention. For example, Leahy and Neary (1997) addressed the strategic relationship between optimal output (or R&D) subsidies and firms' R&D investments. Lee (1998) found the role of research spillovers in the mechanism design of output subsidy policy under asymmetric information. Poyago-Theotoky and Teerasuwannajak (2002) and Lee and Park (2021) analysed the environmental tax (or output subsidy) policy toward firm's investment on abatement technology in private and mixed markets, respectively.[1]

In a formal theoretical operation with regulatory frameworks between the government and firms' R&D rivalry, it is traditional to assume that a firm's decision is sequentially finalized after subsidy policy realization. That is, the output subsidy rate is exogenously fixed when firms determine their R&D decisions in a committed policy setting where the government credibly commits to its policy rule and subsidy rate. As Leahy and Neary (1997) and Chen et al. (2022) highlighted, on the other hand, opportunistic decision in a different timing is critical to economic performance if the firm determines its R&D before the government announces its optimal policy rate. That is, firms could induce the government to adjust the output subsidy rate in a way that favors them in a time-consistency framework.[2] However, previous works have considered a fixed timing relation between output subsidies and firms' R&D investments.

This study investigates strategic relations between government's output subsidies and duopolistic firms' R&D rivalry in an endogenous timing game with research spillovers. We consider an observable delay game with a three-period and three-player model, extending the formulation of the two-period and two-player model by Hamilton and Slutsky (1990) in a homogeneous duopolistic competition. In the presence of research spillovers, each firm determines its cost-reducing R&D investment while the government ascertains the output subsidy rate individually, either simultaneously or sequentially. We adopt the analysis of Amir et al. (2000) in an endogenous decisions of R&D timing game between the firms where both firms compete in a Cournot fashion in the last stage of output choices.[3] This structural enhancement of the model allows us to anticipate when the government or the firm is likely to play either a leader or a follower in decision-making in the interplay between R&D investments and output subsidies.

We show that research spillovers are crucial in determining multiple equilibria of the game, which provides different welfare consequences. First, a simultaneous-move game appears at

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[1] Haruna and Goel (2017), and Lee and Muminov (2021a, 2021b) emphasised the role of public institutions in the presence of research spillovers and examined the effect of output subsidies on R&D investments in mixed markets.
[1] Some related studies on the time-consistent policy can be also found in the recent literature such as Leal et al. (2018) and Garcia et al. (2018).
[2] Some related studies on the time-consistent policy can be also found in the recent literature such as Leal et al. (2018) and Garcia et al. (2018).
[3] Using the framework in Amir et al. (2000), Leal et al. (2021) examined a private duopoly with corporate social responsibility and Lee and Muminov (2021b) analyzed a mixed duopoly with a public firm. They all showed that simultaneous choice of R&D appears when spillovers rate is low, while sequential choice of R&D appears when spillovers rate is high.

equilibrium if the spillovers rate is extremely low, but it is always socially undesirable. Second, the government followership (as a last mover) with either a simultaneous-move or a sequential-move between firms appears unless the spillovers rate is low, but it is socially desirable only when the spillovers rate is sufficiently high. Third, the government leadership (as a first mover) with a simultaneous-move between firms appears if the spillovers rate is intermediate, but it is socially desirable only when the spillovers rate is low enough. Thus, if the spillovers rate is not extremely low, the government plays a leader or a follower at equilibrium, while it could cause welfare loss unless spillovers are either sufficiently high or low enough but not extremely low. Our findings suggest that the appropriate role of the government in providing output subsidies should be based on the rate of research spillovers when the firms strategically choose their endogenous timings of R&D activities.

2. The Model

We consider a duopoly market in which two firms (1 and 2) produce homogeneous goods. The inverse demand function is linear, $P = a - Q$ where P is the market price, $Q = q_1 + q_2$ is the market total output, and q_i is the output of firm *i=1,2*. Then, consumer surplus is $CS = \frac{Q^2}{2}$. Following d'Aspremont et al. (1988) in a standard model of cost-reducing R&D investment with research spillovers, we assume that the cost functions in output production and R&D investment are ex-ante identical between the firms and are given as:

$$C(q_i, x_i) = (c - x_i - \beta x_j)q_i \text{ and } \Gamma(x_i) = x_i^2, \quad for\ i = 1,2 \text{ and } i \neq j. \tag{1}$$

where x_i is the outcome of R&D investment for firm i and $\beta \in [0,1]$ is the rate of spillovers. The initial cost c reduces according to each firm's R&D outcome, x_i, and the opponent's R&D outcome, βx_j, where $a > c > 0$. The firm must spend $\Gamma(x_i) = x_i^2$ to implement cost-reducing R&D investment that causes decreasing returns to scale.

We also assume that each firm is granted an output subsidy, $s > 0$, which is the per-unit subsidy rate to output, financed by the government. Then, the profit function of the firm is:

$$\pi_i = (a - q_i - q_j)q_i - (c - x_i - \beta x_j)q_i - x_i^2 + s\, q_i, \ for\ i = 1,2 \text{ and } i \neq j. \tag{2}$$

Social welfare is the sum of consumer surplus and firms' profit minus total subsidy:

$$W = CS + \pi_1 + \pi_2 - s(q_1 + q_2). \tag{3}$$

We consider a multi-stage game in which both the government and the firms first determine their output subsidy rate and cost-reducing R&D investments, respectively, either simultaneously or sequentially, given the rate of spillovers, and subsequently, firms play in Cournot competition in the last output stage. We solve the subgame perfect Nash equilibrium of these games by backward induction.

3. The Analysis

In the analysis, we extend an observable delay game in a two-period and two-player framework formulated by Hamilton and Slutsky (1990) to a three-period and three-player framework in which the government (G) and both firms (firm F1 and firm F2) choose their timing to move among $T_k = 1,2,3$ where $k = G, F1, F2$ in determining their output subsidy and R&D choices, respectively. If all players choose the same period, the equilibrium of a simultaneous-move game is yielded. If all players select a different period, the equilibrium of a successive sequential-move game is yielded. Otherwise, the various equilibria of both simultaneous-move and sequential-move games with leadership or followership emerges. Table 1 illustrates the matrix of the game.

Table 1: Matrix of an endogenous timing game

Government	$T_G = 1$			$T_G = 2$			$T_G = 3$		
Firm1 \ Firm2	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$	$T_{F1} = 1$	$T_{F1} = 2$	$T_{F1} = 3$
$T_{F2} = 1$	(1,1,1)	(1,2,1)	(1,3,1)	(2,1,1)	(2,2,1)	(2,3,1)	(3,1,1)	(3,2,1)	(3,3,1)
$T_{F2} = 2$	(1,1,2)	(1,2,2)	(1,3,2)	(2,1,2)	(2,2,2)	(2,3,2)	(3,1,2)	(3,2,2)	(3,3,2)
$T_{F2} = 3$	(1,1,3)	(1,2,3)	(1,3,3)	(2,1,3)	(2,2,3)	(2,3,3)	(3,1,3)	(3,2,3)	(3,3,3)

In Table 1, the order of parentheses indicates the timing to move of each player, that is, (T_G, T_{F1}, T_{F2}) where $T_k = 1,2,3$ and $k = G, F1, F2$. There are 27 subgames in choosing the timing of movement in total among three-player and three-timing. Due to its symmetry, we can reduce the total number to eight cases as follows:

(1) Case I: a simultaneous-move game: (T_G, T_{F1}, T_{F2})={(1,1,1), (2,2,2), and (3,3,3)}
(2) Case II: a simultaneous-move game between the government and one firm, and subsequent sequential-move game with the other firm's followership: (T_G, T_{F1}, T_{F2})={(1,1,2), (1,1,3), (2,2,3); (1,2,1), (1,3,1), (2,3,2)}
(3) Case III: the government's leadership with a simultaneous-move game between the firms: (T_G, T_{F1}, T_{F2})={(1,2,2), (1,3,3), (2,3,3)}
(4) Case IV: a successive sequential-move game with the government's leadership: (T_G, T_{F1}, T_{F2})={(1,2,3), (1,3,2)}
(5) Case V: the government's followership with s simultaneous-move game between the firms: (T_G, T_{F1}, T_{F2})={(2,1,1), (3,1,1), (3,2,2)}
(6) Case VI: one firm's leadership with a simultaneous-move game between the government and the other firm: (T_G, T_{F1}, T_{F2})={(2,1,2), (3,1,3), (3,2,3); (2,2,1), (3,3,1), (3,3,2)}
(7) Case VII: a successive sequential-move game with the government's intermediation: (T_G, T_{F1}, T_{F2})={(2,1,3), (2,3,1)}
(8) Case VIII: a successive sequential-move game with the government followership: (T_G, T_{F1}, T_{F2})={(3,1,2), (3,2,1)}

In Appendix A, we examine the equilibrium outcomes with respect to output subsidy, R&D investments, outputs, profits, and social welfare in each case. Note that in the last stage, both firms decide on their outputs simultaneously, given the output subsidy rate and R&D investments. The first order condition of the firm yields the following equilibrium output:[4]

$$q_i = \frac{1}{3}(a - c + s + (2 - \beta)x_i + (2\beta - 1)x_j) \text{ where } i = 1,2 \quad (4)$$

Then, equilibrium output increases as either output subsidy or its own R&D increases while it decreases as its rival's R&D increases if $\beta < 0.5$. Thus, the spillovers rate is crucial in determining the firm's equilibrium output.

We compare equilibrium outcomes and find equilibrium outcomes of an endogenous timing game between both firms and the government. We first compare the profits of the firms, given the choice of the government, and find equilibrium choices of $(\mathrm{T_G}, \mathrm{T_{F1}}, \mathrm{T_{F2}})$ in the subgames.[5]

Lemma 1.

(1) *Suppose* $T_G = 1$. *Then, the equilibrium outcomes between the firms are (1,1,1) if* $0 \leq \beta \leq 0.221$; *(1,2,2), (1,3,3) if* $0.221 < \beta \leq 0.5$; *either (1,2,3) or (1,3,2) if* $0.5 < \beta \leq 1$.

(2) *Suppose* $T_G = 2$. *Then, the equilibrium outcomes between the firms are (2,1,1) if* $0 \leq \beta \leq 0.236$; *(2,1,1), (2,3,3) if* $0.236 < \beta \leq 0.238$ *(2,3,3) if* $0.238 < \beta \leq 1$.

(3) *Suppose* $T_G = 3$. *Then, the equilibrium outcomes between the firms are (3,1,1) if* $0 \leq \beta \leq 0.333$; *(3,1,2), (3,2,1) if* $0.333 < \beta \leq 1$.

Lemma 1 shows that the government activity in different timings of equilibria depends on the rate of spillovers. Note that neither Case II nor Case VI, VII can be an equilibrium.

We then compare the welfare levels of $\mathrm{W}(\mathrm{T_G}, \mathrm{T_{F1}}, \mathrm{T_{F2}})$, given the firms' choices of timing, and examine the government decision of timing.

Lemma 2.

(1) *Suppose that* $T_{F1} = T_{F2} = 1$. *Then,* $W(1,1,1) \gtreqless W(2,1,1) = W(3,1,1)$ *if* $\beta \lesseqgtr 0.267$;

(2) *Suppose that* $T_{F1} = 1, T_{F2} = 2$. *Then,* $W(1,1,2) \gtreqless W(2,1,2)$ *if* $\beta \lesseqgtr 0.293$; $W(2,1,2) \gtreqless W(3,1,2)$ *if* $\beta \lesseqgtr 0.269$; $W(1,1,2) \gtreqless W(3,1,2)$ *if* $\beta \lesseqgtr 0.278$;

(3) *Suppose that* $T_{F1} = 1, T_{F2} = 3$. *Then,* $W(1,1,3) \geq W(2,1,3)$ *if* $0.113 \leq \beta \leq 0.288$; $W(2,1,3) > W(3,1,3)$ *for all* $\beta \in [0,1]$.

(4) *Suppose that* $T_{F1} = 2, T_{F2} = 1$. *Then,* $W(1,2,1) \gtreqless W(2,2,1)$ *if* $\beta \lesseqgtr 0.293$; $W(1,2,1) \gtreqless W(3,2,1)$ *if* $\beta \lesseqgtr 0.278$; $W(2,2,1) \gtreqless W(3,2,1)$ *if* $\beta \lesseqgtr 0.269$;

(5) *Suppose that* $T_{F1} = T_{F2} = 2$. *Then,* $W(2,2,2) \gtreqless W(3,2,2)$ *if* $\beta \lesseqgtr 0.267$; $W(1,2,2) \gtreqless W(3,2,2)$ *if* $\beta \lesseqgtr 0.274$; $W(1,2,2) \geq W(2,2,2)$ *for all* $\beta \in [0,1]$;

(6) *Suppose that* $T_{F1} = 2, T_{F2} = 3$. *Then,* $W(2,2,3) \gtreqless W(3,2,3)$ *if* $\beta \lesseqgtr 0.293$; $W(1,2,3) \gtreqless W(3,2,3)$ *if* $\beta \lesseqgtr 0.297$; $W(1,2,3) \geq W(2,2,3)$ *for all* $\beta \in [0,1]$.

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[4] Note that the second-order and stability conditions are satisfied.

[5] Note that the threshold of the spillovers rate for each equilibrium outcome is represented by numbers with three decimal places for expositional convenience. Proofs of lemmas and propositions are provided in Appendix B.

(7) *Suppose that* $T_{F1} = 3, T_{F2} = 1$. *Then,* $W(1,3,1) \gtreqless W(3,3,1)$ *if* $\beta \lesseqgtr 0.293$*;* $W(1,3,1) \geq W(2,3,1)$ *if* $0.113 \leq \beta \leq 0.288$*;* $W(2,3,1) \geq W(3,3,1)$ *for all* $\beta \in [0,1]$*;*
(8) *Suppose that* $T_{F1} = 3, T_{F2} = 2$. *Then* $W(1,3,2) \gtreqless W(3,3,2)$ *if* $\beta \lesseqgtr 0.297$*;* $W(2,3,2) \gtreqless W(3,3,2)$ *if* $\beta \lesseqgtr 0.293$*;* $W(1,3,2) \geq W(2,3,2)$ *for all* $\beta \in [0,1]$*;*
(9) *Suppose that* $T_{F1} = T_{F2} = 3$. *Then,* $W(1,3,3), (2,3,3) \geq W(3,3,3)$ *for all* $\beta \in [0,1]$.

Using Lemmas 1 and 2, we obtain the following proposition.

Proposition 1. *The equilibrium outcome of an endogenous timing game is as follows:*
(1) *Case I is an equilibrium if* $0 \leq \beta \leq 0.221$.
(2) *Case III is an equilibrium if* $0.221 < \beta \leq 0.5$.
(3) *Case V is an equilibrium if* $0.267 < \beta \leq 0.333$.
(4) *Case VIII is an equilibrium if* $0.333 < \beta \leq 1$.

Proposition 1 shows that a simultaneous-move game (Case I) appears if the spillovers rate is sufficiently low ($0 \leq \beta \leq 0.221$), while the government leadership (as a first mover) with a simultaneous-move game between the firms (Cases III) appears if the spillovers rate is intermediate ($0.221 < \beta \leq 0.5$).[6] It implies that if the spillovers rate is not sufficiently low, the commitment of the output subsidy policy is attainable in an equilibrium while both firms determine R&D investments simultaneously after observing government policy. These findings support the traditional approach in the optimal subsidy policy wherein a firm's decision is sequentially finalized after subsidy policy realization. That is, the output subsidy rate is exogenously fixed when firms determine their R&D decisions in a committed policy setting where the government credibly commits to its policy rule and subsidy rate.

However, unless the spillovers rate is sufficiently low ($0.267 < \beta \leq 1$), the government followership (as a last mover) either with a simultaneous-move game between the firms (Case V if $0.267 < \beta \leq 0.333$) or with a sequential game between firms (Case VIII if $0.333 < \beta \leq 1$) appears This implies that the government does not necessarily commit to the output subsidy rate but chooses the policy option opportunistically after observing firms' R&D investments unless the spillovers rate is sufficiently low. Thus, a time-inconsistency problem in the commitment on government output subsidy policy could occur wherein a different timing between the two competing firms appear either simultaneously or sequentially. Subsequently, a sequential-move game between firms under the time-consistent government policy yields that a leading firm provides a higher output and R&D investment with reduced profit. Therefore, our analysis highlights the feasible role of government policy to improve welfare if the firms determines the strategic timing of R&D activities while the spillovers rate is crucial in the endogenous timing game.

4. Welfare Comparisons

We now compare the welfare consequences of the equilibrium outcomes. We first compare the

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[6] In the absence of the government, Amir et al. (2000) also showed that simultaneous choice of R&D appears when spillovers rate is below 0.5, while sequential choice of R&D appears otherwise.

welfare levels of the eight possible cases in the above analysis.

Lemma 3. *The welfare comparisons present the followings:*
(1) Case III provides the highest welfare if $0 \leq \beta \leq 0.233$*;*
(2) Case IV provides the highest welfare if $0.233 < \beta \leq 0.278$*;*
(3) Case V provides the highest welfare if $0.278 < \beta \leq 0.333$*;*
(4) Case VIII provides the highest welfare if $0.333 < \beta \leq 1$.

Lemma 3 shows that only Cases III, IV, V, and VIII are socially desirable but Case I is not. Thus, from Proposition 1, a simultaneous-move game is always socially undesirable even though it is an equilibrium if the spillovers rate is extremely low. Furthermore, Case IV is socially desirable but it is not an equilibrium.

Using Proposition 1 and Lemma 3, we obtain the following proposition:

Proposition 2. *The equilibrium outcome of an endogenous timing game is socially desirable only when (i) Case III if* $0.221 < \beta \leq 0.223$: *(ii) Case V if* $0.278 < \beta \leq 0.333$: *and (iii) Case VIII if* $0.333 < \beta \leq 1$.

Proposition 2 show that the government leadership with a simultaneous-move between firms (Case III) appears if the spillovers rate is intermediate, but it is socially desirable only when the spillovers rate is low enough, while the government followership with either a simultaneous-move (Case V) or a sequential-move (Case VIII) between firms appears unless the spillovers rate is sufficiently low, but it is not always socially desirable. Therefore, if the spillovers rate is not extremely low, the government plays a leader or a follower at equilibrium, while it could cause a welfare loss unless spillovers are either high or sufficiently low. Therefore, the appropriate role of the government in providing output subsidies should be based on the rate of research spillovers when the firms strategically choose their endogenous R&D timings.

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Appendix A.: Equilibrium outcomes

In the last stage, we can obtain the equilibrium outputs from Eq.(4). Using them, we can examine a fixed timing game in each case among the government who maximizes Eq.(3) and the firms where each maximizes Eq.(2), simultaneously or sequentially. Then, we can provide the following equilibrium outcomes in each case.

Case I. $s^I = \frac{3(a-c)}{4-\beta+\beta^2}$, $x_1^I = x_2^I = \frac{(a-c)(2-\beta)}{4-\beta+\beta^2}$, $q_1^I = q_2^I = \frac{3(a-c)}{4-\beta+\beta^2}$, $\pi_1^I = \pi_2^I = \frac{(a-c)^2(5+4\beta-\beta^2)}{(4-\beta+\beta^2)^2}$, $W^I = \frac{2(a-c)^2(5+4\beta-\beta^2)}{(4-\beta+\beta^2)^2}$.

Case II. $s^{II} = \frac{(a-c)(12+107\beta-3\beta^2-36\beta^3+8\beta^4)}{24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7}$

$x_i^{II} = \frac{(a-c)(16+36\beta-42\beta^2+36\beta^3-27\beta^4+9\beta^5-\beta^6)}{24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7} \geq x_j^{II} = \frac{(a-c)(2-\beta)(4+39\beta-30\beta^2+17\beta^3-7\beta^4+\beta^5)}{24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7}$

$q_i^{II} = \frac{(a-c)(20+71\beta+5\beta^2-34\beta^3+11\beta^4-\beta^5)}{24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7} \geq q_j^{II} = \frac{3(a-c)(4+39\beta-30\beta^2+17\beta^3-7\beta^4+\beta^5)}{24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7}$

$\pi_i^{II} = \frac{(a-c)^2(4+11\beta-7\beta^2+\beta^3)^2(9+56\beta-14\beta^2+8\beta^3-7\beta^4+4\beta^5-\beta^6)}{(24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7)^2}$

$\pi_j^{II} = \frac{(a-c)^2(5+4\beta-\beta^2)(4+39\beta-30\beta^2+17\beta^3-7\beta^4+\beta^5)^2}{(24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7)^2}$ where $\pi_i^{II} \gtrless \pi_j^{II}$ if $\beta \lessgtr 0.5$.

$W^{II} = \frac{(a-c)^2(704+7296\beta+17756\beta^2-19630\beta^3+7889\beta^4+4456\beta^5-10209\beta^6+8800\beta^7-5200\beta^8+2120\beta^9-532\beta^{10}+72\beta^{11}-4\beta^{12})}{2(24+108\beta-109\beta^2+82\beta^3-44\beta^4+23\beta^5-8\beta^6+\beta^7)^2}$

Case III. $s^{III} = \frac{(a-c)(7+11\beta-5\beta^2)}{2(7-7\beta+4\beta^2)}$, $x_1^{III} = x_2^{III} = \frac{3(a-c)(2-\beta)}{2(7-7\beta+4\beta^2)}$, $q_1^{III} = q_2^{III} = \frac{9(a-c)}{2(7-7\beta+4\beta^2)}$, $\pi_1^{III} = \pi_2^{III} = \frac{9(a-c)^2(5+4\beta-\beta^2)}{4(7-7\beta+4\beta^2)^2}$, $W^{III} = \frac{9(a-c)^2}{2(7-7\beta+4\beta^2)}$

Case IV. $s^{IV} = \frac{(a-c)(20+323\beta+1534\beta^2+522\beta^3-1181\beta^4+959\beta^5-807\beta^6+443\beta^7-179\beta^8+50\beta^9-6\beta^{10})}{52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10}}$

$x_i^{IV} = \frac{(a-c)(8+49\beta-9\beta^2+2\beta^3-2\beta^4)(4+10\beta-8\beta^2+7\beta^3-5\beta^4+\beta^5)}{52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10}}$

$x_j^{IV} = \frac{(a-c)(2-\beta)(8+129\beta+441\beta^2-309\beta^3+202\beta^4-106\beta^5+25\beta^6-8\beta^7+2\beta^8)}{52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10}}$

$q_i^{IV} = \frac{(a-c)(40+397\beta+934\beta^2+77\beta^3-361\beta^4+86\beta^5-35\beta^6+16\beta^7-2\beta^8)}{52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10}}$

$q_j^{IV} = \frac{3(a-c)(8+129\beta+441\beta^2-309\beta^3+202\beta^4-106\beta^5+25\beta^6-8\beta^7+2\beta^8)}{52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10}}$

$\pi_i^{IV} = \frac{(a-c)^2(9+56\beta-14\beta^2+8\beta^3-7\beta^4+4\beta^5-\beta^6)(8+73\beta+130\beta^2-74\beta^3+13\beta^4-8\beta^5+2\beta^6)^2}{(52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10})^2}$

$\pi_j^{IV} = \frac{(a-c)^2(5+4\beta-\beta^2)(8+129\beta+441\beta^2-309\beta^3+202\beta^4-106\beta^5+25\beta^6-8\beta^7+2\beta^8)^2}{(52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10})^2}$

$W^{IV} = \frac{(a-c)^2(8+49\beta-9\beta^2+2\beta^3-2\beta^4)^2}{2(52+566\beta+1017\beta^2-1630\beta^3+1737\beta^4-1482\beta^5+1102\beta^6-558\beta^7+210\beta^8-60\beta^9+8\beta^{10})}$

Note that (i) $q_i^{IV} \geq q_j^{IV}$, $x_i^{IV} \geq x_j^{IV}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{IV} \gtrless \pi_j^{IV}$ if $\beta \lessgtr 0.5$.

Case V. $s^{V} = \frac{4(a-c)}{5-2\beta+\beta^2}$, $x_1^{V} = x_2^{V} = \frac{(a-c)(3-\beta)}{5-2\beta+\beta^2}$, $q_1^{V} = q_2^{V} = \frac{4(a-c)}{5-2\beta+\beta^2}$, $\pi_1^{V} = \pi_2^{V} = \frac{(a-c)^2(7+6\beta-\beta^2)}{(5-2\beta+\beta^2)^2}$,
$W^{V} = \frac{2(a-c)^2(7+6\beta-\beta^2)}{(5-2\beta+\beta^2)^2}$

Case VI. $s^{VI} = \frac{(a-c)(-9-56\beta+10\beta^2-10\beta^3+5\beta^4)}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$, $x_i^{VI} = -\frac{2(a-c)(5+14\beta-4\beta^2+11\beta^3-10\beta^4+2\beta^5)}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$,
$x_j^{VI} = \frac{2(a-c)(2-\beta)(-1-11\beta+6\beta^2-6\beta^3+2\beta^4)}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$, $q_i^{VI} = -\frac{2(a-c)(6+23\beta+8\beta^2-8\beta^3+\beta^4)}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$,
$q_j^{VI} = \frac{6(a-c)(-1-11\beta+6\beta^2-6\beta^3+2\beta^4)}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$, , $\pi_i^{VI} = -\frac{4(a-c)^2(-1-3\beta+\beta^2)^2}{-11-70\beta+28\beta^2-18\beta^3+19\beta^4-16\beta^5+4\beta^6}$,
$\pi_j^{VI} = \frac{4(a-c)^2(5+4\beta-\beta^2)(1+11\beta-6\beta^2+6\beta^3-2\beta^4)^2}{(11+70\beta-28\beta^2+18\beta^3-19\beta^4+16\beta^5-4\beta^6)^2}$,
$W^{VI} = -\frac{4(a-c)^2(-16-250\beta-1101\beta^2-88\beta^3+285\beta^4-462\beta^5+505\beta^6-456\beta^7+279\beta^8-80\beta^9+8\beta^{10})}{(11+70\beta-28\beta^2+18\beta^3-19\beta^4+16\beta^5-4\beta^6)^2}$

Note that (i) $q_i^{VI} \geq q_j^{VI}$, $x_i^{VI} \geq x_j^{VI}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VI} \gtrless \pi_j^{VI}$ if $\beta \lessgtr 0.405$.

Case *VII*. $s^{VII} = \frac{(a-c)(-33-276\beta-338\beta^2-9\beta^3+138\beta^4-127\beta^5+105\beta^6-36\beta^7+4\beta^8)}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$x_i^{VII} = -\frac{(a-c)(1+\beta)^2(36+87\beta-64\beta^2+94\beta^3-121\beta^4+61\beta^5-13\beta^6+\beta^7)}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$x_j^{VII} = \frac{(a-c)(2-\beta)(1+\beta)(-12-81\beta+45\beta^2-34\beta^3-17\beta^4+30\beta^5-10\beta^6+\beta^7)}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$q_i^{VII} = \frac{(a-c)(-48-252\beta-290\beta^2+89\beta^3+68\beta^4-61\beta^5+35\beta^6-10\beta^7+\beta^8)}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$q_j^{VII} = \frac{3(a-c)(-12-93\beta-36\beta^2+11\beta^3-51\beta^4+13\beta^5+20\beta^6-9\beta^7+\beta^8)}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$\pi_i^{VII} = -\frac{(a-c)^2(4+15\beta+4\beta^2-6\beta^3+\beta^4)^2}{-63-324\beta-30\beta^2+252\beta^3-210\beta^4+70\beta^5-64\beta^6-2\beta^7+28\beta^8-10\beta^9+\beta^{10}}$

$\pi_j^{VII} = \frac{(a-c)^2(5-\beta)(1+\beta)^3(12+81\beta-45\beta^2+34\beta^3+17\beta^4-30\beta^5+10\beta^6-\beta^7)^2}{(63+324\beta+30\beta^2-252\beta^3+210\beta^4-70\beta^5+64\beta^6+2\beta^7-28\beta^8+10\beta^9-\beta^{10})^2}$

$W^{VII} = ((a-c)^2(1+\beta)^2(-4968-46818\beta-97857\beta^2+101760\beta^3+19075\beta^4-183264\beta^5+209551\beta^6-191234\beta^7+118300\beta^8-38386\beta^9+21894\beta^{10}-24982\beta^{11}+15529\beta^{12}-5168\beta^{13}+964\beta^{14}-96\beta^{15}+4\beta^{16}))/(-2(63+324\beta+30\beta^2-252\beta^3+210\beta^4-70\beta^5+64\beta^6+2\beta^7-28\beta^8+10\beta^9-\beta^{10})^2)$

Note that (i) $q_i^{VII} \geq q_j^{VII}$, $x_i^{VII} \geq x_j^{VII}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VII} \gtrless \pi_j^{VII}$ if $\beta \lessgtr 0.395$.

Case VIII. $s^{VIII} = \frac{(a-c)(-11-90\beta+20\beta^2-22\beta^3+7\beta^4)}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$, $x_i^{VIII} = -\frac{4(a-c)(3+11\beta-4\beta^2+12\beta^3-7\beta^4+\beta^5)}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$

$x_j^{VIII} = \frac{2(a-c)(3-\beta)(-1-14\beta+9\beta^2-8\beta^3+2\beta^4)}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$, $q_i^{VIII} = -\frac{2(a-c)(7+34\beta+16\beta^2-10\beta^3+\beta^4)}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$

$q_j^{VIII} = \frac{8(a-c)(-1-14\beta+9\beta^2-8\beta^3+2\beta^4)}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$, $\pi_i^{VIII} = -\frac{4(a-c)^2(-1-4\beta+\beta^2)^2}{-13-108\beta+54\beta^2-36\beta^3+43\beta^4-24\beta^5+4\beta^6}$

$\pi_j^{VIII} = \frac{4(a-c)^2(7+6\beta-\beta^2)(1+14\beta-9\beta^2+8\beta^3-2\beta^4)^2}{(13+108\beta-54\beta^2+36\beta^3-43\beta^4+24\beta^5-4\beta^6)^2}$,

$W^{VIII} = -\frac{4(a-c)^2(-20-414\beta-2405\beta^2-400\beta^3+849\beta^4-1366\beta^5+1557\beta^6-1260\beta^7+555\beta^8-112\beta^9+8\beta^{10})}{(13+108\beta-54\beta^2+36\beta^3-43\beta^4+24\beta^5-4\beta^6)^2}$

Note that (i) $q_i^{VIII} \geq q_j^{VIII}$, $x_i^{VIII} \geq x_j^{VIII}$ for all $\beta \in [0,1]$; (ii) $\pi_i^{VIII} \gtreqless \pi_j^{VIII}$ if $\beta \lesseqgtr 0.333$.

Appendix B. Proofs.

Proof of Lemma 1: (1) When $T_G = 1$, firm 1's profit ranks are as follows: (i) $\pi_1(1,1,1) \gtreqless \pi_1(1,2,1)$ if $\beta \lesseqgtr 0.221$; (ii) $\pi_1(1,1,2) \gtreqless \pi_1(1,2,2)$ if $\beta \lesseqgtr 0.221$; $\pi_1(1,1,2) \gtreqless \pi_1(1,3,2)$ if $\beta \lesseqgtr 0.261$; $\pi_1(1,2,2) \gtreqless \pi_1(1,3,2)$ if $\beta \lesseqgtr 0.5$; (iii) $\pi_1(1,1,3) \gtreqless \pi_1(1,2,3)$ if $\beta \lesseqgtr 0.239$; $\pi_1(1,1,3) \gtreqless \pi_1(1,3,3)$ if $\beta \lesseqgtr 0.221$; $\pi_1(1,3,3) \gtreqless \pi_1(1,2,3)$ if $\beta \lesseqgtr 0.5$. Also, firm 2's profit ranks are as follows: (i) $\pi_2(1,1,1) \gtreqless \pi_2(1,1,2)$ if $\beta \lesseqgtr 0.221$; (ii) $\pi_2(1,2,1) \gtreqless \pi_2(1,2,2)$ if $\beta \lesseqgtr 0.221$; $\pi_2(1,2,1) \gtreqless \pi_2(1,2,3)$ if $\beta \lesseqgtr 0.261$; $\pi_2(1,2,2) \gtreqless \pi_2(1,2,3)$ if $\beta \lesseqgtr 0.5$; (iii) $\pi_2(1,3,1) \gtreqless \pi_2(1,3,3)$ if $\beta \lesseqgtr 0.221$; $\pi_2(1,3,1) \gtreqless \pi_2(1,3,2)$ if $\beta \lesseqgtr 0.239$; $\pi_2(1,3,3) \gtreqless \pi_2(1,3,2)$ if $\beta \lesseqgtr 0.5$. Hence, comparing the profits ranks between the firms provides an equilibrium choice of timing between the firms when $T_G = 1$.

(2) When $T_G = 2$, firm 1's profit ranks are as follows: (i) $\pi_1(2,1,1) \gtreqless \pi_1(2,2,1)$ if $\beta \lesseqgtr 0.816$; $\pi_1(2,1,1) \gtreqless \pi_1(2,3,1)$ if $\beta \lesseqgtr 0.238$; $\pi_1(2,2,1) \geq \pi_1(2,3,1)$ if $0.037 \leq \beta \leq 0.186$; (ii) $\pi_1(2,1,2) > \pi_1(2,2,2)$ for all β; $\pi_1(2,1,2) \gtreqless \pi_1(2,3,2)$ if $\beta \lesseqgtr 0.246$; $\pi_1(2,2,2) \gtreqless \pi_1(2,3,2)$ if $\beta \lesseqgtr 0.221$; (iii) $\pi_1(2,1,3) > \pi_1(2,2,3)$ for all $\beta \in [0,1]$; $\pi_1(2,1,3) \gtreqless \pi_1(2,3,3)$ if $\beta \lesseqgtr 0.236$; $\pi_1(2,2,3) \gtreqless \pi_1(2,3,3)$ if $\beta \lesseqgtr 0.221$. Also, firm 2's profit ranks are as follows: (i) $\pi_2(2,1,1) \gtreqless \pi_2(2,1,2)$ if $\beta \lesseqgtr 0.816$; $\pi_2(2,1,1) \gtreqless \pi_2(2,1,3)$ if $\beta \lesseqgtr 0.238$; $\pi_2(2,1,2) \geq \pi_2(2,1,3)$ if $0.037 \leq \beta \leq 0.186$; (ii) $\pi_2(2,2,1) > \pi_2(2,2,2)$ for all β; $\pi_2(2,2,3) \gtreqless \pi_2(2,2,1)$ if $\beta \gtreqless 0.246$; $\pi_2(2,2,2) \gtreqless \pi_2(2,2,3)$ if $\beta \lesseqgtr 0.221$; (iii) $\pi_2(2,3,1) > \pi_2(2,3,2)$ for all β; $\pi_2(2,3,3) \gtreqless \pi_2(2,3,1)$ if $\beta \gtreqless 0.236$; $\pi_2(2,3,2) \gtreqless \pi_2(2,3,3)$ if $\beta \lesseqgtr 0.221$. Hence, comparing the profits ranks between the firms provides an equilibrium choice of timing between the firms when $T_G = 2$.

(3) When $T_G = 3$, firm 1's profit ranks are as follows: (i) $\pi_1(3,1,1) \gtreqless \pi_1(3,2,1)$ if $\beta \lesseqgtr 0.333$; $\pi_1(3,1,1) \gtreqless \pi_1(3,3,1)$ if $\beta \lesseqgtr 0.816$; $\pi_1(3,2,1) > \pi_1(3,3,1)$ for all β; (ii) $\pi_1(3,1,2) > \pi_1(3,2,2)$ for all $\beta \in [0,1]$; $\pi_1(3,1,2) > \pi_1(3,3,2)$ for all β; $\pi_1(3,3,2) \gtreqless \pi_1(3,2,2)$ if $\beta \gtreqless 0.816$; (iii) $\pi_1(3,1,3), (3,2,3) > \pi_1(3,3,3)$ for all β. Also, firm 2's profit ranks are as follows: (i)$\pi_2(3,1,1) \gtreqless \pi_2(3,1,2)$ if $\beta \lesseqgtr 0.333$; $\pi_2(3,1,1) \gtreqless \pi_2(3,1,3)$ if $\beta \lesseqgtr 0.816$; $\pi_2(3,1,2) > \pi_2(3,1,3)$ for all β; (ii) $\pi_2(3,2,1) > \pi_2(3,2,2)$ for all β; $\pi_2(3,2,1) \gtreqless \pi_2(3,2,3)$ for all β; $\pi_2(3,2,2) \gtreqless \pi_2(3,2,3)$ if $\beta \lesseqgtr 0.816$; (iii) $\pi_2(3,3,1), (3,3,2) > \pi_2(3,3,3)$ for all β. Hence, comparing the profits ranks between the firms provides an equilibrium choice of timing between the firms when $T_G = 3$.

Proof of Lemma 2 and 3: It is easy to compare welfare rankings and thus omitted.

Proof of Proposition 1: Using Lemma 1 and Lemma 2, we can obtain the following relations where the government and both firms do not want to deviate from their choices

(1) If $0 \leq \beta \leq 0.221$, $(T_G, T_{F1}, T_{F2}) = (1,1,1)$
(2) If $0.221 < \beta \leq 0.236$, $(T_G, T_{F1}, T_{F2}) = \{(1,2,2), (1,3,3)\}$
(3) If $0.236 < \beta \leq 0.238$, $(T_G, T_{F1}, T_{F2}) = \{(1,2,2), (2,3,3)\}$
(4) If $0.238 < \beta \leq 0.267$, $(T_G, T_{F1}, T_{F2}) = \{(1,2,2), (2,3,3), (1,3,3)\}$
(5) If $0.267 < \beta \leq 0.274$, $(T_G, T_{F1}, T_{F2}) = \{(2,3,3), (3,1,1), (1,2,2), (1,3,3)\}$
(6) If $0.274 < \beta \leq 0.333$, $(T_G, T_{F1}, T_{F2}) = \{(2,3,3), (3,1,1), (1,3,3)\}$
(7) If $0.333 < \beta \leq 0.5$, $(T_G, T_{F1}, T_{F2}) = \{(1,3,3), (2,3,3), (3,2,1), (3,1,2)\}$
(8) If $0.5 < \beta \leq 1$, $(T_G, T_{F1}, T_{F2}) = \{(3,2,1), (3,1,2)\}$

Arranging these results provides the equilibrium of endogenous timing game.

Proof of Proposition 2: Comparing Proposition 1 and Lemma 3, we can obtain the followings:

(i) $(T_G, T_{F1}, T_{F2}) = (1,2,2)$ *if* $0.221 < \beta \leq 0.223$
(ii) $(T_G, T_{F1}, T_{F2}) = (3,1,1)$ *if* $0.278 < \beta \leq 0.333$,
(iii) $(T_G, T_{F1}, T_{F2}) = \{(3,1,2), (3,2,1)\}$ *if* $0.333 < \beta \leq 1$.