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Time aggregation and unemployment volatility

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Abstract

This paper explores the importance of time aggregation for labor market fluctuations. Instead of correcting time aggregation bias in the data, we develop a simple search-matching model in which some individuals lose and find a job within a period to artificially generate the bias. The magnitude of time aggregation bias is highly procyclical. An increase in the degree of time aggregation bias is associated with a significantly lower unemployment volatility.

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1 Introduction

Worker flow data such as the Current Population Survey (CPS) contain measurement errors known as time aggregation bias. Because an individual's employment status in a certain month is defined by his or her status during a reference week at the end of that month, those who lose and find (or find and lose) a job within a month cannot be reported. Shimer (2012) develops a method for correcting the bias to measure the job-finding probability for the U.S. labor market.[1]

Instead of correcting time aggregation bias in the published data, this paper develops an *equilibrium unemployment model* containing time aggregation bias. Specifically, we present a simple search-matching model in which some individuals lose and find a job within a period. When a job is found, with a certain probability, a job seeker immediately becomes employed, rather than waits until the next period. We show that (i) the magnitude of time aggregation bias is highly procyclical and (ii) an increase in the immediate-employment probability significantly reduces unemployment volatility, worsening the unemployment volatility puzzle (Shimer, 2005).

2 The Model

2.1 Environment

The model is nearly identical to (the discrete-time version of) the textbook search-matching model of unemployment (Pissarides, 2000). In the textbook model, those who find a job must all wait until the next period to be employed. In contrast, the novel assumption we consider is that the timing of employment is randomly determined for those who find a job. Ex post, some are *immediately* employed within the period and the others are employed in the next period. This is the source of time aggregation bias.

The number of matches made within period t is determined by the matching technology $m_0 U^{\xi} V^{1-\xi}$, where $m_0 > 0$ and $0 < \xi < 1$ are parameters, U_t is the total number of job seekers, and V_t is the number of aggregate job vacancies. Let $V_t/U_t \equiv \theta_t$ denote the labor market tightness. A vacancy is matched to a worker during a period with probability q_t, where $q_t = m_0 U_t^{\xi} V_t^{1-\xi}/V_t = m_0 \theta_t^{-\xi} \equiv q(\theta_t)$. Similarly, the probability

[1] Related contributions include Elsby et al. (2009) and Fujita and Ramey (2009). Shimer (2012) provides a review of the literature.

that a worker is matched with a vacancy, or the job-finding rate, is given by f_t, where $f_t = m_0 U_t^{\xi} V_t^{1-\xi} / U_t = m_0 \theta_t^{1-\xi} = \theta_t q(\theta_t) \equiv f(\theta_t)$.

While all job seekers face the same matching technology and the job-finding rate, the timing of employment is assumed to be random. Given that a match is formed, with probability $0 \leq \phi \leq 1$, a job seeker is employed immediately *within the period*. With the remaining probability, he or she will be employed in the next period, as in the textbook model. The fact that a proportion of job seekers change their employment status within the period cannot be reported in labor statistics because measurement takes place *at the end of each period*.

All individuals discount the future by the common discount factor, $\beta < 1$. Each employed worker supplies one unit of labor to produce p_t units of output.[2] Let w_t and $b > 0$ denote the wage rate and the unemployment benefit, respectively. The values of being employed and unemployed, J_t^E and J_t^U, respectively satisfy

$$J_t^E = w_t + \lambda \beta \mathbb{E}_t J_{t+1}^U + (1-\lambda) \beta \mathbb{E}_t J_{t+1}^E, \tag{1}$$

$$J_t^U = b + f(\theta_t) \left[\phi J_t^E + (1-\phi) \beta \mathbb{E}_t J_{t+1}^E \right] + [1 - f(\theta_t)] \beta \mathbb{E}_t J_{t+1}^U, \tag{2}$$

where $0 < \lambda < 1$ is the exogenous separation rate. Separation occurs at the end of each period. Similarly, the values of a filled job and a vacancy, J_t^F and J_t^V, respectively satisfy

$$J_t^F = p_t - w_t + \lambda \beta \mathbb{E}_t J_{t+1}^V + (1-\lambda) \beta \mathbb{E}_t J_{t+1}^F, \tag{3}$$

$$J_t^V = -c + q(\theta_t) \left[\phi J_t^F + (1-\phi) \beta \mathbb{E}_t J_{t+1}^F \right] + [1 - q(\theta_t)] \beta \mathbb{E}_t J_{t+1}^V, \tag{4}$$

where $c > 0$ is the cost of posting a vacancy.

The wage rate is determined by asymmetric Nash bargaining, which requires

$$\eta \left(J_t^F - J_t^V \right) = (1-\eta) \left(J_t^E - J_t^U \right), \tag{5}$$

where $0 \leq \eta \leq 1$. Finally, the free entry condition, $J_t^V = 0$, determines the number of vacancies.

2.2 Time Aggregation Bias in Worker Flows

Let $\ell_t = 1 - u_t$ denote the number of employees in period t. Worker flows satisfy

$$\ell_t = (1-\lambda) \ell_{t-1} + (1-\phi) f_{t-1} u_{t-1} + \phi f_t u_t, \tag{6}$$

[2] This paper ignores variations in work hours. See Kudoh et al. (2019) and Kudoh and Miyamoto (2023) for business cycle models with endogenous work hours.

which captures the fact that a proportion of job seekers in period t are employed immediately to engage in production in period t. The term $\phi f_t u_t$ is the number of current job seekers who are classified as "employees" in period t.

The magnitude of *time aggregation bias*, denoted by τ_t, is

$$\tau_t = \phi f_t \lambda \ell_{t-1}. \tag{7}$$

This is the number of individuals who are incorrectly classified as those who did not change their employment status between $t-1$ and t. Note that $\phi f_t u_t$ is not the correct measure of time aggregation bias because it includes those who spent more than one period to find a job at the beginning of period t. Such individuals are correctly classified as those who changed their status from "unemployment" in period $t-1$ to "employment" in period t. Because the job-finding rate f_t is an increasing function of θ_t, the magnitude of time aggregation bias τ_t is procyclical, which supports the evidence (Nordmeier, 2014).

Given the magnitude of time aggregation bias, the *observed* flows from unemployment to employment during period t, denoted by UE_t, is $(1-\phi) f_t u_t$ because the remaining flows $\phi f_t u_t$ are (incorrectly) classified as job-to-job transitions. Thus, the observed job-finding rate is given by

$$f_t^* = \frac{UE_t}{UE_t + UU_t} = \frac{(1-\phi)\, f_t u_t}{(1-\phi)\, f_t u_t + (1-f_t)\, u_t} = \frac{(1-\phi)\, f_t}{1-\phi f_t} \leq f_t. \tag{8}$$

Evidently, the observed job-finding rate coincides with the actual rate if $\phi = 0$. Because f_t^* decreases with ϕ, time aggregation bias induces an underestimation of the job-finding rate.

2.3 Equilibrium

The following equilibrium conditions are straightforward extensions of those for the textbook search-matching model.[3] First, the job-creation condition is given by

$$\frac{c}{\Lambda q_t} + \frac{\Phi\,(p_t - w_t)}{\Lambda} = p_t - w_t + (1-\lambda)\,\beta \mathbb{E}_t \left[\frac{c}{\Lambda q_{t+1}} + \frac{\Phi\,(p_{t+1} - w_{t+1})}{\Lambda} \right], \tag{9}$$

[3]The details of the model equations are presented in Appendix A.

where $\Phi = (1-\phi)/(1-\lambda)$ and $\Lambda = \Phi + \phi$. Second, the wage equation for this economy is given by

$$\frac{\eta}{1-\eta}\left[\frac{c}{\Lambda q_t} + \frac{\Phi\left(p_t - w_t\right)}{\Lambda}\right] = \left(1 - f_t\phi\right) w_t - b + \left[\left(1 - f_t\phi\right)\left(1-\lambda\right) - f_t\left(1-\phi\right)\right] \times \beta\frac{\eta}{1-\eta}\mathbb{E}_t\left[\frac{c}{\Lambda q_{t+1}} + \frac{\Phi\left(p_{t+1} - w_{t+1}\right)}{\Lambda}\right]. \tag{10}$$

Finally, worker flows satisfies

$$1 - u_t = \left(1-\lambda\right)\left(1 - u_{t-1}\right) + \left(1-\phi\right) f_{t-1} u_{t-1} + \phi f_t u_t. \tag{11}$$

Evidently, the system of equations will reduce to those for the textbook model if we set $\phi = 0$.

3 Numerical Analysis

3.1 Calibration

We calibrate our model to match the published U.S. labor market facts. The description here is kept minimum as we follow the standard calibration strategy and targets in the literature (Shimer, 2005; Pissarides, 2009). The parameter values for β, ξ, λ, η, and the steady-state level of p, are exogenously given, while the values for m_0, b, and c are determined using the model equations and calibration targets. The model parameters are summarized in Table 1.

Consistent with labor statistics such as the CPS, the model period is one month. We set the monthly discount factor at $\beta = 0.996$ to match the annual real interest rate of approximately 4 percent. The matching elasticity with respect to the number of job seekers is set at $\xi = 0.5$. We set $\lambda = 0.036$ to match the average monthly U.S. separation rate. Worker's bargaining power is set at $\eta = 0.5$. The steady-state level of labor productivity is normalized at $p = 1$.

We target the steady-state labor market tightness of 0.72 and the monthly job finding rate of 0.594. These targets with $\xi = 0.5$ imply $m_0 = 0.7$. For robustness, we consider two calibration targets for the unemployment benefit b. We first follow Shimer (2005) to target $b/w = 0.4$. We then consider an alternative target at $b/w = 0.71$ (Hall and Milgrom, 2008; Pissarides, 2009). The vacancy cost c is obtained from the steady-state solution of the model.

Table 1: Parameters

Parameter	$b/w = 0.4$	$b/w = 0.71$	Description
β	0.996	0.996	Discount factor
ξ	0.5	0.5	Matching elasticity
λ	0.036	0.036	Exogenous separation rate
η	0.5	0.5	Worker's bargaining power
p	1.0	1.0	Steady-state productivity level
m_0	0.7	0.7	Matching efficiency
b	0.385	0.697	Unemployment benefits
c	0.752	0.371	Vacancy cost

3.2 Unemployment Volatility

We focus on fluctuations in the unemployment rate driven by productivity shocks. We assume that labor productivity p_t follows a first order autoregressive process such that $\log p_t - \log p = \rho(\log p_{t-1} - \log p) + \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma^2)$. We set $\rho = 0.97$ and $\sigma = 0.007$ to match the first-order autocorrelation and standard deviation of U.S. labor productivity in the data. Table 2 presents standard deviations of selected variables of our log-linearized model under different levels of ϕ for the two calibration targets.[4]

Table 2: Unemployment Volatility

		$b/w = 0.4$				$b/w = 0.71$			
	$\hat{p}$	$\hat{\tau}$	$\hat{f}$	$\hat{\theta}$	$\hat{u}$	$\hat{\tau}$	$\hat{f}$	$\hat{\theta}$	$\hat{u}$
$\phi = 0$	0.022	0.033	0.018	0.035	0.016	0.067	0.036	0.071	0.033
$\phi = 0.1$	0.022	0.030	0.016	0.032	0.015	0.058	0.031	0.062	0.028
$\phi = 0.3$	0.022	0.024	0.013	0.026	0.012	0.044	0.023	0.047	0.021
$\phi = 0.5$	0.022	0.020	0.011	0.021	0.010	0.033	0.018	0.035	0.016
$\phi = 0.7$	0.022	0.015	0.008	0.016	0.007	0.024	0.013	0.026	0.012
$\phi = 1$	0.022	0.010	0.005	0.011	0.005	0.015	0.008	0.016	0.007

Consider the benchmark case in which $b/w = 0.4$ and $\phi = 0$. The standard deviations of the variables resemble those of Shimer (2005) as expected. The inability of the standard search-matching model to generate a high standard deviation is known in the literature

[4] All hat variables represent the percentage deviations from their steady-state levels. The log-linearized verion of the model is presented in Appendix B.

as the unemployment volatility puzzle. Strikingly, however, as ϕ becomes larger, the standard deviation of unemployment gets even smaller. The standard deviation of unemployment is 0.016 under $\phi = 0$, and it reduces to 0.010 under $\phi = 0.5$. Similarly, in the case with $b/w = 0.71$, the standard deviation of unemployment is 0.033 under $\phi = 0$, and it reduces to 0.016 under $\phi = 0.5$.

The magnitude of time aggregation bias is highly pro-cyclical, consistent with the evidence found by Nordmeier (2014). In all cases we consider, the correlation between (the cyclical components of) the magnitude of time aggregation bias and labor productivity is in between 0.974 and 0.982.

The Beveridge curve is preserved. In all cases we consider, the correlation between unemployment and vacancies is in between -0.816 and -0.911. As ϕ increases (and hence time aggregation bias becomes more serious), the correlation increases in absolute value.

4 Conclusion

The published worker flow data contain time aggregation bias. Instead of correcting the bias to measure the transition rates, this paper developed an equilibrium unemployment model containing the bias by allowing some job seekers to be employed immediately after finding a job without waiting for the next period. We quantitatively showed that the magnitude of fluctuations in the unemployment rate significantly decreases as the immediate-employment probability increases, worsening the unemployment volatility puzzle.

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Appendix

A Equilibrium Conditions

This section derives the equilibrium conditions. Let us first derive the job-creation condition (9). Substitute the free entry condition $J_t^V = 0$ into (4) and (3) to obtain

$$c = q(\theta_t)\left\{\phi J_t^F + (1-\phi)\beta \mathbb{E}_t J_{t+1}^F\right\}, \tag{12}$$

$$J_t^F = p_t - w_t + (1-\lambda)\beta \mathbb{E}_t J_{t+1}^F. \tag{13}$$

Eliminate J_{t+1}^F from these two expressions to obtain

$$J_t^F = \frac{c + q(\theta_t)\Phi(p_t - w_t)}{\Lambda q(\theta_t)}, \tag{14}$$

where $\Phi = (1-\phi)/(1-\lambda)$ and $\Lambda = \Phi + \phi$. Substitute (14) back into (13) to obtain the job-creation condition:

$$\frac{c + q(\theta_t)\Phi(p_t - w_t)}{\Lambda q(\theta_t)} = p_t - w_t + (1-\lambda)\beta \mathbb{E}_t\left[\frac{c + q(\theta_{t+1})\Phi(p_{t+1} - w_{t+1})}{\Lambda q(\theta_{t+1})}\right],$$

which is (9).

We now derive the wage equation (10). Subtract (2) from (1) to obtain

$$\begin{aligned} J_t^E - J_t^U = {} & w_t - b + (1-\lambda)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right) \\ & - f(\theta_t)(1-\phi)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right) \\ & - f(\theta_t)\phi\left(J_t^E - J_t^U\right) \\ & + f(\theta_t)\phi\left(\beta \mathbb{E} J_{t+1}^U - J_t^U\right). \end{aligned} \tag{15}$$

Similarly, subtract J_t^U from both sides of (1) to obtain

$$J_t^E - J_t^U = w_t + (1-\lambda)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right) + \left(\beta \mathbb{E}_t J_{t+1}^U - J_t^U\right). \tag{16}$$

Eliminate $\beta \mathbb{E}_t J_{t+1}^U - J_t^U$ from (15) and (16) to obtain

$$\begin{aligned} J_t^E - J_t^U = {} & w_t - b + (1-\lambda)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right) \\ & - f(\theta_t)(1-\phi)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right) \\ & - f(\theta_t)\phi\left(J_t^E - J_t^U\right) \\ & + f(\theta_t)\phi\left[\left(J_t^E - J_t^U\right) - w_t - (1-\lambda)\beta \mathbb{E}_t\left(J_{t+1}^E - J_{t+1}^U\right)\right], \end{aligned}$$

which reduces to

$$J_t^E - J_t^U = [1 - f(\theta_t)\phi]\, w_t - b + \{[1 - f(\theta_t)\phi]\, (1-\lambda) - f(\theta_t)\,(1-\phi)\}\, \beta \mathbb{E}_t \left(J_{t+1}^E - J_{t+1}^U \right). \tag{17}$$

Observe that (14), $J_t^V = 0$, and (5) imply

$$J_t^E - J_t^U = \frac{\eta}{1-\eta}\left(J_t^F - J_t^V\right) = \frac{\eta}{1-\eta}\frac{c + q\,(\theta_t)\,\Phi\,(p_t - w_t)}{\Lambda q\,(\theta_t)}.$$

Substitute this into (17) to finally obtain

$$\begin{aligned}\frac{\eta}{1-\eta}\frac{c + q\,(\theta_t)\,\Phi\,(p_t - w_t)}{\Lambda q\,(\theta_t)} &= [1 - f(\theta_t)\phi]\, w_t - b \\ &+ \{[1 - f(\theta_t)\phi]\,(1-\lambda) - f(\theta_t)\,(1-\phi)\} \\ &\times \beta \mathbb{E}_t \left(\frac{\eta}{1-\eta}\frac{c + q\,(\theta_{t+1})\,\Phi\,(p_{t+1} - w_{t+1})}{\Lambda q\,(\theta_{t+1})} \right),\end{aligned}$$

which is the wage equation (10).

A steady state of this model is determined by

$$\begin{aligned}[1 - (1-\lambda)\,\beta]\,\frac{c}{q} &= [\phi + (1-\lambda)\,\beta\Phi]\,(p - w)\,, \\ \{1 - [1 - \lambda - f\,(1-\lambda\phi)]\,\beta\}\,\frac{\eta}{1-\eta}\left[\frac{c}{\Lambda q} + \frac{\Phi\,(p-w)}{\Lambda}\right] &= (1 - f\phi)\,w - b, \\ u &= \frac{\lambda}{f+\lambda}.\end{aligned}$$

Interestingly, the steady-state unemployment rate is not influenced by the presence of time aggregation bias.

B Log-linearized Model

From

$$\frac{c}{\Lambda q_t} + \frac{\Phi\,(p_t - w_t)}{\Lambda} = p_t - w_t + (1-\lambda)\,\beta \mathbb{E}_t \left[\frac{c}{\Lambda q_{t+1}} + \frac{\Phi\,(p_{t+1} - w_{t+1})}{\Lambda}\right],$$

we obtain

$$-\frac{c}{\Lambda q}\hat{q}_t + \frac{\Phi\,(p\hat{p}_t - w\hat{w}_t)}{\Lambda} = p\hat{p}_t - w\hat{w}_t + (1-\lambda)\,\beta \mathbb{E}_t \left[-\frac{c}{\Lambda q}\hat{q}_{t+1} + \frac{\Phi\,(p\hat{p}_{t+1} - w\hat{w}_{t+1})}{\Lambda}\right], \tag{18}$$

From

$$\frac{\eta}{1-\eta}\left[\frac{c}{\Lambda q_t}+\frac{\Phi\left(p_t-w_t\right)}{\Lambda}\right]=\left(1-f_t\phi\right)w_t-b$$
$$+\left[\left(1-f_t\phi\right)\left(1-\lambda\right)-f_t\left(1-\phi\right)\right]$$
$$\times\beta\frac{\eta}{1-\eta}\mathbb{E}_t\left[\frac{c}{\Lambda q_{t+1}}+\frac{\Phi\left(p_{t+1}-w_{t+1}\right)}{\Lambda}\right],$$

we obtain

$$\frac{\eta}{1-\eta}\left[-\frac{c}{\Lambda q}\hat{q}_t+\frac{\Phi\left(p\hat{p}_t-w\hat{w}_t\right)}{\Lambda}\right]=w\hat{w}_t-f\phi w\left(\hat{f}_t+\hat{w}_t\right)$$
$$-f\hat{f}_t\left(1-\lambda\phi\right)\beta\frac{\eta}{1-\eta}\left[\frac{c}{\Lambda q}+\frac{\Phi\left(p-w\right)}{\Lambda}\right]$$
$$+\left[1-\lambda-f\left(1-\lambda\phi\right)\right]\beta\frac{\eta}{1-\eta}$$
$$\times\mathbb{E}_t\left[-\frac{c}{\Lambda q}\hat{q}_{t+1}+\frac{\Phi\left(p\hat{p}_{t+1}-w\hat{w}_{t+1}\right)}{\Lambda}\right]. \quad (19)$$

From

$$1-u_t=\left(1-\lambda\right)\left(1-u_{t-1}\right)+\left(1-\phi\right)f_{t-1}u_{t-1}+\phi f_t u_t,$$

we obtain

$$\hat{u}_t=\left(1-\lambda\right)\hat{u}_{t-1}-\left(1-\phi\right)f\left(\hat{f}_{t-1}+\hat{u}_{t-1}\right)-\phi f\left(\hat{f}_t+\hat{u}_t\right). \quad (20)$$

Finally, from

$$\tau_t=\phi f_t\lambda\left(1-u_{t-1}\right),$$

we obtain

$$\tau\hat{\tau}_t=\tau\hat{f}_t-\phi f\lambda u\hat{u}_{t-1}. \quad (21)$$