Economics Bulletin

Volume 43, Issue 1

On the stationary distribution of income and wealth in a growing economy with endogenous labor supply

Kazuo Mino
Institute of Economic Research, Kyoto University

Abstract

In the context of a perpetual youth model with capital, we explore the effect of the labor supply behavior of households on the stationary distributions of income and wealth. Assuming that the households have Greenwood--Hercowitz--Huffman preferences, we show that inequality in income and wealth distributions increase with the elasticity of labor supply.

Citation: Kazuo Mino, (2023) "On the stationary distribution of income and wealth in a growing economy with endogenous labor supply", *Economics Bulletin*, Volume 43, Issue 1, pages 108-115
Contact: Kazuo Mino - mino@kier.kyoto-u.ac.jp.
Submitted: October 23, 2022. **Published:** March 30, 2023.

1 Introduction

Using neoclassical growth models with heterogeneous agents and idiosyncratic shocks, several authors reveal that stationary distribution of income and wealth exhibits Pareto profile: see, for example, Benhabib et al. (2011 and 2016), Jones (2014 and 2015), Hiraguchi (2019), Nirei and Aoki (2015), and Moll et al. (2022)[1].Those contributions assume that the labor supply of households is fixed over time. In this note, we explore the effect of endogenous labor supply on the stationary distribution of income and wealth. In the context of a perpetual youth model[2], we assume that households have Greenwood–Hercowitz–Huffman (GHH) preferences under which labor supply is independent of income effect. We show that stationary distribution of income and wealth becomes more unequal as the elasticity of labor supply rises.

2 Model

Time is continuous. The number of households born at time t is B_t. We assume that B_t changes at a constant rate of b so that $B_t = B_0 e^{bt}$. Each household may die in each moment according to a Poisson process with an intensity $m\,(>0)$. Thus, the number of households born at s who survive at $t\,(\geq s)$ is $N_{s,,t} = B_s e^{-m(t-s)}$, meaning that the total population at t is

$$N_t = \int_{-\infty}^{t} N_{s,t} ds = e^{-mt} \int_{-\infty}^{t} B_0 e^{-(b+m)s} ds.$$

As a result, the total population changes according to

$$\dot{N}_t = B_t - mN_t.$$

We focus on the steady state of the population dynamics where $\dot{N}_t/N_t = \dot{B}_t/B_t = b$, and thus it holds that

$$N_t = \frac{B_0 e^{bt}}{b+m}.$$

We allow a negative birth rate $(b < 0)$ but we assume that $b + m > 0$ to keep the total population positive. In addition, we normalize $B_0 = b + m$. Therefore, the total population follows

$$N_t = e^{bt}. \tag{1}$$

Faced with the probability of death, the objective function of the households born at time s is

$$U_s = \int_{s}^{\infty} e^{-(\rho+m)(t-s)} \log \left(c_{s,t} - \frac{n_{s,t}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) dt, \quad \gamma > 0,$$

where $c_{s,t}$ and $n_{s,t}$ respectively denote consumption and labor supply of the households born at s, and $\rho\,(>0)$ is a time discount rate. We assume that the instantaneous utility function takes a GHH form given by Greenwood et al. (1998) that have been frequently used in the

[1] Jones (2014 and 2015) discusses models with death and birth processes. Benhabib et al.(2011 and 2016) and Hiraguchi (2019) utilize overlapping generations models. Nirei and Aoki (2015) explore a neoclassical growth model with idiosyncratic investment shocks. Moll et al. (2022) study the impacts of automation on income and wealth distribution. Those studies rely on the mechanisms that generate power law: see Gabaix (2009) for a useful survey.

[2] Our analytical framework is close to the model studied by Buiter (1989) who assumes that the total population may change.

business cycle litrature[3]. The households maximize U_s by choosing $\{c_{s,t}, n_{s,t}\}_{t=s}^{\infty}$ subject to the flow budget constraint

$$\dot{a}_{s,t} = (r_t + m)\, a_{s,t} + w_t n_{s,t} - c_{s,t}, \tag{2}$$

where $a_{s,t}$, r_t, and w_t denote the asset holding, the net rate of return to assets, and the real wage, respectively. Following Yaari (1969) and Blanchard (1985), we assume the presence of fair insurance, and, hence, the return to asset received by the households involves a risk premium, m. The optimal choice must satisfy the no-Ponzi-game constraint such that $\exp\left(-\int_t^v (r_\mu + m)\, d\mu\right) a_{s,v} \geq 0$. We assume that households have no bequest motive, so that the initial condition on asset holding is $a_{s,s} = 0$.

Denoting the utility value of asset by $q_{s,t}$, the first-order conditions for an optimum include the following:

$$\begin{aligned}
\left(c_{s,t} - \frac{n_{ns,t}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right)^{-1} &= q_{s,t}, && (3)\\
n_{s,t}^{\frac{1}{\gamma}} \left(c_{s,t} - \frac{n_{s,t}^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right)^{-1} &= w_t q_{s,t}, && (4)\\
\dot{q}_{s,t} &= q_{s,t}\, (\rho - r_t)\, q_{s,t}, && (5)
\end{aligned}$$

together with the transversality condition: $\lim_{t\to\infty} e^{-(\rho+m)t} q_{s,t} a_{s,t} = 0$. Conditions (3) and (4) give

$$n_{s,t} = w_t^{\gamma}, \tag{6}$$

which represents the labor supply of each household. The labor supply is independent of the income effect, and the elasticity of labor supply is γ. When $\gamma = 0$, each household supplies one unit of labor in each moment.

Using (6), we define the 'net' consumption in the following manner:

$$\widetilde{c}_{s,t} = c_{s,t} - \frac{\gamma w_t^{1+\gamma}}{1+\gamma}. \tag{7}$$

We restrict our attention to the case where $\widetilde{c}_{s,t} > 0$. From (3) and (5), the Euler equation of the net consumption is

$$\frac{d}{dt} \widetilde{c}_{s,t} = (r_t - \rho)\, \widetilde{c}_{s,t}. \tag{8}$$

Using $\widetilde{c}_{s,t}$, the flow budget constraint (2) is rewritten as

$$\dot{a}_{s,t} = (r_t + m)\, a_{s,t} + \frac{1}{1+\gamma} w_t^{1+\gamma} - \widetilde{c}_{s,t}. \tag{9}$$

Hence, when both the no-Ponzi-game and transverality conditions are held, the intertemporal

[3] Ascari, and Rankin (2007) study a perpetual youth model in which households have GHH preferences. The central concern of their study is to examine the effects of fiscal and monetary policies, and the authors do not discuss income and wealth distribution.

budget constraint at time t is expressed as

$$\int_t^\infty \exp\left(-\int_t^v (r_\mu + m)\, d\mu\right) \widetilde{c}_{s,v} dv = a_{s,t} + \int_t^\infty \exp\left(-\int_t^v (r_\mu + m)\, d\mu\right) \frac{1}{1+\gamma} w_v^{1+\gamma} dv. \tag{10}$$

Using (8) and (10), we obtain

$$\widetilde{c}_{s,v} = (\rho + m)(a_{s,t} + h_t), \tag{11}$$

where h_t is a modified human wealth defined as

$$h_t = \int_t^\infty \exp\left(-\int_t^v (r_\mu + m)\, d\mu\right) \frac{w_v^{1+\gamma}}{1+\gamma} dv. \tag{12}$$

Consequently, the optimal consumption at time t is

$$c_{s,t} = (\rho + m)(a_{s,t} + h_t) + \frac{\gamma w_t^{1+\gamma}}{1+\gamma}. \tag{13}$$

The production side of the model is standard. There is a continuum of identical firms with a unit mass. The aggregate production function is

$$Y_t = A K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where L_t is the aggregate labor input. Condition (6) means that L_t is determined by

$$L_t = w_t^\gamma N_t. \tag{14}$$

Factor markets are competitive, and the factor prices are given by

$$r_t = \alpha \frac{Y_t}{K_t} - \delta, \tag{15}$$

$$w_t = (1-\alpha) \frac{Y_t}{L_t}, \tag{16}$$

where $\delta \in [0, 1)$ is the depreciation rate of capital.

Define the aggregate consumption and asset:

$$C_t = \int_{-\infty}^t c_{s,t} N_{s,t} ds, \quad A_t = \int_{-\infty}^t a_{s,t} N_{s,t} ds.$$

The aggregate asset changes according to

$$\dot{A}_t = (r_t + m) A_t + w_t N_t - C_t - m A_t = r_t A_t + w_t L_t - C_t, \tag{17}$$

and from (13), C_t satisfies

$$C_t = (\rho + m)(A_t + h_t N_t) + \frac{\gamma w_t^{1+\gamma}}{1+\gamma} N_t. \tag{18}$$

Note that $m A_t$ is transferred from the households who die at t to the existing households, meaning that the aggregate net revenue from asset holding is $r_t A_t$. The market equilibrium condition for the asset market is

$$A_t = K_t. \tag{19}$$

From (15) and (16), and (19), we see that (17) also represents the market equilibrium condition for final goods: $Y_t = \dot{K}_t + \delta K_t + C_t$.

3 Income and Wealth Distribution in the Steady State

In what follows, we focus on the balanced-growth equilibrium where Y_t, K_t, and L_t change at a common rate of $\dot{N}_t/N_t = b$. Thus, (15) and (16) mean that r_t and w_t stay constant over time. Therefore, (12) becomes

$$h = \frac{w^{1+\gamma}}{(1+\gamma)(r+m)}. \tag{20}$$

Using (18) and (20), we see that in the balanced-growth equilibrium (17) yields

$$bK_t = rK_t + w^{1+\gamma}N_t - (\rho+m)\left[K_t + \frac{w^{1+\gamma}}{(1+\gamma)(r+m)}N_t\right] - \frac{\gamma w^{1+\gamma}}{1+\gamma}N_t. \tag{21}$$

Define $K_t/w_t^{1+\gamma}N_t = x_t$, which is constant in the steady state. Then (21) can be written as

$$x = \frac{1}{(1+\gamma)(b+m+\rho-r)}\left[1 - \frac{\rho+m}{(r+m)}\right]. \tag{22}$$

Moreover, (15) and (16) yield

$$\frac{w_t}{r_t+\delta} = \left(\frac{1-\alpha}{\alpha}\right)\frac{K_t}{L_t}. \tag{23}$$

From (6), it holds that $L_t = w_t^{\gamma}N_t$, so that in the balanced-growth equilibrium (23) is expressed as

$$x = \frac{\alpha}{(1-\alpha)(r+\delta)}. \tag{24}$$

Combining (22) and (24), we obtain

$$\frac{\alpha}{(1-\alpha)(r+\delta)} = \frac{1}{(1+\gamma)(b+m+\rho-r)}\left[1 - \frac{\rho+m}{(r+m)}\right]. \tag{25}$$

Figure 1 depicts the graphs of the left-hand side (LHS) and the right-hand side (RHS) of (25). As the figure shows, (25) has a unique solution denoted by r^*. Note that a rise in γ shifts the graph of RHS downward, which leads to a higher r^*.

Figure 1

Following Moll et al. (2021), we define the effective wealth of cohort s as $\omega_{s,t} = a_{s,t} + h$. Then we see that

$$\dot{\omega}_{s,t} = \dot{a}_{s,t} = (r-\rho)\left(a_{s,t} + \frac{w^{1+\gamma}}{(1+\gamma)(r+m)}\right) = (r-\rho)\,\omega_{s,t}. \tag{26}$$

Define the complementary cumulative distribution function (CCDF) of effective wealth in the following manner:

$$G(\omega, t) = \Pr(\omega_{s,t} \geq \omega) \quad \text{for } \omega \in [h, \infty),$$

which expresses the share of households whose effective wealth is larger than ω. The density function given by $g(\omega,t) = -\frac{\partial}{\partial\omega}G(\omega,t)$ satisfies the following Kolmogorov forward equation:

$$\frac{\partial}{\partial t}g(\omega,t) = -\frac{\partial}{\partial\omega}\left[(\rho-r)\,\omega g(\omega,t)\right] - (b+m)\,g(\omega,t).$$

The stationary density function is independent of t, and it fulfills

$$(r-\rho)\left[g(\omega)+\omega g'(\omega)\right] + (b+m)\,g(\omega) = 0. \tag{27}$$

By the use of guess and verify method[4], we find that the solution of (27) is written as $g(\omega) = \frac{b+m}{r-\rho}\left(\frac{1}{h}\right)\left(\frac{\omega}{h}\right)^{-\frac{b+m}{r-\rho}-1}$. Hence, the stationary CCDF is given by

$$G(\omega) = \left(\frac{\omega}{h}\right)^{-\frac{b+m}{r-\rho}} \quad \text{for } \omega \in [h,\infty). \tag{28}$$

meaning that the cumulative distribution function (CDF) is $1-G(\omega) = 1-\left(\frac{\omega}{h}\right)^{-\frac{b+m}{r-\rho}}$. That is, CDF of ω exhibits a Pareto profile with a shape parameter $\zeta = \frac{b+m}{r-\rho}$. Note that the distribution functions of asset, $a\,(=\omega-h)$, and income, $y=(r+m)\,a+wn$, respectively satisfy the following:

$$\begin{aligned}
\Pr(a_{s,t} \geq a) &= \Pr(a_{s,t}+h \geq a+h) = G(\omega),\\
\Pr(y_{s,t} \geq y) &= \Pr\left(\frac{y_{s,t}}{r+m} \geq \frac{y}{r+m}\right) = \Pr(\omega_{s,t} \geq \omega) = G(\omega).
\end{aligned}$$

Therefore, the stationary distributions of asset and income have the same profiles as that of the effective wealth.

The reciprocal of the shape parameter (tail index) given by

$$\frac{1}{\zeta} = \frac{r^*-\rho}{b+m}$$

is a measure of inequality. Hence, a higher r^* means a higher degree of inequality. Since we have found that r^* increases with the elasticity of labor supply, γ, flexible labor supply raises inequality of income and wealth in the long run. Intuitively, the term $\gamma {w_t}^{1+\gamma}/(1+\gamma)$ in (7) plays the same role as the subsistence consumption in the Stone-Geary utility function. Thus, its aggregate level, $\gamma w^{1+\gamma}/(1+\gamma)\,N_t$, involved in the right hand side of (21) corresponds to the aggregate subsistence consumption. This additional consumption depresses capital accumulation, which yields a higher rate of return to capital in the steady state than in the model with fixed labor supply. Furthermore, other things being equal, a higher γ yields a larger subsistence consumption, which suggests that r^* increases with the elasticity of labor supply.

4 Conclusion

This note studies the effect of labor supply behavior of the households on the long-run distribution of income and wealth . To keep the model tractable, we assume that households

[4]Suppose that $g(\omega) = \psi\zeta\omega^{-\zeta-1}$. Subsitituting this into (27) shows that $\zeta = \frac{b+m}{r-\rho}$. In addition, $\int_h^\infty g(\omega)\,d\omega = 1$ leads to $\psi = -\left(\frac{1}{h}\right)^{-\frac{b+m}{r-\rho}-1}$.

have GHH preferences in which labor supply is free from income effect. We have shown that long-run inequality of income and wealth increases with the elasticity of labor supply. Our finding reveals that log-run distribution of income and wealth would be sensitive to the households' labor supply behavior.

References

Ascari, G. and Rankin, N. 2007, Perpetual youth and endogenous labor supply: a problem and a possible solution, *Journal of Macroeconomics* 29(4): 708-723.

Behnabib, J., Bisin, A., and Zhu, S. 2011, The distribution of wealth and fiscal policy in wconomies with finitely-lived agents, *Econometrica* 79(1):, 123- 157.

Benhabib, J., Bisin, A., and Zhu, S. 2016, The distribution of wealth In the Blanchard–Yaari model, *Macroeconomic Dynamics* 20(2): 466-481.

Blanchard, O. 1985, Debt, deficits, and finite horizons, *Journal of Political Economy* 93(2): 223-247.

Buiter, W. 1989, Death, birth, productivity growth, and debt neutrality, *Economic Journal* 98(1): 279-293.

Gabaix, X. 2009, Power laws in economics and finance, *Annual Review of Economics* 1, 255-293.

Greenwood, J. Hercowitz, Z., and Huffman, G., 1988, Investment, capacity utilization, and the real business cycle, *American Economic Review* 78(3): 402–17.

Jones, C. 2014, Simple models of income and wealth inequality, unpublished manuscript.

Jones, C. 2015, Pareto and Piketty: the macroeconomics of top income and wealth inequality, *Journal of Economic Perspectives* 29(1): 29-46.

Hiraghuchi, R. 2019, Wealth inequality, or r-g, in the economic growth model, *Macroeconomics Dynamics* 23(2): 479-488.

Moll, B., Rachel, L., and Restrepo, P. 2021, Uneven growth: automation's impacts on income and wealth distribution", *Econometrica* 90, 2645-2663.

Nirei, M., & Aoki, S. 2016. Pareto distribution of income in neoclassical growth models,. *Review of Economic Dynamics*, 20, 25-42.

Yaari, M. 1965, Uncertain lifetime, life insurance, and the theory of the consumer, *Review of Economic Studies* 32(2): 137-160.

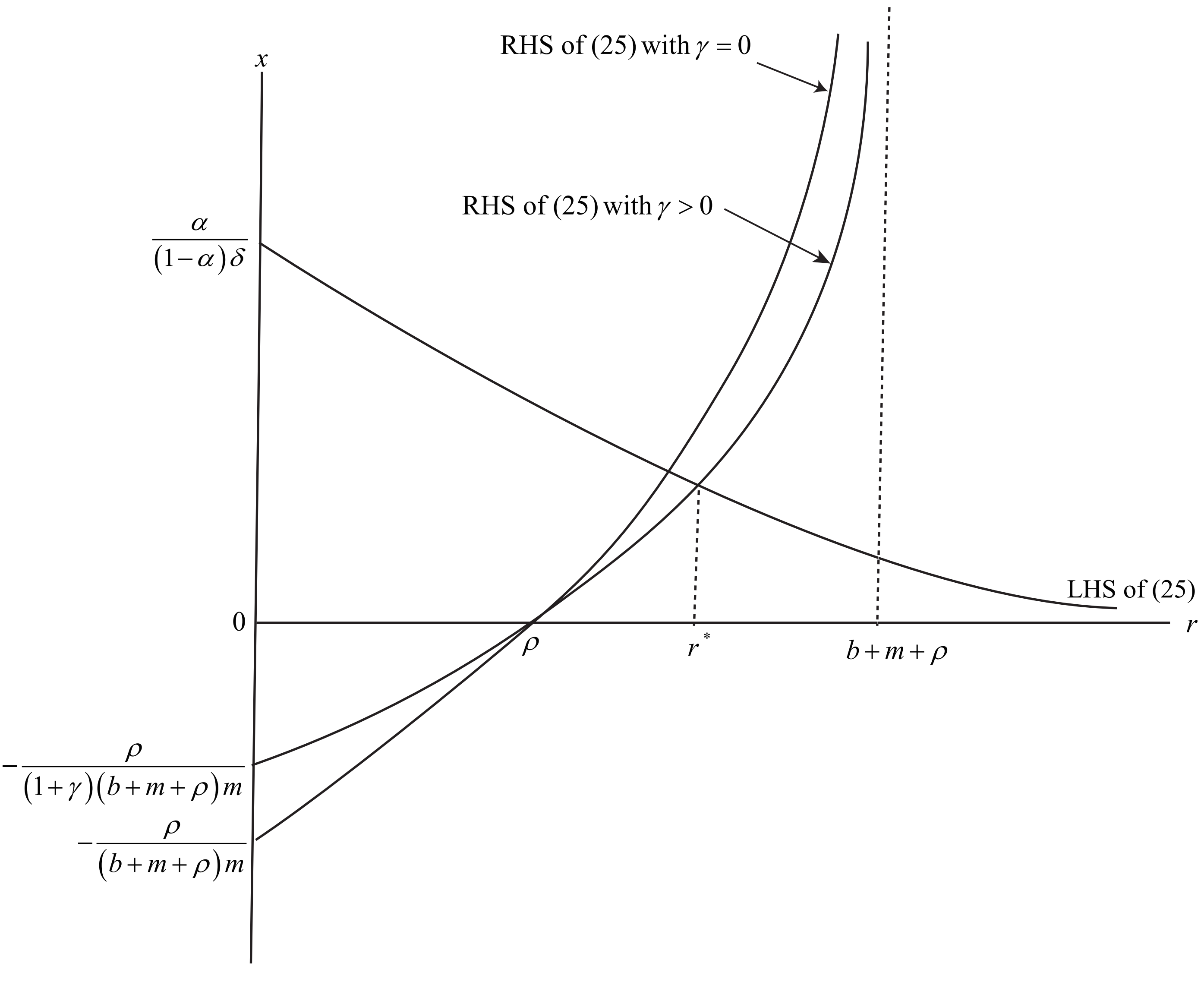

Figure 1 Determination of the steady-state rate or return to capital