Economics Bulletin

Volume 43, Issue 2

Monopoly profit lower than oligopoly due to risk aversion

Jim Jin
University of St Andrews

Shinji Kobayashi
Nihon University

Abstract

The industry profit is usually maximized under monopoly and falls with the number of firms in a Cournot oligopoly. However, demand uncertainty and risk aversion reduce firms' outputs, thus raising oligopoly profits and reducing monopoly one. Given a liner demand and costs and a mean-variance utility, we obtain the necessary and sufficient condition for a monopoly's profit and utility to be lower than an oligopoly. We also find such a condition for collusion to yield a lower profit. Finally, we provide a sufficient condition for a monopoly profit to be lower than an oligopoly given a general non-linear demand function.

Citation: Jim Jin and Shinji Kobayashi, (2023) "Monopoly profit lower than oligopoly due to risk aversion", *Economics Bulletin*, Volume 43, Issue 2, pages 1010-1015
Contact: Jim Jin - jyj@st-andrews.ac.uk, Shinji Kobayashi - kobayashi.shinji@nihon-u.ac.jp.
Submitted: June 28, 2022. **Published:** June 30, 2023.

1. Introduction

According to the standard economic theory, given the same demand and cost functions, a monopoly charges a higher price and enjoys a larger profit than independent oligopoly firms do. Since monopoly power results in both higher prices and profits, it can be measured by either the Lerner index (1934) or Bain's profit rates (1941). In Cournot competition the industry price and profit both fall with the number of firms and the two measures are always consistent. This remains largely true when firms merge. While Salant et al. (1983) and Levin (1990) show that profits may not rise for participants of a partial merger, it always does so when all firms in one market merge into a monopoly. In Bertrand oligopoly Deneckere and Davidson (1985) find that merged firms always charge higher prices and earn more profits. The literature usually uses profits to measure firms' incentives to merge. Empirical studies also use profitability to measure merger performance. Some find little evidence of higher profitability (Ravenscraft and Scherer, 1989; Cosh et al., 1989), others obtain positive results (Healy et al., 1992; Matsusaka, 1993; Akhavein et al., 1997). Comprehensive reviews provide mixed conclusions (Andrade et al., 2001; Pautler, 2003; and Whinston, 2006). These studies focus on partial mergers, not a total merger into monopoly. It has not been shown that a monopoly makes less profit than independent oligopoly firms do.

Our paper compares the monopoly profit with oligopoly one under demand uncertainty and risk aversion. The literature has shown that a monopoly chooses a lower output under these conditions (Baron, 1971; Leland, 1972; and Hau, 2004), but also chooses a lower price (Baron, 1971). This result has been generalized to asymmetric duopoly (Asplund, 2002) and oligopoly (Jin and Kobayashi, 2016), i.e., outputs (prices) are always lower in Cournot (Bertrand) competition due to risk aversion.

In quantity competition lower outputs would reduce the monopoly profit but raise the total oligopoly profit. This effect may reverse the profit ranking of monopoly and oligopoly. Intuitively, the stronger demand uncertainty and risk aversion, the more likely for the monopoly profit to be lower than oligopoly one. The question is if and under which condition this will happen. This paper provides the necessary and sufficient conditions for the reversion of the profit ranking in a simple symmetric Cournot oligopoly.

2. Model

We first consider a linear Cournot oligopoly with n identical firms facing an uncertain price $p = a + \varepsilon - bX$, where a (> 0) is the demand intercept, b (> 0) is the slope of inverse demand curve, X is the total output, ε is a random shock with $E(\varepsilon) = 0$, and $V(\varepsilon) = \sigma^2$. We assume $a + \varepsilon \geq 0$ as it indicates the product's marginal utility to consumers, thus should not be negative. This excludes distributions with infinite domains such as normal distributions which are unrealistic, but it does not guarantee a positive price margin $p - c > 0$. Hence, firms could make negative profits under some realization of ε.

Every firm has a constant marginal cost c (< a) and chooses output x_i to maximize its mean-variance utility function $u_i = E(\pi_i) - \delta V(\pi_i)$, where expected profit $E(\pi_i) = E[(p-c)x_i]$ $= (a - c - bX)x_i$ and profit variance $V(\pi_i) = x_i^2 V(p-c) = x_i^2\sigma^2$. Hence, we have:

$$u_i = (a - c - bX)x_i - \delta\sigma^2 x_i^2 \tag{1}$$

The first-order condition for x_i to maximize (1) is $a - c - bX - b x_i - 2\delta\sigma^2 x_i = 0$. In the symmetric Cournot equilibrium $X = n x_i$ and the Cournot equilibrium output for each firm is $x_i^* = \frac{a-c}{(n+1)b + 2\delta\sigma^2}$. The corresponding total output X is:

$$n x_i^* = \frac{(a-c)n}{(n+1)b + 2\delta\sigma^2} \tag{2}$$

The expected price-cost margin, $p^* - c = a - c - bX = (b + 2\delta\sigma^2)\, x_i^* = \frac{(a-c)(b+2\delta\sigma^2)}{(n+1)b + 2\delta\sigma^2}$. So, the expected total industry profit is:

$$E(n\pi_i^*) = \frac{(a-c)^2 (b+2\delta\sigma^2)n}{[(n+1)b + 2\delta\sigma^2]^2} \tag{3}$$

Substituting the equilibrium output x_i^* into (1), we get each firm's mean-variance utility $u_i^* = \frac{(a-c)^2 (b+\delta\sigma^2)}{[(n+1)b + 2\delta\sigma^2]^2}$. The total utility of all oligopoly firms is:

$$n u_i^* = \frac{(a-c)^2 (b+\delta\sigma^2)n}{[(n+1)b + 2\delta\sigma^2]^2} \tag{4}$$

In the next section we compare the outcomes in oligopoly vs. monopoly ($n = 1$) and obtain necessary and sufficient conditions for a monopoly to have a lower profit or utility.

3. Main Result

From (2), we see that the total output increases with n, which is treated as a continuous variable. As the output rises, the price must fall, so does the Lerner index $(p - c)/p$. The monopoly power according to this measure always falls with the number of firms.

However, differentiating (3) with respect to n, we find that the derivative is positive if and only if $(n - 1)b < 2\delta\sigma^2$. To simplify our notation, we let $\theta \equiv \delta\sigma^2/b$. Then (3) is maximized when $n = 1 + 2\theta$. As $1/b$ indicates the market size, θ rises with the market size as well as demand uncertainty and risk aversion. When $\theta = 0$, the monopoly has the highest profit as usually happens. However, if $\theta \geq 0.5$, the maximum profit occurs when $n \geq 2$. This is a sufficient condition for a monopoly profit being lower than an oligopoly (at least duopoly). To obtain a necessary and sufficient condition, we need to consider the precise threshold.

From (3) we see that an oligopoly and a monopoly profit depend on $\frac{n}{(n+1+2\theta)^2}$ and $\frac{1}{4(1+\theta)^2}$ ($n = 1$) respectively. So, the former is larger if and only if:

$$\frac{n}{(n+1+2\theta)^2} > \frac{1}{4(1+\theta)^2} \tag{5}$$

(5) implies that $\frac{\sqrt{n}}{n+1+2\theta} > 0.5/(1 + \theta)$, which simplifies to $(\sqrt{n} - 1)^2 < 2(\sqrt{n} - 1)\theta$, i.e. $\theta > 0.5(\sqrt{n} - 1)$ for any $n > 1$. Hence, we get our result below.

Proposition 1*: The Lerner index falls with n; the industry profit is maximized when n* = 1 + 2θ *and the monopoly profit is lower than an oligopoly if and only if* $\theta > 0.5(\sqrt{n} - 1)$.

This result implies that a monopoly makes less profit than a duopoly if and only if $\theta > 0.5(\sqrt{2} - 1)$. If $\theta = 0.5$, the industry profit is maximized under duopoly and the monopoly profit is equal to that of a quadropoly. In general, a monopoly profit is more likely to be lower than an oligopoly given a higher θ, because it implies more significant output reductions, thus reduces monopoly profit and raises oligopoly one more, more likely to reverse their ranking.

Intuitively, stronger risk aversion δ and demand uncertainty σ^2 imply a firm's output x_i imposes a significant loss in its mean-variance utility (1), and thus reduce its output. The value of b indicates how sensitive the market price is to the total output. When it is large, a firm's output mainly depends on b, and the impact of δ and σ^2 is limited. When b is small, the impact of δ and σ^2 will be significant. Hence, a higher θ implies more significant output reductions, and more likely to reverse the profit ranking.

On the other hand, θ must be bounded, because the market size $1/b$ cannot be infinite, the price variance is finite as ε has a limited range as assumed, and risk aversion must be limited for a firm to participate in the market. Hence, for a very large n, the inequality in Proposition 1 cannot hold and a large number of firms make a lower profit than a monopoly.

Moreover, a monopoly may have a lower utility than oligopoly firms. Comparing (3) with (4), we see that $nu_i^* = \dfrac{b + \delta\sigma^2}{b + 2\delta\sigma^2} E(n\pi_i^*)$. This relation between nu_i^* and $E(n\pi_i^*)$ is independent of n. Thus, we have the following result.

Proposition 2*: The monopoly has a lower mean-variance utility than oligopoly firms if and only if it has a lower profit.*

If a mean-variance utility represents shareholders' interests as often assumed in the finance literature, it can be directly linked to firms' stock values. Then a monopoly can have a lower value than the sum of oligopoly firms. This result raises a question whether we should always use profits as firms' incentive to merge.

Given the same value of $\delta\sigma^2$, a monopolist has a bigger loss in its utility (1) than an oligopoly firm due to its larger output. This provides a stronger incentive for a monopolist to reduce its output and it is more likely to reverse the profit ranking. This disadvantage will be reduced if a monopoly is not fully integrated, but all firms retain their independence and jointly maximize their total utility. Without demand uncertainty and risk aversion such a collusion would generate an identical profit as a monopoly. However, they are different here due to different objective functions. Colluding firms still reduce outputs due to uncertainty and risk aversion, but less than a monopoly. It is interesting to know if and under which condition the total profit under collusion is less than an independent oligopoly.

We now assume the sum of firms' mean-variance utility (1) is jointly maximized. Hence, each firm must have a higher utility than an independent oligopoly firm. Nonetheless its profit might be lower. Each firm's output x_i is chosen to maximize $\sum_{i=1}^{n} E(\pi_i) - \delta\sigma^2 \sum_{i=1}^{n} x_i^2$. As $x_i = X/n$, the objective becomes $(p - c)X - (\delta\sigma^2/n)X^2$. Differentiating this function, we get the FOC $a - c - 2bX - 2\delta\sigma^2 X/n = 0$. So, $X^* = \dfrac{(a-c)n}{2(nb + \delta\sigma^2)}$, which increases with n and is smaller than nx_i^* in (2). Hence, the Lerner index under collusion must fall with n and is always higher than it is in an independent oligopoly. Then, substituting X^* into the total profit function $(p - c)X^* = (a - c - bX^*)X^*$, we obtain the total profit as $(a - c)^2 \frac{(nb+2\delta\sigma^2)n}{4(nb+\delta\sigma^2)^2}$. It is less than the oligopoly profit in (3) if and only if:

$$\frac{n+2\theta}{4(n+\theta)^2} < \frac{1+2\theta}{(n+1+2\theta)^2} \tag{6}$$

(6) can be written as $(n + 2\theta)(n + 1 + 2\theta)^2 - 4(1 + 2\theta)(n + \theta)^2 < 0$. The left hand side is $n(n-1)^2 - 2(n+1)(n-1)\theta - 4(n-1)\theta^2 = (n-1)[n(n-1) - 2(n+1)\theta - 4\theta^2]$. Hence, for any $n > 1$, (6) holds if and only if $4\theta^2 + 2(n+1)\theta - n(n-1) > 0$, i.e., $\theta > 0.25(\sqrt{5n^2 - 2n + 1} - n - 1)$.

Proposition 3*: Collusion yields a lower profit than independent oligopoly if and only if* $\theta > 0.25(\sqrt{5n^2 - 2n + 1} - n - 1)$.

This threshold again rises with n. In a 6-firm oligopoly, if $\delta\sigma^2 > 1.5b$, collusion yields a lower profit. Note for any $n > 1$, the threshold in Proposition 3 is less than $0.5(n - 1)$. Hence, (6) is guaranteed if $\theta > 0.5(n - 1)$. If $\theta > 0.5$, collusion profit must be smaller than an independent duopoly. One can verify that the threshold here is more than $0.5(\sqrt{n} - 1)$ in Proposition 1. So, in comparison with a fully integrated monopoly, collusion profit is less likely to be lower than an oligopoly. Nonetheless, with a sufficiently large θ, in both cases the Lerner index and Bain's profitability measurements may give different rankings for monopoly power.

4. Non-linear Demand[1]

Finally, we extend our key result to a non-linear demand. The market price $p(X)$ is decreasing and continuously twice differentiable in total output X. Novsheck (1985) provided a sufficient condition for any n-firm Cournot equilibrium, which is assumed here:

$$p'(X) + Xp''(X) < 0 \qquad \text{for any } X > 0 \tag{7}$$

Given (7), the total industry profit $[p(X) - c]X$ must be concave in X as its second-order derivative $2p'(X) + Xp''(X) < 0$, thus has a unique maximum at a certain output denoted by X^0 satisfying the FOC $p(X^0) - c + X^0p'(X^0) = 0$. If this X^0 is obtained under oligopoly rather than monopoly, the normal profit ranking will be reversed. When each firm maximizes its mean-variance utility $u_i = [p(X) - c]x_i - \delta\sigma^2 x_i^2$, the FOC is: $p(X) - c + x_i p'(X) - 2\delta\sigma^2 x_i = 0$, or

$$p(X) - c + [p'(X) - 2\delta\sigma^2]\frac{X}{n} = 0. \tag{8}$$

Equation (8) implies that X is a function of n, i.e., $X(n)$. Differentiating (8) with respect to n, we get $\{p'(X) + [p'(X) + Xp''(X) - 2\delta\sigma^2]/n\}X'(n) = [p'(X) - 2\delta\sigma^2]X/n^2$. Given $p'(X) < 0$ and $p'(X) + Xp''(X) < 0$, we see $X'(n) > 0$. Hence, $p(X)$ falls with n. The monopoly power measured by the Lerner index falls with the number of firms, but not necessarily the profit.

(8) will satisfy the FOC for maximizing the industry profit at X^0, i.e., $p(X^0) - c + X^0p'(X^0) = 0$ if $[p'(X^0) - 2\delta\sigma^2]X^0/n = X^0p'(X^0)$, i.e., $n^0 = 1 - 2\delta\sigma^2/p'(X^0)$. This is the number of firms maximizing the total industry profit.

Proposition 4*: With a non-linear demand, the Lerner index falls with n, but the industry profit reaches its maximum when* $n = 1 - 2\delta\sigma^2/p'(X^0)$.

---

[1] We thank an anonymous referee for the suggestion to consider this case.

In our linear case, we have $p'(X^0) = -b$, so the corresponding $n^0 = 1 + 2\theta$. In a general non-linear case, if this profit maximizing $n^0 \geq 2$, the monopoly profit must be lower than a duopoly. Different from the linear case, however, n^0 will not only depend on θ, but also other factors as well, as we explain below.

For instance, we consider a special inverse demand function $p(X) = a + \varepsilon - bX^\alpha$ with $\alpha > 0$. (7) holds. The FOC for total profit maximization is $a - c - bX^\alpha - b\alpha X^\alpha = 0$. So, $X^0 = [\frac{a-c}{(1+\alpha)b}]^{1/\alpha}$. Substituting it into $p'(X^0) = -b\alpha X^{\alpha-1}$, we get $n^0 = 1 + \frac{2\delta\sigma^2}{b\alpha}[\frac{a-c}{(1+\alpha)b}]^{(1-\alpha)/\alpha}$. This n^0 rises (falls) with $a - c$ if $\alpha <$ (>) 1, i.e., the inverse demand function is convex (concave). In a linear case, $\alpha = 1$, $n^0 = 1 + 2\theta$, independent of $a - c$. If we have $\alpha = 0.5$, $n^0 = 1 + \frac{8\theta(a-c)}{3b}$, rising with $a - c$. If $\alpha = 2$, $n^0 = 1 + \theta\sqrt{\frac{3b}{a-c}}$, decreasing with $a - c$. Intuitively, X^0 always rises with $a - c$, but $-1/p'(X^0)$ rises with X^0 if and only if $\alpha < 1$. So does n^0. On the other hand, n^0 rises with $1/b^{1/\alpha}$, thus always falls with b, as a larger market size requires more firms to maximize the total profit.

5. Conclusion

Given the same demand, costs, and mean-variance utility function, we find that monopoly and collusion profits can be lower than an independent oligopoly. Thus, the Lerner index and Bain's profitability measurements may give different rankings of monopoly power. These results suggest that using profits to measure the monopoly power and the incentives for mergers may not be totally innocuous in quantity competition with risk aversion. However, none of the above would happen if firms choose prices instead of quantities, as prices will fall due to risk aversion as shown by Asplund (2002) and Jin and Kobayashi (2016).

References

Akhavein, J., J. Berger and D. Humphrey (1997) "The Effects of Megamergers on Efficiency and Prices: Evidence from a Bank Profit Function" *Review of Industrial Organization*, **12**, 95-139.

Andrade, G., M. Mitchell, and E. Stafford (2001) "New Evidence and Perspectives on Mergers" *Journal of Economic Perspectives*, **15**, 103-120.

Asplund, M. (2002) "Risk-averse Firms in Oligopoly" *International Journal of Industrial Organization*, **20**, 995-1012.

Bain, J. (1941) "The Profit Rate as a Measure of Monopoly Power" *Quarterly Journal of Economics*, **55**, 271–293.

Baron, D. (1971) "Demand Uncertainty in Imperfect Competition" *International Economic Review*, **12**, 196-208.

Cosh, A., A. Hughes, and K. Lee (1989) "Institutional Investment, Mergers and the Market for Corporate Control" *International Journal of Industrial Organization*, **7**, 73-100.

Deneckere, R. and C. Davidson (1985) "Incentive to Form Coalitions with Bertrand Competition" *Rand Journal of Economics*, **16**, 473-486.

Hau, A. (2004) "Theory of the Firm Facing Uncertain Demand Revisited" *Economic Theory*, **24**, 457-464.

Healy, P., K. Palepu, and R. Ruback (1992) "Does Corporate Performance Improve after Mergers?" *Journal of Financial Economics*, **31**, 135-175.

Jin, J. and S. Kobayashi (2016) "Impact of Risk Aversion and Countervailing Tax in Oligopoly" *Annals of Finance*, **12**, 393-408.

Leland, H. E. (1972) "Theory of the Firm Facing Uncertain Demand" *American Economic Review*, **62**, 278-291.

Lerner, A. (1934) "The Concept of Monopoly and the Measurement of Monopoly Power" *Review of Economic Studies*, **1**,157- 175.

Levin, D. (1990) "Horizontal Mergers: The 50-Percent Benchmark" *American Economic Review*, **80**, 1238- 1245.

Matsusaka, J. (1993) "Target Profits and Managerial Discipline during the Conglomerate Merger Wave" *Journal of Industrial Economics*, **16**, 179-189.

Novshek, W. (1985) "On the Existence of Cournot Equilibrium" *Review of Economic Studies*, **52**, 85-98.

Pautler, P. (2003) "Evidence on Mergers and Acquisitions" *Antitrust Bulletin*, **48**, 119-222.

Ravenscraft, D. and F. Scherer (1989) "The Profitability of Mergers" *International Journal of Industrial Organization*, **7**, 101-116.

Salant, S., S. Switzer, and R. Reynolds (1983) "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium" *Quarterly Journal of Economics*, **98**, 185-199.

Whinston, M. (2006) *Lectures on Antitrust Economics*, MIT Press: Cambridge, MA.