Economics Bulletin

Volume 42, Issue 4

PAYG pensions and endogenous retirement revisited

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Abstract

This paper presents an OLG model with endogenous retirement and endogenous growth. The purpose is to show analytically the effects of a PAYG pension system on the economy in such a setting. Firstly, it is shown that a PAYG system is neutral in capital intensity. Secondly, we analytically characterize the conditions that determine the effect of a PAYG system on welfare, and show that a PAYG system can be welfare improving. Thirdly, the analysis and the results apply to a non-steady-state equilibrium path.

I am indebted to Steinar Holden, Pan Liu, Øystein Thøgersen and two anonymous referees for valuable comments and suggestions.
Citation: Joachim Thøgersen, (2022) "PAYG pensions and endogenous retirement revisited", *Economics Bulletin*, Volume 42, Issue 4, pages 1826-1835
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Submitted: April 19, 2022. **Published:** December 30, 2022.

1 Introduction

The present analysis is motivated by theoretical contributions that study endogenous retirement in OLG models with production, as d'Autume (2003), Michel and Pestieau (2013), Chen (2018), Miyazaki (2019) and Liu and Thøgersen (2020). Starting from these articles our purpose is to revisit the topic of how a PAYG pension system affect the capital-labor ratio and welfare, when old individuals make a choice on retirement.

The current paper departs from the classical textbook presentation (Azariadis, 2000; de la Croix and Michel, 2002) in two respects. Firstly, we extend the standard OLG model ala Diamond (1965) with endogenous retirement. Secondly, we consider an endogenous growth model where productivity depends on cumulated aggregate investment per worker. To keep it tractable, the model applies a logarithmic utility function and a Cobb-Douglas production function. This formulation provides an interesting simplification that makes it possible to achieve analytical results and study the non-steady-state equilibrium path.

The results in the current paper are twofold. Firstly, we use a simple set-up to show that capital intensity and thus factor prices are independent of the PAYG pension scheme, thus the PAYG scheme is neutral in the capital-labor ratio. This result corresponds to Proposition 1 in Liu and Thøgersen (2020). Within this set-up we can also derive analytically the conditions that determine how a PAYG program will affect welfare. In particular we demonstrate that a PAYG program may increase welfare, if the value for leisure in old age is above a specific critical value.

Secondly, it is shown that these results also hold in an endogenous growth model, and in an equilibrium outside the steady-state. Thus, the analysis applies to non-steady-state equilibrium, and is not confined to steady-state as in Michel and Pestieau (2013) and Liu and Thøgersen (2020). Moreover, we derive the optimal and time invariant PAYG contribution rate.

The rest of the paper is organized as follows. Section 2 presents the model set-up. Section 3 investigates the effect of a PAYG system on the capital-labor ratio, capital accumulation, labor supply and the welfare, along a non-steady-state path. Section 4 concludes.

2 The model

2.1 Consumers and endogenous retirement

In any period, population is assumed to be stationary and the number of individuals in each generation is normalized to one. Individuals live for two periods. In their first period they are young and inelastically supply one unit of labor. The budget constraint of the young in period t reads as:

$$c_{1,t} = (1-\tau_t)w_t - s_t \, , \tag{1}$$

where $c_{1,t}$ is consumption as young, w_t is labor income, s_t is savings , and $\tau_t \in (0,1)$ denotes the contribution rate.

In their second period individuals are old and endogenously choose their retirement age. The budget constraint of the elderly states that consumption equals accumulated savings, net labor income earned as old $(1-\tau_{t+1})w_{t+1}l_{t+1}$, where $l_{t+1} \in (0,1)$ is labor supply as old, and pension benefits $P_{t+1}(1-l_{t+1})$:

$$c_{2,t+1} = s_t R_{t+1} + (1-\tau_{t+1})w_{t+1}l_{t+1} + P_{t+1}(1-l_{t+1}) \, , \tag{2}$$

where R_{t+1} is the real interest factor. The representative individual chooses savings and old-age labor supply to maximize the lifecycle utility function:

$$u(c_{1,t}, c_{2,t+1}, 1-l_{t+1}) = \ln c_{1,t} + \rho \left[\ln c_{2,t+1} + \mu \ln(1-l_{t+1})\right] \, , \tag{3}$$

subject to the constraints in (1) and (2), where $\rho \in (0,1)$ is the subjective discount factor, and μ is a parameter that measures preference for retirement.

In every period, the government balances the tax income and the social security benefit in a PAYG program. The balanced budget scheme is thus:

$$P_{t+1}(1-l_{t+1}) = (1+l_{t+1})\tau_{t+1}w_{t+1} \, . \tag{4}$$

Exploiting the first order conditions together with the budget constraint of the government in (4), optimal savings and optimal old-age labor supply are, respectively:

$$s_t = \frac{\rho\,(1+\mu)\,(1-\tau_t)\,R_{t+1}w_t - (1-\tau_{t+1})\,w_{t+1}}{(1+\rho+\rho\mu)\,R_{t+1}} \, , \tag{5}$$

$$l_{t+1} = \frac{1+\rho - [2(1+\rho)+\rho\mu]\,\tau_{t+1} - \rho\mu R_{t+1}(1-\tau_t)\frac{w_t}{w_{t+1}}}{1+\rho+\rho\mu}. \quad (6)$$

2.2 Firms

Firms are identical and the representative firm produces according to a Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ where Y_t, K_t and L_t are output, capital and labor respectively, A_t is total factor productivity, and $\alpha \in (0,1)$. The capital stock is assumed to depreciate completely in each period. As firms are assumed to maximize profits in competitive markets, the input factors are rewarded by their marginal products, taking A_t as given:

$$w_t = (1-\alpha)\,A_t K_t^{\alpha} L_t^{-\alpha} \quad (7)$$

$$R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \quad (8)$$

Economic growth is endogenized by using an Arrow-Romer approach, where productivity is determined by cumulated aggregate investment per worker. Specifically, the productivity index A_t is endogenized by assuming that there exists a concave relationship between productivity and cumulated aggregate investment per individual:[1]

$$A_t = A\left(\frac{K_t}{L_t}\right)^{\beta}, \quad (9)$$

where $A > 0$ is a technological parameter. Inserting this formulation of A_t into the production function, gives $Y_t = Ak_t^{\alpha+\beta} L_t$. Production per unit of labor is then given by the production function $y_t = Ak_t^{\alpha+\beta}$, where $y_t := Y_t/L_t$ and $k_t := K_t/L_t$.

Substituting (9) into (7) and (8) yields:

$$w_t = (1-\alpha)Ak_t^{\alpha+\beta}, \quad (10)$$

$$R_t = \alpha Ak_t^{\alpha+\beta-1}. \quad (11)$$

[1] See Arrow (1962) and Romer (1986) for the original set-up.

3 Equilibrium analysis

The market-clearing condition in the capital market is $K_{t+1} = s_t$. By invoking the definition of k, and that equilibrium in the labor market implies $L_t = 1+l_t$, equilibrium in the capital market becomes:

$$(1 + l_{t+1})k_{t+1} = s_t \,. \tag{12}$$

3.1 Capital dynamics, labor supply and savings

The model is now described by the discrete dynamic system characterized by optimal savings, optimal labor supply and the capital market equilibrium. To solve this system in terms of capital intensity, we substitute for savings and labor supply into (12). Also plugging in (10) and (11), this becomes:

$$k_{t+1} = \frac{\alpha\rho A\,(1-\alpha+\mu)\,(1-\tau_t)}{(1+\alpha+2\alpha\rho+\alpha\rho\mu)\,(1-\tau_{t+1})} k_t^{\alpha+\beta} \,. \tag{13}$$

If the government implement a constant policy, that is, $\tau_t = \tau$, then the above equation becomes:

$$k_{t+1} = \frac{\alpha\rho A(1-\alpha+\mu)}{1+\alpha(1+2\rho+\rho\mu)} k_t^{\alpha+\beta} \,. \tag{14}$$

Thus, in equilibrium, the old-age labor supply in period $t+1$ equals:

$$l_{t+1} = \frac{1-\alpha-\alpha\mu-(2+\mu)\,(1-\alpha)\,\tau}{1-\alpha+\mu} \,.$$

So the total labor supply in period $t+1$ is:

$$L_{t+1} = 1 + l_{t+1} = \frac{(2+\mu)\,(1-\alpha)\,(1-\tau)}{1-\alpha+\mu} \,, \tag{15}$$

and the capital stock at the beginning of period $t+1$ is:

$$K_{t+1} = s_t = \frac{\alpha\rho A\,(2+\mu)\,(1-\alpha)\,(1-\tau)}{1+\alpha+2\alpha\rho+\alpha\rho\mu} k_t^{\alpha+\beta} \,. \tag{16}$$

From equation (14) we have the following proposition:

Proposition 1 *In an OLG model with logarithmic utility, concave production, endogenous retirement, and endogenous growth, the PAYG contribution rate does not affect the evolution of capital intensity, i.e. the PAYG system is neutral.*

Accordingly, the neutrality result in Liu and Thøgersen (2020) still holds in a model with endogenous growth and without restricting the analysis to steady-state. Besides, in the current model the set-up is simpler and the intuition more transparent. An increase in the contribution rate will reduce savings, and reduce old-age labor supply. Lower savings reduce capital intensity, while lower labor supply increase capital intensity. But, as shown in proposition 1, these contradictory effects will offset each other within the present model.

Even though capital intensity is independent of the PAYG contribution rate, it will affect capital accumulation, labor supply and output in the economy. To study this, we compare two economies with different contribution rates.

3.2 A two country illustration

Consider two economies, A and B, where both countries implement a constant contribution rate pension program, but with $\tau^A < \tau^B$. Then along the path, the capital-labor ratio in both countries evolve according to (14). However, the capital accumulation and labor supply in country A and B are different.

Obviously, from equation (15) and (16), with $\tau^A < \tau^B$, if $k_t^A = k_t^B$ at t (which implies $k_{t+j}^A = k_{t+j}^B$ for all $j \geq 1$), the labor supply and capital accumulation and thus the aggregate production are higher in country A, that is, $L_{t+1}^A > L_{t+1}^B$, $K_{t+1}^A > K_{t+1}^B$ and $Y_{t+1}^A > Y_{t+1}^B$. A larger size of the pension program (a higher contribution rate) gives consumers a disincentive to work. In this set-up, since the quantity of labor supply $(1 + l_{t+1})$ doesn't equal the number of individuals $(1 + 1)$, the "output per unit of labor" in countries A and B is the same, but the "output per capita" in each period is different.

Let us also look at consumption. It can easily be shown that for generation t, their young-age consumption and old-age consumption are given by:

$$c_{1,t} = \frac{A(1+\alpha)(1-\alpha)(1-\tau)}{1+\alpha+2\alpha\rho+\alpha\rho\mu} k_t^{\alpha+\beta},$$

$$c_{2,t+1} = \frac{A(1+\alpha)(1-\alpha)(1-\tau)}{1-\alpha+\mu} k_{t+1}^{\alpha+\beta}.$$

Hence, since $\tau^A < \tau^B$ and both countries have the same capital intensity in each period, consumers in country A consume more both when young and when old. However, they also work longer when old and retire later. Thus, the effect on welfare is ambiguous. However, as will be shown in the next section, it is possible to characterize the conditions that determine the welfare implications.

3.3 Welfare

In this section we study welfare effects of the PAYG program. In particular, we consider whether utility is higher in a country with a relatively large contribution rate, compared to a country with a relatively small contribution rate, given the same preferences, and not constrained to the steady-state. We still assume $k_t^A = k_t^B$ at t, which implies $k_{t+j}^A = k_{t+j}^B$ for all j. Notice that in opposition to several other studies, we compare two countries with different policies in a non-steady-state environment, instead of the dynamics before and after a policy change within the same country. Besides, we derive the optimal contribution rate.

Since we are comparing two cases with the same capital-labor ratio, and only consider pension programs with constant contribution rate, we can insert the solutions for $c_{1,t}, c_{2,t+1}$ and l_{t+1} into (3) and rewrite the utility function as:

$$\begin{aligned} u_t(\cdot) = {} & (1+\rho)\ln(1-\tau) + \rho\mu\ln\left[\mu(1+\alpha) + (2+\mu)(1-\alpha)\tau\right] + h(t) \\ & - \rho\mu\ln(1-\alpha+\mu), \end{aligned}$$

where

$$h(t) = \ln\frac{A(1+\alpha)(1-\alpha)}{1+\alpha+2\alpha\rho+\alpha\rho\mu} k_t^{\alpha+\beta} + \rho\ln\frac{A(1+\alpha)(1-\alpha)}{1-\alpha+\mu} k_{t+1}^{\alpha+\beta},$$

is a function of time and k_0 since the capital-labor ratio evolves according to (14).

Define $\phi(\tau) = (1+\rho)\ln(1-\tau) + \rho\mu \ln[\mu(1+\alpha) + (2+\mu)(1-\alpha)\tau]$. Then

$$\frac{\partial \phi}{\partial \tau} = \frac{\mu[\rho(2+\mu)(1-\alpha) - (1+\rho)(1+\alpha)] - (1+\rho+\rho\mu)(2+\mu)(1-\alpha)\tau}{(1-\tau)[\mu(1+\alpha) + (2+\mu)(1-\alpha)\tau]},$$

and

$$\frac{\partial^2 \phi}{\partial \tau^2} = -\frac{1+\rho}{(1-\tau)^2} - \rho\mu \frac{(2+\mu)^2(1-\alpha)^2}{[\mu(1+\alpha) + (2+\mu)(1-\alpha)\tau]^2} < 0,$$

i.e. ϕ is concave in τ. Evaluating utility near $\tau = 0$ yields:

$$\left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=0} = \frac{\rho(2+\mu)(1-\alpha) - (1+\rho)(1+\alpha)}{1+\alpha}$$

$$\begin{cases} > 0 & \text{if } \rho(2+\mu)(1-\alpha) - (1+\rho)(1+\alpha) > 0 \\ < 0 & \text{if } \rho(2+\mu)(1-\alpha) - (1+\rho)(1+\alpha) < 0 \end{cases}$$

And

$$\left.\frac{\partial \phi}{\partial \tau}\right|_{\tau \to 1} = -\infty$$

Accordingly, we can deduce the following proposition.

Proposition 2 *In an OLG model with logarithmic utility, concave production, endogenous retirement, and endogenous growth, the effect of the PAYG contribution rate on welfare depends on certain conditions determined by $\partial\phi/\partial\tau$ evaluated near $\tau = 0$. That is*

(i) If $\left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=0} < 0$, then $\frac{\partial \phi}{\partial \tau} < 0$ for all $\tau \in (0,1)$. An increase in the contribution rate reduce utility, thus the optimal choice is no pension program, i.e. $\tau = 0$.

(ii) If $\left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=0} > 0$, then there exists a $\tau^* \in (0,1)$ where $\left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=\tau^*} = 0$ such that $\phi(\tau^*) = \max \phi(\tau)$. The optimal contribution rate is when $\frac{\partial \phi}{\partial \tau} = 0$, that is,

$$\tau^* = \frac{\mu[\rho(2+\mu)(1-\alpha) - (1+\rho)(1+\alpha)]}{(1+\rho+\rho\mu)(2+\mu)(1-\alpha)}.$$

When $\tau < \tau^*$, $\frac{\partial \phi}{\partial \tau} > 0$, i.e., an increase in the PAYG contribution rate will increase welfare. When $\tau > \tau^*$, $\frac{\partial \phi}{\partial \tau} < 0$, i.e., an increase in the PAYG contribution rate will reduce welfare.

The optimal contribution rate is affected by the preference for retirement, the individual's discount factor and the capital share in the production function. The effect of these factors on the optimal contribution rate is ambiguous.

Proposition 2 shows the effect of the PAYG contribution rate on welfare. In particular, it characterize the conditions for welfare gains. Let us elaborate this result. Note that $\frac{\partial\phi}{\partial\tau}\big|_{\tau=0} > 0$, if $\rho\,(2+\mu)\,(1-\alpha) - (1+\rho)\,(1+\alpha) > 0$. Which is equivalent to:

$$\mu > \frac{(1+\rho)\,(1+\alpha)}{\rho\,(1-\alpha)} - 2\,.$$

For reasonable values of α (for example, $\alpha = 1/3$), $\frac{(1+\rho)(1+\alpha)}{\rho(1-\alpha)} - 2 > 0$. So the above inequality requires that the weight consumers put on the leisure is above a critical value, that is, the consumer cares relatively a lot about leisure. Then, with a larger size of the pension program, the utility gains from the increased leisure can compensate the utility loss from reduced consumption.

4 Conclusion

In this paper we examine the effect of a PAYG pension system on capital dynamics and welfare in an endogenous growth model, when retirement is endogenous. The formulation of the model makes it an analytically tractable setting which allows us to study non-steady-state equilibrium, and explicitly characterize conditions for welfare implications.

Our first finding is that capital intensity is unaffected by the PAYG system. This result is similar to proposition 1 in Liu and Thøgersen (2020), however our analysis is not confined to steady-state. Thus, the originality of this result is that the neutrality of the PAYG system on capital intensity is not restricted to steady-state, and not only holds under exogenous but also endogenous growth. The second finding is that countries with a relatively low pension contribution rate have higher capital accumulation, labor supply and output. Our third finding is that if a consumer values leisure and retirement beneath a critical value, an increase in the pension contribution rate will reduce welfare and no PAYG system is optimal. But, if the consumer values leisure and retirement above a critical value, there exists an optimal pension contribution rate, and if the contribution rate is below this level, a higher contribution rate will increase welfare. This result also applies to non-steady-state equilibrium.

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