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Designing linear inflation contracts in the New Keynesian model

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Abstract

This paper studies how to design linear inflation contracts to shape the incentive structure faced by the central bank in the New Keynesian model with positive optimal output gap and inflation target. Such contracts are known to be able to deal with the time-inconsistency problem in the Barro-Gordon framework, arising from incentives for the central bank to exploit the inflation-output trade-off induced by an "overambitious" output-gap target. We show that linear inflation contracts help reduce inflation undershooting and partially eliminate the inflation bias in the New Keynesian model. They are significantly different from those designed in the Barro-Gordon model.

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1 Introduction

Several studies associate New-Keynesian DSGE models with a micro-founded welfare function affected by steady-state distortions due to nominal rigidities and monopolistic competition (Woodford 1999, 2003; Eusepi et al. 2018). This implies a time-inconsistency problem and an inflation bias if the central bank (CB) conducts policy under discretion. Few attention has been paid to this bias in New Keynesian models. This contrasts with a large literature on time-inconsistency pioneered by Kydland and Prescott (1977) and that on monetary policy delegation initiated by Thompson (1981), Barro and Gordon (1983) and Rogoff (1985) in the so-called Barro-Gordon framework, within which various delegation schemes have been proposed that exactly implement the commitment optimum when the CB operates under discretion. They include appointing a conservative central banker (Rogoff 1985, Lohmann 1992), performance contracts (Persson and Tabellini 1993, Walsh 1995), inflation targeting (Svensson 1997), and nominal income growth targeting (Beetsma and Jensen 1999).

A strand of literature studies the virtues of delegation in New Keynesian models but focuses on the stabilization bias that mainly pertains to the lack of inertia induced by the discretionary policy response to persistent cost-push shocks. Various delegation schemes such as interest-rate smoothing (Woodford 1999), nominal income growth targeting (Jensen 2002), output-gap growth targeting (Walsh 2003a), inflation targeting with the government penalizing the CB for deviating from targets (Walsh 2003b), and price level targeting (Vestin 2006) can induce inertia and improve upon the discretionary equilibrium. When the discretionary CB is delegated with an appropriate objective function, it is possible to exactly implement the timeless optimal commitment policy under various delegation schemes (Bilbiie 2014).

This paper examines if the prescriptions for designing linear inflation contracts to reduce the inflation bias in the Barro-Gordon model may apply to the New Keynesian model. It also shows how they help reduce inflation undershooting when the inflation target is not null. While Dai and Spyromitros (2012) and Nakata and Schmidt (2019) study inflation contracts in the New Keynesian model, they ignore these issues. Following Candel-Sánchez and Campoy-Miñarro (2004), we consider a class of linear inflation contracts where Walsh's (1995) contract is a particular case. The government designs and implements a transfer mechanism, called inflation contract that cannot be rejected by the CB, to achieve the efficient equilibrium. It aims at fighting against the inflation bias by providing an incentive, i.e., a penalty for missing the inflation goal, for the CB to limit its short-run opportunism.

In the following, section 2 sets up the model. Section 3 examines the equilibrium and derives optimal inflation contracts. Section 3 compares the results across monetary policy regimes and with those obtained in the Barro-Gordon model. Section 4 concludes.

2 The model

The supply side of the economy is represented by a standard New Keynesian Phillips curve (Clarida et al. 1999):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \tag{1}$$

where π_t is the inflation rate, $\beta \in (0,\ 1)$ the discount factor, x_t the output gap, and E_t the rational expectations (RE) operator. The composite parameter $\kappa \equiv \frac{(1-\vartheta)(1-\vartheta\beta)}{\vartheta}(1+\varphi)$ stands for the output-gap elasticity of inflation. Here, φ and ϑ are respectively the inverse of the steady-state Frisch elasticity of labor supply and the share of firms that do not optimally adjust but simply update in period t their price by the steady-state inflation rate. Eq. (1) is derived assuming that monopolistic competitive firms set the nominal price of their product to maximize profits subject to the constraint on the frequency of price adjustments as in Calvo (1983). The noise $e_t \sim \mathcal{N}(0, \sigma_e^2)$ is an *i.i.d.* cost-push shock.

Society's preferences are captured by a quadratic loss function that is a weighted sum of variances of inflation and the output gap and represents a second-order Taylor series approximation to the exact welfare measure (Woodford (2003, chap. 6):

$$L_t^s = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T \right)^2 + \alpha \left(x_{t+i} - \widetilde{x} \right)^2 \right], \tag{2}$$

where $\alpha > 0$ is the relative weight placed on output-gap stabilization, π^T the inflation target, and $\widetilde{x}$ the optimal output gap with $\widetilde{x} \equiv \log \frac{Y^*}{\overline{Y}} > 0$. A positive $\widetilde{x}$ means that the efficient level of output, Y^*, is at all times a certain number of percentage points above the natural rate of output, $\overline{Y}$. It results from the distortions induced by delays in price adjustment and a constant level of market power of the producers of differentiated goods, if an offsetting output subsidy is not implemented (Woodford 1999). Notice that a positive $\widetilde{x}$ has the same implications for optimal monetary policy as an overambitious output-gap target in the literature on time inconsistency.

An independent CB conducting discretionary policy, as it cares about social welfare, would not avoid the time-inconsistency problem due to positive $\widetilde{x}$ and the associated inflation bias. To mitigate this bias, the government imposes on the CB an inflation contract stipulating that the CB receives a monetary transfer payment, T, from the government, depending on the CB's efficiency in stabilizing inflation with respect to its target:

$$T = \tau_0 - \tau \left(\pi_t - \pi^T \right) \tag{3}$$

where τ_0 is set to ensure the CB's participation and τ is the penalty rate the CB would face if it failed to control inflation (Walsh 1995). This payment can affect the central banker's income or the CB's budget. The weak aspect of this theoretical scheme emerges with issues induced by its implementation, e.g., there is a potential public relations problem if the central banker's income rises following a deflationary recession. For Chortareas and Miller (2010), criticism about its effectiveness and implementability can emerge only under a narrow interpretation of inflation contracts. Indeed, central bankers may have other motives such as altruism towards society, e.g., their interest in reducing social losses, and the prestige and possible career opportunities that come with holding office. The results obtained with an inflation contract can be achieved if other components of their utility such as the chance to be re-appointed or the public's perception of their competence depend on inflation performance.

The delegation through the transfer payment mechanism (3) allows the government to regain some control over how the CB operates. The CB conducts policy to maximize

$$U_t^{CB} = -\frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left\{ \left(\pi_{t+i} - \pi^T \right)^2 + \alpha \left(x_{t+i} - \widetilde{x} \right)^2 - \xi \left[\tau_0 - \tau \left(\pi_{t+i} - \pi^T \right) \right] \right\}, \tag{4}$$

subject to (1). Here, ξ indicates the extent to which the CB cares about the incentive scheme versus social loss. The value of τ_0 is set to ensure that the participation constraint, $u_{t+i}^{CB} \equiv \left(\pi_{t+i} - \pi^T\right)^2 + \alpha\left(x_{t+i} - \widetilde{x}\right)^2 - \xi\left[\tau_0 - \tau\left(\pi_{t+i} - \pi^T\right)\right] \geq u_0^{CB}$ with u_0^{CB} being the reservation utility of the CB, is satisfied so that the CB accepts the contract, which is designed in a way that the higher the penalty rate, the costlier it is for the CB to adjust inflation to achieve the "overambitious" output-gap target. We focus on the case where the participation constraint is not binding, i.e., $u_{t+i}^{CB} > u_0^{CB}$. The case where it is binding, i.e., $u_{t+i}^{CB} = u_0^{CB}$, implies that the central banker is not offered an additional incentive to accept the contract. Even though the central banker may accept the contract because of the prestige, non-pecuniary benefits or large future pecuniary benefits from serving as a central banker, there is no reason to expect him to perform (optimize) with respect to the contract (Chortareas and Miller 2007). Hence, the binding participation constraint should only be considered as a limit case, showing to which extent the government can reduce τ_0.

The government cares about both social welfare and the transfer it pays to the CB. Its utility function is modeled following Candel-Sánchez and Campoy-Miñarro (2004):

$$U_t^G = -\frac{1}{2}E_t\sum_{i=0}^{+\infty}\beta^i\left\{\left(\pi_{t+i} - \pi^T\right)^2 + \alpha\left(x_{t+i} - \widetilde{x}\right)^2 + \varsigma\left[\tau_0 - \tau\left(\pi_{t+i} - \pi^T\right)\right]\right\}, \tag{5}$$

where ς denotes the weight on the transfer, which represents a cost for the government.

The sequence of events is as follows: 1. The government designs a linear inflation contract, taking into account the CB's response to any penalty it may impose. 2. Private agents form expectations. 3. The realization of shocks is known. 4. The CB conducts optimal policy.

3 The equilibrium and optimal inflation contracts

This section derives the equilibrium solutions and optimal linear inflation contracts in the New Keynesian model. Their counterparts for the Barro-Gordon (Appendix A.1) can then be derived by setting $\beta = 1$ in some solutions obtained for the New Keynesian model.

3.1 Optimal policies under discretion

Under discretion, given private expectations, the CB maximizes its utility function (4) subject to (1). The first-order conditions yield the optimal inflation targeting rule:

$$\pi_t - \pi^T = -\frac{\alpha}{\kappa}\left(x_t - \widetilde{x}\right) - \frac{1}{2}\xi\tau. \tag{6}$$

The rule (6) yields a "leaning against the wind" policy, i.e., the CB should contract demand below capacity to disinflate the economy whenever inflation is above its target. In addition, a positive (instead of zero) penalty rate implies a lower output gap by making it more costly for the CB to adjust inflation to achieve the output-gap target.

The system of Eqs. (1) and (6) has a unique non-explosive RE equilibrium solution, called the "minimal state variable" solution (McCallum 1983), in terms of state variables π^T, $\widetilde{x}$, e_t and τ. The solution of π_t takes the form: $\pi_t = \zeta_0 + \zeta_1 e_t$. Knowing that $E_t e_{t+1} = 0$, we get $E_t\pi_{t+1} = \zeta_0 + \zeta_1 E_t e_{t+1} = \zeta_0$. Using the method of undetermined coefficients yields $\zeta_0 = \frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}\left(\pi^T - \frac{1}{2}\xi\tau + \frac{\alpha}{\kappa}\widetilde{x}\right)$ and $\zeta_1 = \frac{\alpha}{\alpha+\kappa^2}$. Here, for $\tau = 0$ and $\widetilde{x} = 0$, inflation expectations are given by $E_t\pi_{t+1} = \frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}\pi^T$, which undershoots the inflation target. This is due to the features of the New Keynesian model (i.e., nominal rigidities, monopolistic competition, and the forward-looking nature of firms' pricing decisions). To avoid inflation undershooting, the inflation target is set to zero when studying inflation targeting in New Keynesian models.

Replacing $E_t\pi_{t+1}$ by the solution of ζ_0 into (1) and solving the resulting equation together with (6) lead to the RE equilibrium solutions:

$$\pi_t = \frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}\pi^T + \frac{\alpha\kappa}{\alpha(1-\beta)+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}\xi\tau + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{7}$$

$$x_t = \frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2}\pi^T + \frac{\alpha(1-\beta)}{\alpha(1-\beta)+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2}\xi\tau - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{8}$$

These solutions suffer from the classic inflation bias, first identified by Kydland and Prescott (1977). For $\tau = 0$ and $\pi^T = 0$, the long-run inflation rate is not only positive, but also substantially higher than the one obtained under commitment (see below). The inflation-targeting regime is not effective in dealing with the inflation bias caused by the "overambitious" output-gap target in the New-Keynesian model.

Under discretion, the inflation bias can be reduced by implementing an optimal inflation contract. Maximizing (5) subject to (7)-(8) yields:

$$\tau = \frac{-2\alpha(1-\beta)\left\{\beta\kappa^2\xi + \varsigma\left[\alpha(1-\beta)+\kappa^2\right]\right\}\pi^T + 2\alpha\kappa\left\{\beta\xi\kappa^2 + \varsigma\left[\alpha(1-\beta)+\kappa^2\right]\right\}\widetilde{x}}{\kappa^2\xi\left\{(\xi+2\varsigma)\left[\alpha(1-\beta)^2+\kappa^2\right] + 2\varsigma\alpha\beta(1-\beta)\right\}}. \tag{9}$$

The optimal penalty rate is negatively (positively) related to the inflation target, π^T (the output-gap target, $\widetilde{x}$). Inserting τ given by in (9), into (7)-(8) yields:

$$\pi_t = \frac{\left\{\kappa^2(\xi+\varsigma) + \varsigma\left[\alpha(1-\beta)+\kappa^2\right]\right\}\pi^T + \alpha\kappa\left[\xi(1-\beta)+\varsigma\right]\widetilde{x}}{(\xi+2\varsigma)\left[\alpha(1-\beta)^2+\kappa^2\right] + 2\varsigma\alpha\beta(1-\beta)} + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{10}$$

$$x_t = \frac{(1-\beta)}{\kappa}\frac{\left\{\kappa^2(\xi+\varsigma) + \varsigma\left[\alpha(1-\beta)+\kappa^2\right]\right\}\pi^T + \alpha\kappa\left[\xi(1-\beta)+\varsigma\right]\widetilde{x}}{(\xi+2\varsigma)\left[\alpha(1-\beta)^2+\kappa^2\right] + 2\varsigma\alpha\beta(1-\beta)} - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{11}$$

Thus, under discretion, even an optimal inflation contract cannot fully offset inflation undershooting and the effects of the "overambitious" output-gap target on the equilibrium. This is due to the impatience of agents reflected by a discount factor less than unity ($\beta < 1$) and the fact that the transfer payment enters the government's utility function with a weight $\varsigma > 0$. The impatience of agents is the key to explain why the optimal inflation contract cannot fully offset the effects of distortions on the equilibrium in this New-Keynesian model even if $\varsigma = 0$, contrary to what happens in the Barro-Gordon framework. Indeed, a lower β leads the CB to pay less attention to inflation expectations that are function of steady-state distortions and to be more focused on current inflation stabilization. This implies an inflation bias that the government cannot fully offset with an optimal inflation contract.

3.2 Optimal policies under commitment

The equilibrium under discretion differs from that under constrained commitment (commitment to a rule) or unconstrained commitment (from a timeless perspective).

3.2.1 Constrained commitment

The CB uses a rule contingent on state variables π^T, $\widetilde{x}$, τ and e_t for the target variable x_t:

$$x_t = a_x \pi^T + b_x \widetilde{x} + c_x \tau + d_x e_t, \tag{12}$$

where a_x, b_x, c_x and d_x are undetermined coefficients. Inserting (12) into (1) and iterating forwardly to eliminate inflation expectations yields a solution of π_t with undetermined coefficients. Inserting the assumed solutions of π_t and x_t into (4) and maximizing the resulting function with respect to a_x, b_x c_x and d_x. Solving the first-order conditions with the method of undetermined coefficients gives the equilibrium solutions:

$$\pi_t = \frac{\kappa^2}{\alpha\left(1-\beta\right)^2+\kappa^2}\pi^T + \frac{\alpha\kappa(1-\beta)}{\alpha(1-\beta)^2+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{\kappa^2}{\alpha(1-\beta)^2+\kappa^2}\xi\tau + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{13}$$

$$x_t = \frac{\kappa\left(1-\beta\right)}{\alpha\left(1-\beta\right)^2+\kappa^2}\pi^T + \frac{\alpha(1-\beta)^2}{\alpha(1-\beta)^2+\kappa^2}\widetilde{x} - \frac{1}{2}\frac{\kappa(1-\beta)}{\alpha(1-\beta)^2+\kappa^2}\xi\tau - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{14}$$

Substituting these solutions into (5), we find that the government should optimally set

$$\tau = \frac{-2\alpha\varsigma\left(1-\beta\right)^2\pi^T + 2\alpha\varsigma\kappa(1-\beta)\widetilde{x}}{\kappa^2\xi\left(\xi+2\varsigma\right)}. \tag{15}$$

If $\beta = 1$, the optimal penalty rate is zero, i.e., its level in the Barro-Gordon model. For $\beta < 1$, it becomes positive if $\varsigma > 0$ because, while commitment eliminates the need for an inflation contract to discipline the CB, the cost of this contract leads the government to desire to reduce inflation under the level that is socially optimal under constrained commitment by setting a positive τ for all $\varsigma > 0$. Inserting τ given by (15) into (13)-(14) yields:

$$\pi_t = \frac{\left\{\kappa^2\left(\xi+\varsigma\right)+\varsigma\left[\alpha(1-\beta)^2+\kappa^2\right]\right\}\pi^T + \alpha\kappa(1-\beta)\left(\xi+\varsigma\right)\widetilde{x}}{\left(\xi+2\varsigma\right)\left[\alpha(1-\beta)^2+\kappa^2\right]} + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{16}$$

$$x_t = \frac{\left(1-\beta\right)\left\{\kappa^2\left(\xi+\varsigma\right)+\varsigma\left[\alpha(1-\beta)^2+\kappa^2\right]\right\}\pi^T + \alpha\kappa(1-\beta)^2\left(\xi+\varsigma\right)\widetilde{x}}{\kappa\left(\xi+2\varsigma\right)\left[\alpha(1-\beta)^2+\kappa^2\right]} - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{17}$$

Eqs. (16)-(17) show that, under constrained commitment, optimal inflation contracts do not fully eliminate inflation undershooting and the inflation bias even if $\varsigma = 0$.

3.2.2 Unconstrained commitment

The constrained commitment is not fully optimal compared to the unconstrained (or optimal) commitment, i.e., the commitment from a timeless perspective (Clarida et al. 1999, Woodford 1999, 2003). The latter implies that the policymaker should respect the same optimality condition for any periods, including the current one such that, for $i \geq 0$,

$$\pi_{t+i} - \pi^T = -\frac{1}{2}\xi\tau - \frac{\alpha}{\kappa}(x_{t+i} - x_{t+i-1}). \tag{18}$$

The equilibrium solutions under unconstrained commitment respecting (18) are:

$$\pi_t = \rho_0^\pi x_{t-1} + \rho_1^\pi \pi^T + \rho_2^\pi \widetilde{x} + \rho_3^\pi \tau + \rho_4^\pi e_t, \tag{19}$$

$$x_t = \rho_0^x x_{t-1} + \rho_1^x \pi^T + \rho_2^x \widetilde{x} + \rho_3^x \tau + \rho_4^x e_t. \tag{20}$$

where $\rho_0^x = \frac{\alpha(1+\beta)+\kappa^2 \pm \sqrt{[\alpha(1+\beta)+\kappa^2]^2 - 4\alpha^2\beta}}{2\alpha\beta}$, $\rho_0^\pi = \rho_0^\pi = \frac{\kappa\rho_0^x}{1-\beta\rho_0^x}$, $\rho_1^\pi = \frac{1-\rho_0^x}{1-\beta\rho_0^x}$, $\rho_2^\pi = 0$, $\rho_3^\pi = -\frac{1}{2}\frac{\xi\left(1-\rho_0^x\right)}{\left(1-\beta\rho_0^x\right)}$, $\rho_4^\pi = \rho_0^x$, $\rho_1^x = \frac{\kappa(1-\beta)\rho_0^x}{\alpha\left(1-\beta\rho_0^x\right)}$, $\rho_2^x = 0$, $\rho_3^x = -\frac{1}{2}\frac{\kappa\xi(1-\beta)\rho_0^x}{\alpha\left(1-\beta\rho_0^x\right)}$, and $\rho_4^x = -\frac{\kappa}{\alpha}\rho_0^x$ (Appendix A.2). Solutions (19)-(20) show inertia and are invariant to $\widetilde{x}$. In the steady state, we get

$$\pi = -\frac{1}{2}\xi\tau + \pi^T \tag{21}$$

$$x = -\frac{1}{2}\frac{1-\beta}{\kappa}\xi\tau + \frac{1-\beta}{\kappa}\pi^T. \tag{22}$$

In the steady state, we have $\lim_{t\to+\infty} \pi_t = \pi^T$, for $\tau = 0$. Thus, an inflation contract is useless. Moreover, inflation undershooting (below the target), observed under discretion and constrained commitment, is fully eliminated by the commitment mechanism since inflation and inflation expectations are equal to the inflation target. However, the steady state equilibrium defined by (21)-(22) implies that the value of τ that maximizes the government's utility (5) depends on $\widetilde{x}$ and π^T (Table 2 and Appendix A.2). Such an inflation contract could reintroduce inflation undershooting and the dependence of inflation on the "overambitious" output gap and would not be socially optimal. Hence, unconstrained commitment represents the ideal policy regime that the government should not search to improve.

4 Comparison and discussion

The comparison is first done across policy regimes in the New Keynesian model. The conclusions are then compared with those obtained in the Barro-Gordon model. To facilitate the comparison, Table 1 recapitulates the coefficients on π^T and $\widetilde{x}$ in the (steady state) equilibrium solutions under different policy regimes in these two types of model.

In the New Keynesian model, the optimal long-run inflation rate positively depends on the inflation target and the "overambitious" output-gap target. Without an inflation contract, inflation undershooting observed under discretion is not improved with constrained commitment. In contrast, constrained commitment reduces the inflation bias due to $\widetilde{x}$ compared to discretion. However, constrained commitment does not fully eliminate this inflation bias. This does not result from the CB's inability to commit but is due to the impatience of agents. Notice that when the discount factor β goes to unity, inflation stabilization is optimal in the long run under constrained commitment without implementing an optimal inflation contract, while a residual inflation bias due to $\widetilde{x}$ remains under discretion because the government cares about the cost of the inflation contract as shown in (10). Compared to unconstrained commitment, the time inconsistency under constrained commitment comes from the difference in the optimality condition for the current period (Woodford 1999).

Model	Policy regime	variable	coefficient on π^T		coefficient on $\widetilde{x}$	
			no contract	with contract	no contract	with contract
New Keynesian (NK)	Discretion	π_t	$\frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}$	$\frac{\kappa^2(\xi+\varsigma)+\varsigma\left[\alpha(1-\beta)+\kappa^2\right]}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\alpha\beta(1-\beta)}$	$\frac{\alpha\kappa}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha\kappa[\xi(1-\beta)+\varsigma]}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\alpha\beta(1-\beta)}$
		x_t	$\frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2}$	$\frac{(1-\beta)\left\{\kappa^2(\xi+\varsigma)+\varsigma\left[\alpha(1-\beta)+\kappa^2\right]\right\}}{\kappa\{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\alpha\beta(1-\beta)\}}$	$\frac{\alpha(1-\beta)}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha(1-\beta)[\xi(1-\beta)+\varsigma]}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\alpha\beta(1-\beta)}$
	Constrained commitment	π_t	$\frac{\kappa^2}{\alpha(1-\beta)+\kappa^2}$	$\frac{\kappa^2(\xi+\varsigma)+\varsigma\left[\alpha(1-\beta)^2+\kappa^2\right]}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$	$\frac{\alpha\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha\kappa(1-\beta)(\xi+\varsigma)}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$
		x_t	$\frac{\kappa(1-\beta)}{\alpha(1-\beta)+\kappa^2}$	$\frac{(1-\beta)\left\{\kappa^2(\xi+\varsigma)+\varsigma\left[\alpha(1-\beta)^2+\kappa^2\right]\right\}}{\kappa(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$	$\frac{\alpha(1-\beta)^2}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha\kappa(1-\beta)^2(\xi+\varsigma)}{\kappa(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$
	Unconstrained commitment	π_t	1 (benchmark)	$\frac{(\xi+2\varsigma)\kappa^2}{[\alpha(1-\beta)^2+\kappa^2]\xi+2\varsigma\kappa^2}$	0 (benchmark)	$\frac{\alpha\kappa\xi(1-\beta)}{\left[\alpha(1-\beta)^2+\kappa^2\right]\xi+2\varsigma\kappa^2}$
		x_t	$\frac{1-\beta}{\kappa}$	$\frac{(1-\beta)(\xi+2\varsigma)\kappa}{\xi[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\kappa^2}$	0	$\frac{\alpha\xi(1-\beta)^2}{\xi[\alpha(1-\beta)^2+\kappa^2]+2\varsigma\kappa^2}$
Barro-Gordon (B-G)	Discretion	π_t	1	1	$\frac{\alpha}{\kappa}$	$\frac{\alpha\varsigma}{\kappa(\xi+2\varsigma)}$
		x_t	0	0	0	0
	Commitment	π_t	1 (benchmark)	1	0 (benchmark)	0
		x_t	0	0	0	0

Table 1: Comparison of steady state equilibrium solutions.

Another observation is that optimal inflation penalty rates given in (9) and (15) are negatively (positively) related to π^T ($\widetilde{x}$). For β close to unity, the optimal penalty rate under constrained commitment given by (15) tends to zero. If the government does not care about the transfer payment so that $\varsigma = 0$, the optimal penalty rate is positive and increases with $\widetilde{x}$ under discretion while under constrained commitment, we obtain $\tau = 0$.

Using inflation undershooting and biases compared to the benchmark equilibrium inflation reported in Table 2, it is easy to show that optimal inflation contracts reduce inflation undershooting, if $\alpha < \frac{\kappa^2(\xi+2\varsigma)}{(1-\beta)^2\xi}$, and the inflation bias more under constrained commitment than under discretion in the New Keynesian model. The fact that $\beta < 1$ explains that such contracts cannot fully eliminate inflation undershooting and bias while a positive ς causes the residual inflation biases to be different under these policy regimes.

Model	Policy regime	Inflation undershooting compared to the benchmark		Inflation bias compared to benchmark	
		no contract	with contract	no contract	with contract
NK	Discretion	$\frac{-\alpha(1-\beta)\pi^T}{\alpha(1-\beta)+\kappa^2}$	$\frac{-\alpha(1-\beta)[\xi(1-\beta)+\varsigma]\pi^T}{\Xi}$	$\frac{\alpha\kappa\widetilde{x}}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha\kappa[\xi(1-\beta)+\varsigma]\widetilde{x}}{\Xi}$
	Constrained commitment	$\frac{-\alpha(1-\beta)\pi^T}{\alpha(1-\beta)+\kappa^2}$	$\frac{-\alpha(1-\beta)^2(\xi+\varsigma)\pi^T}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$	$\frac{\alpha(1-\beta)\kappa\widetilde{x}}{\alpha(1-\beta)+\kappa^2}$	$\frac{\alpha\kappa(1-\beta)(\xi+\varsigma)\widetilde{x}}{(\xi+2\varsigma)[\alpha(1-\beta)^2+\kappa^2]}$
B-G	Discretion	0	0	$\frac{\alpha\widetilde{x}}{\kappa}$	$\frac{\alpha\varsigma\widetilde{x}}{\kappa(\xi+2\varsigma)}$

Note: $\Xi \equiv (\xi + 2\varsigma)\left[\alpha(1-\beta)^2 + \kappa^2\right] + 2\varsigma\alpha\beta(1-\beta)$.

Table 2: Comparison of inflation undershooting and bias across policy regimes and models.

Compared to the Barro-Gordon model, four differences emerge: 1) There are three policy regimes in the New Keynesian model but only two in the Barro-Gordon model. 2) The benchmark for computing inflation undershooting and bias in the New Keynesian model is the unconstrained commitment equilibrium with $\tau = 0$ that is optimal for society but not for the government, and the commitment equilibrium in the Barro-Gordon model, where $\tau = 0$ is optimal for society and the government. 3) The features of the New Keynesian model (i.e., intertemporal decisions, nominal rigidities and monopolistic competition) imply that the discount factor β is a key determinant of the design of inflation contracts and their effects under discretion and constrained commitment. If $\beta = 1$, such contracts would be identical to those obtained under discretion and commitment in the Barro-Gordon model. 4) Once the optimal inflation contract is implemented, for $\varsigma > 0$, there are inflation undershooting and residual inflation biases under discretion and constrained commitment in the New Keynesian model, while only a residual inflation bias is a concern in the Barro-Gordon model.

The above results are obtained assuming that the participation constraint is not binding. Following Chortareas and Miller (2007) to define the CB's iso-expected utility when the participation constraint is binding and then rearranging the terms, we get

$$\tau_0 = \frac{1}{\xi} E_t \left\{ \left[\left(\pi_{t+i} - \pi^T\right)^2 + \alpha \left(x_{t+i} - \widetilde{x}\right)^2 - u_0^{CB} \right] + \tau \left(\pi_{t+i} - \pi^T\right) \right\}. \tag{23}$$

Substituting τ_0 given by (23) into the government's utility function (5) yields

$$U_t^G = -\frac{1}{2}E_t\sum_{i=0}^{+\infty}\beta^i\left\{\left(1+\frac{\varsigma}{\xi}\right)\left[\left(\pi_{t+i}-\pi^T\right)^2+\alpha\left(x_{t+i}-\widetilde{x}\right)^2-u_0^{CB}\right]\right\}, \tag{24}$$

The government's new utility function (24) is homothetic to the society's loss function (2), implying that a government imposing a binding participation constraint can ignore the costs due to inflation contracts when optimally setting τ, and hence the Walsh's contracts are optimal after all (Chortareas and Miller 2007). This is equivalent to assume that $\varsigma = 0$.

5 Conclusion

Optimal linear inflation contracts reduce inflation undershooting and partially eliminate the inflation bias in the New Keynesian model. They are quite different from those obtained in the Barro-Gordon framework. The difference is due to the role of the discount factor in the Phillips curve and the existence of three policy regimes (discretion, constrained commitment and unconstrained commitment) in the New Keynesian model while there are only two (discretion and commitment) in the Barro-Gordon framework.

The policy implications of our results are clear. The institutional arrangement that grants the central bank instrument independence and makes it accountable for its monetary policy performance by imposing a linear inflation contract allows improving the equilibrium outcome compared to pure inflation targeting. In times of unwelcome high inflation and uncertainty, the effectiveness of inflation contracts can be further explored in the New Keynesian framework by considering the possibility that agents deviate from rational expectations and are actually learning in rapidly changing economic environment. At the 2020 Jackson Hole Economic Policy Symposium, Jerome Powell, Federal Reserve Chair, announced a revision to the Fed's long-run monetary policy framework, i.e., replacing inflation targeting with average inflation targeting to achieve its dual mandate. It would be interesting to study how linear inflation contracts could deal with the time inconsistency that might arise in this new policy regime.

A Appendix

A.1 Inflation contracts in the Barro-Gordon framework

The Phillips curve is augmented of inflation expectations formed at the end of the last period:

$$\pi_t = \pi_t^e + \kappa x_t + e_t. \tag{A.1}$$

where $\pi_t^e = E_{t-1}\pi_t$, the expected inflation rate conditional on information set available at time $t-1$.

A.1.1 Optimal policies under discretion

Under discretion, the CB does not seek to shape private expectations and maximizes its utility (4) subject to (A.1). This gives the optimal targeting rule:

$$\pi_t - \pi^T = -\frac{\alpha}{\kappa}\left(x_t - \widetilde{x}\right) - \frac{1}{2}\xi\tau, \tag{A.2}$$

The above rule is identical to (6). Eqs. (A.1)-(A.2) yields

$$\pi_t = \frac{\kappa^2}{\alpha+\kappa^2}\pi^T + \frac{\alpha\kappa}{\alpha+\kappa^2}\widetilde{x} + \frac{\alpha}{\alpha+\kappa^2}E_{t-1}\pi_t - \frac{\kappa^2\xi}{2\left(\alpha+\kappa^2\right)}\tau + \frac{\alpha}{\alpha+\kappa^2}e_t, \tag{A.3}$$

$$x_t = \frac{\kappa}{\alpha+\kappa^2}\pi^T + \frac{\alpha}{\alpha+\kappa^2}\widetilde{x} - \frac{\kappa}{\alpha+\kappa^2}E_{t-1}\pi_t - \frac{\kappa\xi}{2\left(\alpha+\kappa^2\right)}\tau - \frac{\kappa}{\alpha+\kappa^2}e_t. \tag{A.4}$$

Taking expectations of (A.3) yields the expected inflation rate, i.e., $E_{t-1}\pi_t = \pi^T + \frac{\alpha}{\kappa}\widetilde{x} - \frac{1}{2}\xi\tau$. Inserting the latter into the (A.4)-(A.3) yields the equilibrium solutions:

$$\pi_t = \pi^T + \frac{\alpha}{\kappa}\widetilde{x} - \frac{1}{2}\xi\tau + \frac{\alpha}{\alpha+\kappa^2}e_t \tag{A.5}$$

$$x_t = -\frac{\kappa}{\alpha+\kappa^2}e_t. \tag{A.6}$$

These solutions suffer from the classic inflation bias, first identified by Kydland and Prescott (1977). For $\tau = 0$, the long-run inflation rate is not only positive, but also substantially higher than the one obtained under commitment (see below).

Under discretion, the inflation bias can be reduced by implementing an optimal inflation contract. Maximizing (5) subject to (A.5)-(A.6) yields:

$$\tau = \frac{2\alpha\left(\xi+\varsigma\right)}{\kappa\xi\left(\xi+2\varsigma\right)}\widetilde{x}. \tag{A.7}$$

Inserting (A.7) into (A.5) implies

$$\pi_t = \pi^T + \frac{\alpha\varsigma}{\kappa\left(\xi+2\varsigma\right)}\widetilde{x} + \frac{\alpha}{\alpha+\kappa^2}e_t \tag{A.8}$$

$$x_t = -\frac{\kappa}{\alpha+\kappa^2}e_t. \tag{A.9}$$

If $\varsigma = 0$, the inflation bias is completely eliminated by imposing an optimal inflation contract in the Barro-Gordon framework.

A.1.2 Optimal policies under commitment

The equilibrium under discretion differs from that under commitment to a rule. Under commitment, the CB follows a rule contingent on $\widetilde{x}$, τ and e_t for the target variable x_t:

$$\pi_t = a_\pi\widetilde{x} + b_\pi\tau + c_\pi e_t + d_\pi\pi^T, \tag{A.10}$$

where a_π, b_π, c_π and d_π are undetermined coefficients. Taking expectations of (A.10) yields:

$$\pi_t^e = a_\pi \widetilde{x} + b_\pi \tau + d_\pi \pi^T. \tag{A.11}$$

Inserting π_t^e given by (A.11) into (A.1) gives:

$$\pi_t = a_\pi \widetilde{x} + b_\pi \tau + d_\pi \pi^T + \kappa x_t + e_t. \tag{A.12}$$

Equaling π_t given respectively by (A.10) and (A.12), il follows that

$$x_t = \frac{1}{\kappa}\left(c_\pi - 1\right) e_t. \tag{A.13}$$

Inserting π_t given by (A.10) and x_t given by (A.13) into the CB's utility function (4) and differentiating with respect to undetermined coefficients a_π, b_π, and c_π, we get two independent first-order conditions:

$$a_\pi \widetilde{x} + b_\pi \tau + \left(d_\pi - 1\right) \pi^T = -\frac{1}{2}\xi\tau, \tag{A.14}$$

$$c_\pi = \frac{\alpha}{\alpha + \kappa^2}. \tag{A.15}$$

It follows from (A.14) that $a_\pi = 0$, $b_\pi = -\frac{1}{2}\xi$ and $d_\pi = 1$. Using these results and c_π given by (A.15) into (A.10) and (A.13), we obtain the equilibrium solution under commitment:

$$\pi_t = \pi^T - \frac{1}{2}\xi\tau + \frac{\alpha}{\alpha + \kappa^2} e_t, \tag{A.16}$$

$$x_t = -\frac{\kappa}{\alpha + \kappa^2} e_t \tag{A.17}$$

Substituting these solutions into (5), we find that the government should optimally set

$$\tau = 0. \tag{A.18}$$

This means that commitment eliminates the need for an inflation contract. Inserting τ given by (A.18) into (A.16)-(A.17) yields:

$$\pi_t = \pi^T + \frac{\alpha}{\alpha + \kappa^2} e_t, \tag{A.19}$$

$$x_t = -\frac{\kappa}{\alpha + \kappa^2} e_t \tag{A.20}$$

Finally, for $\widetilde{x} > 0$, the residual inflation bias that cannot be eliminated by implementing the optimal inflation contract under discretion is obtained by comparing (A.8) and (A.19) as

$$\mathcal{O}^{\mathcal{BG}} = \frac{\alpha\varsigma}{\kappa\left(\xi + 2\varsigma\right)} \widetilde{x},$$

which is positive if the government cares about the cost of the inflation contract, i.e., $\varsigma > 0$.

A.2 Equilibrium with timeless perspective (unconstrained commitment)

The full intertemporal optimum, usually called the unconstrained commitment solution, is obtained by maximizing (4) subject to (1) for all periods. The first-order conditions are

$$\pi_t - \pi^T = -\frac{1}{2}\xi\tau - \frac{\alpha}{\kappa}(x_t - \widetilde{x}). \tag{A.21}$$

$$\pi_{t+i} - \pi^T = -\frac{1}{2}\xi\tau - \frac{\alpha}{\kappa}(x_{t+i} - x_{t+i-1}) \quad \text{for } i \geq 1. \tag{A.22}$$

The time inconsistency of the commitment solution is evident from (A.21) since this places a requirement that is specific to the current period and is different from the corresponding requirement (A.22) for later periods. The timeless perspective resolution to the problem of the time inconsistency of optimal policy is that the policymaker should respect the optimality conditions (A.22) even for the current period when the optimization is done. This yields the commitment optimality condition, which overrides (A.21):

$$\pi_t = \pi^T - \frac{1}{2}\xi\tau - \frac{\alpha}{\kappa}(x_t - x_{t-1}). \tag{A.23}$$

The dynamic system defined by (1) and (18) has a unique non-explosive solution in terms of state variables. It is obtained by using the method of undetermined coefficients. We guess the following linear functions for π_t and x_t:

$$\pi_t = \rho_0^\pi x_{t-1} + \rho_1^\pi \pi^T + \rho_2^\pi \widetilde{x} + \rho_3^\pi \tau + \rho_4^\pi e_t, \tag{A.24}$$

$$x_t = \rho_0^x x_{t-1} + \rho_1^x \pi^T + \rho_2^x \widetilde{x} + \rho_3^x \tau + \rho_4^x e_t. \tag{A.25}$$

Using (A.24)-(A.25) and $E_t e_{t+1} = 0$, we obtain

$$E_t\pi_{t+1} = \rho_0^\pi \rho_0^x x_{t-1} + (\rho_0^\pi \rho_1^x + \rho_1^\pi)\pi^T + (\rho_0^\pi \rho_2^x + \rho_2^\pi)\widetilde{x} + (\rho_0^\pi \rho_3^x + \rho_3^\pi)\tau + \rho_0^\pi \rho_4^x e_t. \tag{A.26}$$

Substituting $E_t\pi_{t+1}$ given by (A.26) and x_t given by (A.25) into (1) leads to:

$$\begin{aligned}\pi_t = {} & (\beta\rho_0^\pi\rho_0^x + \kappa\rho_0^x)\, x_{t-1} + [\beta(\rho_0^\pi\rho_1^x + \rho_1^\pi) + \kappa\rho_1^x]\,\pi^T + [\beta(\rho_0^\pi\rho_2^x + \rho_2^\pi) + \kappa\rho_2^x]\,\widetilde{x} \\ & + [\kappa\rho_3^x + \beta(\rho_0^\pi\rho_3^x + \rho_3^\pi)]\,\tau + [\beta\rho_0^\pi\rho_4^x + \kappa\rho_4^x + 1]\, e_t.\end{aligned} \tag{A.27}$$

Rearranging the terms in (A.23) and substituting π_t given by (A.24) yields

$$\begin{aligned}x_t = {} & \left[1 - \frac{\kappa}{\alpha}(\beta\rho_0^\pi\rho_0^x + \kappa\rho_0^x)\right] x_{t-1} + \frac{\kappa}{\alpha}[1 - \beta(\rho_0^\pi\rho_1^x + \rho_1^\pi) - \kappa\rho_1^x]\,\pi^T - \frac{\kappa}{\alpha}[\beta(\rho_0^\pi\rho_2^x + \rho_2^\pi) + \kappa\rho_2^x]\,\widetilde{x} \\ & - \frac{\kappa}{\alpha}\left[\kappa\rho_3^x + \beta(\rho_0^\pi\rho_3^x + \rho_3^\pi) + \frac{1}{2}\xi\right]\tau - \frac{\kappa}{\alpha}[\beta\rho_0^\pi\rho_4^x + \kappa\rho_4^x + 1]\, e_t.\end{aligned} \tag{A.28}$$

Comparing the coefficients in (A.27) and (A.28) with their respective counterpart in

(A.24) and (A.25) leads to

$$\begin{aligned}
\rho_0^\pi &= \beta\rho_0^\pi\rho_0^x + \kappa\rho_0^x, && \text{(A.29)}\\
\rho_1^\pi &= \beta\left(\rho_0^\pi\rho_1^x + \rho_1^\pi\right) + \kappa\rho_1^x, && \text{(A.30)}\\
\rho_2^\pi &= \beta\left(\rho_0^\pi\rho_2^x + \rho_2^\pi\right) + \kappa\rho_2^x, && \text{(A.31)}\\
\rho_3^\pi &= \kappa\rho_3^x + \beta\left(\rho_0^\pi\rho_3^x + \rho_3^\pi\right), && \text{(A.32)}\\
\rho_4^\pi &= \beta\rho_0^\pi\rho_4^x + \kappa\rho_4^x + 1, && \text{(A.33)}\\
\rho_0^x &= 1 - \frac{\kappa}{\alpha}\left(\beta\rho_0^\pi\rho_0^x + \kappa\rho_0^x\right), && \text{(A.34)}\\
\rho_1^x &= \frac{\kappa}{\alpha}\left[1 - \beta\left(\rho_0^\pi\rho_1^x + \rho_1^\pi\right) - \kappa\rho_1^x\right], && \text{(A.35)}\\
\rho_2^x &= -\frac{\kappa}{\alpha}\left[\beta\left(\rho_0^\pi\rho_2^x + \rho_2^\pi\right) + \kappa\rho_2^x\right], && \text{(A.36)}\\
\rho_3^x &= -\frac{\kappa}{\alpha}\left[\kappa\rho_3^x + \beta\left(\rho_0^\pi\rho_3^x + \rho_3^\pi\right) + \frac{1}{2}\xi\right], && \text{(A.37)}\\
\rho_4^x &= -\frac{\kappa}{\alpha}\left[\beta\rho_0^\pi\rho_4^x + \kappa\rho_4^x + 1\right], && \text{(A.38)}
\end{aligned}$$

Extracting ρ_0^π from (A.29) and (A.34) leads to

$$\rho_0^\pi = \frac{\kappa\rho_0^x}{1 - \beta\rho_0^x}, \qquad \text{(A.39)}$$

$$\rho_0^\pi = \frac{\alpha\left(1 - \rho_0^x\right) - \kappa^2\rho_0^x}{\kappa\beta\rho_0^x}, \qquad \text{(A.40)}$$

Equaling ρ_0^π given by (A.39) and (A.40), we get

$$\alpha\left(1 - \rho_0^x\right)\left(1 - \beta\rho_0^x\right) = \kappa^2\rho_0^x. \qquad \text{(A.41)}$$

Solving (A.41) leads to the solution of ρ_0^x. The other coefficients are solved in terms of ρ_0^x using (A.30)-(A.33) and (A.35)-(A.38). These solutions are reported in the main text (sub-subsection 3.2.2).

Using (A.30), (A.32), (A.35) and (A.37), we get some equivalences:

$$\begin{aligned}
\rho_1^\pi &= \frac{\beta\rho_0^\pi + \kappa}{1 - \beta}\rho_1^x && \text{(A.42)}\\
\rho_1^x &= \frac{\kappa\left(1 - \beta\right)\rho_0^x}{\alpha\left(1 - \beta\rho_0^x\right)} && \text{(A.43)}\\
\rho_3^\pi &= \frac{\beta\rho_0^\pi + \kappa}{1 - \beta}\rho_3^x, && \text{(A.44)}\\
\rho_3^x &= -\frac{1}{2}\frac{\kappa\xi\left(1 - \beta\right)\rho_0^x}{\alpha\left(1 - \beta\rho_0^x\right)} && \text{(A.45)}
\end{aligned}$$

In the steady state, $x_t = x_{t-1} = x$, we get using (A.24)-(A.25) that

$$\pi = \rho_0^\pi\tau + \rho_1^\pi x + \rho_3^\pi\pi^T \qquad \text{(A.46)}$$

$$x = \frac{\rho_0^x\tau + \rho_3^x\pi^T}{\left(1 - \rho_1^x\right)}, \qquad \text{(A.47)}$$

Using (A.40)-(A.45), Eqs. (A.46)-(A.47) can be solved as

$$\pi = -\frac{1}{2}\xi\tau + \pi^T \quad (A.48)$$

$$x = -\frac{1}{2}\frac{1-\beta}{\kappa}\xi\tau + \frac{1-\beta}{\kappa}\pi^T. \quad (A.49)$$

Inserting π and x by (A.48)-(A.49) into the government's utility function (5) yields

$$\tau = \frac{2\alpha(1-\beta)^2\pi^T - 2\alpha\kappa(1-\beta)\widetilde{x}}{\left[\alpha(1-\beta)^2 + \kappa^2\right]\xi + 2\varsigma\kappa^2}. \quad (A.50)$$

Inserting the optimal solution of τ given by (A.50) into (A.48)-(A.49) implies

$$\pi = \frac{(\xi + 2\varsigma)\kappa^2}{\left[\alpha(1-\beta)^2 + \kappa^2\right]\xi + 2\varsigma\kappa^2}\pi^T + \frac{\alpha\kappa\xi(1-\beta)}{\left[\alpha(1-\beta)^2 + \kappa^2\right]\xi + 2\varsigma\kappa^2}\widetilde{x}, \quad (A.51)$$

and

$$x = \frac{(1-\beta)(\xi + 2\varsigma)\kappa}{\xi\left[\alpha(1-\beta)^2 + \kappa^2\right] + 2\varsigma\kappa^2}\pi^T + \frac{\alpha\xi(1-\beta)^2}{\xi\left[\alpha(1-\beta)^2 + \kappa^2\right] + 2\varsigma\kappa^2}\widetilde{x}. \quad (A.52)$$

In the steady state, we have $\lim_{t\to+\infty} \pi_t = \pi^T$, for $\tau = 0$, meaning an inflation contract for the CB is useless for fighting inflation bias due to the "overambitious" output gap target. Even it is not optimal from the government's point of view, we will consider that the government will not impose an inflation penalty different from zero to avoid reintroducing inflation bias through inflation contract.

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