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A start-up problem in a pure network good market and the role of a stand-alone effect with a monopoly: A revisit of Rohlfs (1974, 2001)

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Abstract

We consider the start-up problem in the case of a monopoly providing a pure network good, such as in telecommunications and Internet businesses. This is a fundamental problem previously examined by Rohlfs (1974) and an issue closely related to coordination failure. As addressed in Lambertini and Orsini (2004), the coordination problem is not associated with network effects and depends on consumer expectations. We demonstrate the perspectives of Rohlfs (1974, 2001), finding the start-up problem is associated with critical mass under passive expectations and the stand-alone effect can be used to solve the problem.

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1. Introduction

The purpose of this paper is to reconsider the start-up problem (or in other words, coordination failure) in the case of a monopoly providing a pure network good such as telecommunications and Internet businesses, in which the critical mass of the expected network size is very important for the network industry to be viable. Since seminal work by Rohlfs (1974) focusing on the telecommunications industry, many subsequent studies have examined the start-up problem relating to the issue of critical mass in markets with network effects.[1] Most recently, Lambertini and Orsini (2004, p. 124) reconsider Rohlfs (1974), as follows.

A recurrent theme in the literature on network goods is the start-up problem, i.e., how to attract a significant number of customers so as to offer an appealing good or service to additional consumers. Intuitively, joining the network is more valuable to the generic consumer the larger is the size of the network. This may give rise to a coordination problem, since the market performance of a service/product depends upon the achievement of a critical mass of adopters/consumers. *The most widely used illustration of this issue dates back to Rohlfs (1974), assuming that the utility associated with consumption is fully determined by the network effect. This can be the case, e.g., of telecommunication networks, which is the example used by Rohlfs himself.* Thereafter, the coordination problem related to the issue of the critical mass has been generally associated to the presence of network effects. *Yet, this is not true in general, since there exist many goods which exhibit network externalities but carry also an intrinsic utility justifying by itself consumption.* These considerations suggest that the issue of a critical mass is crucial only for a subset of all the goods yielding network effects (italics mine).

As examples, Lambertini and Orsini (2004, footnote 1, p. 124) take the case of personal computers, compact disc players, televisions, etc. As first emphasized in their quote, Rohlfs (1974) focuses on the case in which "… the utility associated with consumption is fully determined by the network effect," such as telecommunication devices (e.g., telephones and telefax machines). Following the definitions of Amir and Lazzati (2011, p. 2395), we understand that this corresponds to the case of a pure network good, which characterizes that the intrinsic utility from consumption depends on the expected network size.

Furthermore, we should also consider that consumer expectations play an important role in markets with network effects. Following the definitions in Hurkens and López (2014, p. 1007), we address responsive and passive expectations.[2] Responsive expectations are where firms first compete in prices (or quantities). Consumers then form expectations about network sizes and make optimal purchasing decisions, given the prices and their expectations. Alternatively, passive expectations are where consumers first form expectations about network sizes and firms then compete in prices (or quantities), so consumers make optimal purchasing decisions, given their expectations. These decisions lead to actual market shares and network sizes. Thus, realized and expected network sizes are equal in equilibrium. That is, Lambertini and Orsini (2004) assume the case of responsive expectations. Rohlfs (1974) also implicitly assumes this case. As demonstrated by Lambertini and Orsini (2004, p. 127), a stable equilibrium exists, and thus, the

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[1] For example, see Shy (1998, pp. 256–259) who concisely summarizes Rohlfs' (1974) model. Furthermore, Economides (1996) also points out the critical mass problem of a network good.

[2] Suleymanova and Wey (2012) consider the role of consumer expectations in a Hotelling model of price competition when products exhibit network effects. In their paper, they distinguish between weak and strong expectations, where the former (latter) corresponds to the case of responsive (passive) expectations in this paper.

coordination problem relating to critical mass does not arise given the presence of network effects. This is true when consumer expectations are responsive. However, in this note, we demonstrate that the coordination problem relating to critical mass is associated with expected network sizes when consumers have passive expectations.

The next aspect we emphasize in Lambertini and Orsini's (2004) quote is "… many goods which exhibit network externalities but carry also an intrinsic utility justifying by itself consumption." Lambertini and Orsini (2004) use this to define that a parameter s, being an intrinsic utility (or satisfaction) from consumption, is independent of the network size. They then introduce the parameter into consumer surplus. Using the definitions of Amir and Lazzati (2011, p. 2395), their model is then the case of a network good with a strictly positive stand-alone value. With respect to the stand-alone effect, Rohlfs (2001, p. 197) describes the start-up problem as follows.

Suppliers must have extraordinarily good products or services to reach a critical mass. All the successful products and services that we examined constituted major technological advances and met important customer needs. At the other extreme, Picturephone was a hard product to sell—even apart from the Bell System's failure to solve the start-up problem. In between are several products that constituted important technological advances but could not reach a critical mass. These include analog fax machines, digital compact cassette players, minidisc players, digital videodisc players, and commercial e-mail (before the rapid growth of the Internet).

A valuable stand-alone application is extremely helpful in solving the start-up problem. Such an application can generate a large initial user set before suppliers have to do anything to solve the start-up problem. If the stand-alone application is sufficiently valuable, the start-up problem solves itself. That is precisely what happened to VCRs, because the stand-alone application of time-shifting of television programs was so valuable. *The start-up problem is much more difficult for pure bandwagon products, which have no such stand-alone application* (italics mine).

We thus note that the start-up problem is the same as the coordination problem and pure bandwagon products are the same as pure network goods. As explained in the emphasized section of the quote, we can see that the role of a stand-alone value is very important in solving the start-up problem for a pure network good. By introducing a stand-alone value into the utility function of a pure network good, we demonstrate here that there exists a stable equilibrium under passive expectations and that the start-up problem can then be solved.

2. The Model

2.1 An inverse demand function of a pure network good

Assuming the presence of a direct network effect as already observed in the information and communications technology industry, such as telecommunications and Internet businesses, we consider a unit-linear market where there is a continuum of consumers, indexed $\theta \in [0,1]$. To simplify, consumers are uniformly distributed with a density of one in the market, and the utility function (willingness to pay) of consumer θ is given by: $u(\theta) = N(S^e)\theta$, where $N(S^e)$ denotes a network effect function of expected network sizes, $S^e \in [0,1]$.[3] Given the price, a consumer purchases at most either one unit of the product or none. Hence, the surplus of consumer θ is expressed as: $v(\theta) = \max\{u(\theta) - P, 0\}$. The index of the marginal consumer with the same surplus from purchasing either one unit of the product or none is $\hat{\theta} = \frac{P}{N(S^e)}$.[4] The quantity demanded of the network product is given by: $X = 1 - \hat{\theta}$, $X \in [0,1]$. We obtain the following inverse demand function:

$$P(X, S^e) = N(S^e)(1 - X). \tag{1}$$

Given (1), if consumers expect that any consumers do not purchase the products, i.e., $S^e = 0$, it holds that $N(0) = 0$. In particular, we have $P(X, S^e = 0) = 0$. This implies that the product is a pure network good (see Amir and Lazzati, 2011, p. 2395).

We assume that a marginal cost of production is constant. For example, the marginal costs of production, such as running costs, in network industries are either negligible or zero, i.e., $C(X) = cX$, $N(S^e) > c \geq 0$, for $S^e \in (0,1]$.[5] Thus, the profit function of monopoly is expressed as:

$$\Pi(X, S^e) = \{P(X, S^e) - c\}X = \{N(S^e)(1 - X) - c\}X. \tag{2}$$

2.2 The monopoly equilibrium under passive expectations

Given the expected network sizes under passive expectations, the monopolist decides the output to maximize the profit. Using equations (1) and (2), the first-order condition (FOC) of profit maximization is given by:

$$\frac{\partial \Pi}{\partial X} = P(X, S^e) - c - N(S^e)X = N(S^e)(1 - 2X) - c = 0, \tag{3}$$

where $X \in [0,1]$ and $S^e \in [0,1]$. The second-order condition (SOC) is given by: $\frac{\partial^2 \Pi}{\partial X^2} = -2N(S^e) \leq 0$. It also holds that $\left.\frac{\partial \Pi}{\partial X}\right|_{X=0} = N(S^e) - c > 0$ for $S^e \in (0,1]$[6] and $\left.\frac{\partial \Pi}{\partial X}\right|_{X=1} = -N(S^e) - c < 0$ for $S^e \in [0,1]$.

Using equation (3) and the concept of Katz and Shapiro (1985), we have the fulfilled expectation equilibria which are satisfied with the following conditions:

$$X = \frac{1}{2}\left\{1 - \frac{c}{N(S^e)}\right\} \text{ and } S^e = X, \text{where } X \in [0,1] \text{ and } S^e \in [0,1]. \tag{4}$$

For example, we assume that $N(S^e) = \beta S^e$, $\beta = 0.1$, and $c = 0.01$. We can use this to plot Figure 1, which illustrates that there are two stable equilibria and one unstable equilibrium, i.e., $X^* = 0$,

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[3] We assume $N'(S^e) \equiv \frac{\partial N(S^e)}{\partial S^e} > 0$ for $\theta \in [0,1]$. Furthermore, it holds that $\frac{\partial u(\theta)}{\partial N(S^e)} = \theta \geq 0$. That is, the larger the value of θ, the higher becomes the marginal utility of the network effects.

[4] S^e implies the expected number of consumers acquiring the product.

[5] This is a necessary condition for a positive output equilibrium to exist.

[6] When $S^e = 0$, $\left.\frac{\partial \Pi}{\partial X}\right|_{X=0} = N(0) - c = -c < 0$.

$\tilde{X} \simeq 0.138$, and $X^{**} \simeq 0.361$.[7] In particular, the unstable equilibrium, $\tilde{X}$, corresponds to the critical mass. With respect to consumer expectations, if $\tilde{X} > S^e$, the pure network good market cannot be viable, i.e., $X^* = 0$. This is a coordination problem, or in other words, a start-up problem. Conversely, if $\tilde{X} < S^e$, the market is viable, i.e., $X^{**} \simeq 0.361$. See Appendix.

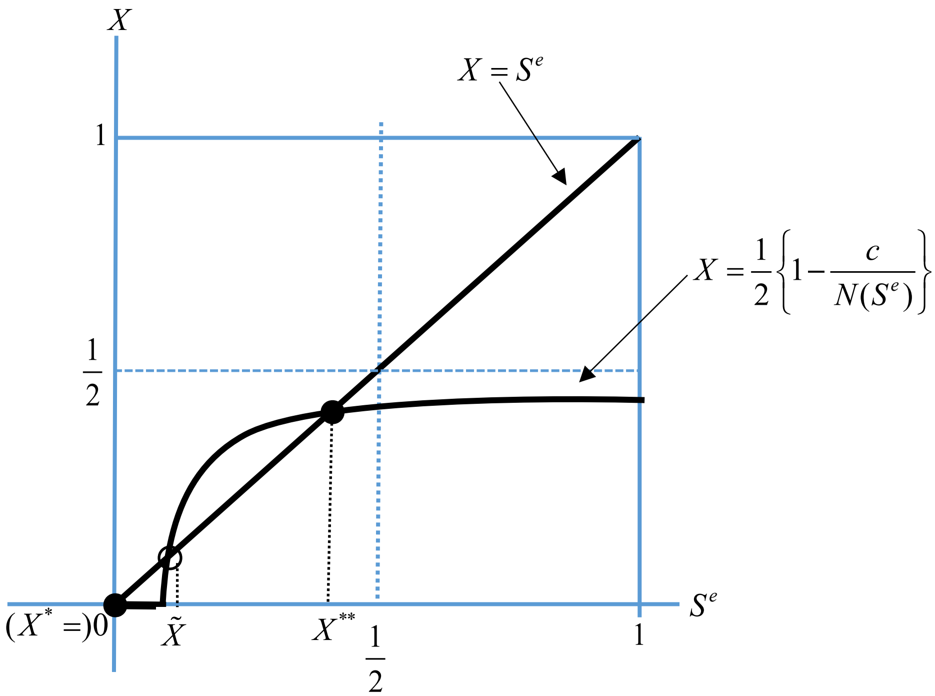

Figure 1. A start-up (coordination) problem in the case of a pure network good

Remark 1. Zero marginal costs, i.e., $c = 0$.
We assume that production (running) costs are zero. This is because we observe low and even negligible marginal running costs in network industries, e.g., telecommunication and Internet businesses. Given equation (3), assuming zero marginal cost, the FOC is given by: $\frac{\partial \Pi}{\partial X} = N(S^e)\left(1-2X\right) = 0$. We have the fulfilled expectation equilibrium which is satisfied with the following conditions: $N(S^e)\left(1-2X\right) = 0$ and $S^e = X$, where $X \in [0,1]$ and $S^e \in [0,1]$.

We can then draw Figure 2. In this case, $\tilde{X} = 0$ is an unstable equilibrium, i.e., the critical mass, and

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[7] Belleflamme and Peitz (2021, Figure 3.4, pp. 86–89), which is closely related to ours, examine dynamic behavior of user expectations and the equilibrium stability. Following their definitions, $X^* = 0$ is a “null network” equilibrium, $\tilde{X} \simeq 0.138$ is a “small network” equilibrium, and $X^{**} \simeq 0.361$ is a “large network” equilibrium. In our model, however, we demonstrate that the stability condition depends on the network effect elasticity in relation to the network sizes.

$X^{**} = \frac{1}{2}$ is a stable equilibrium. Thus, if $S^e > 0$, the market is viable.

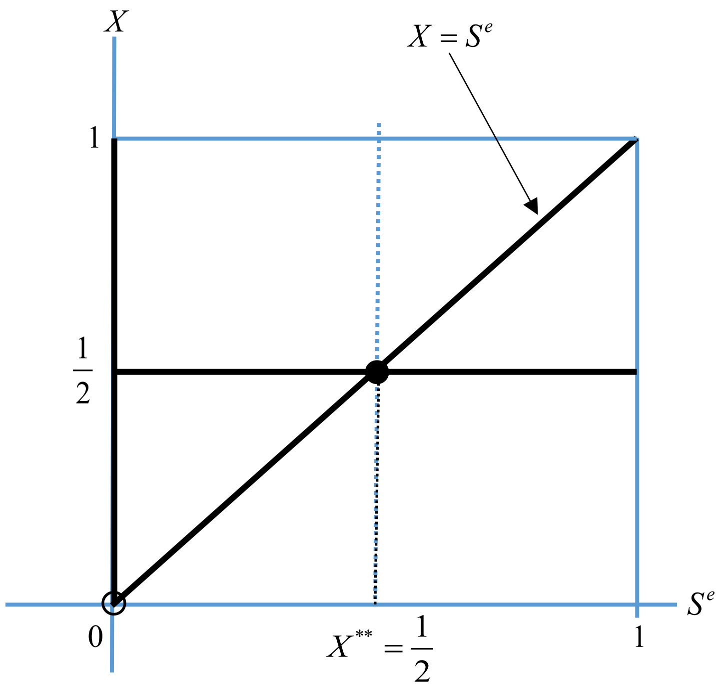

Figure 2. The case of zero costs

Remark 2. The case of responsive expectations
Under responsive expectations, consumers believe that the expected network size is equal to the announced output level of the monopolist, i.e., $S^e = X$. In other words, the monopolist can commit to the announced output level before its decision about actual output production. In this case, the inverse demand function is rewritten as $P(X, X) = N(X)(1-X)$. For example, we assume the following linear network effect function: $N(X) = \beta X$, where $1 > \beta > 0$. In this case, the inverse demand function is concave in X. Given this function, we obtain the same results as Shy (1998, pp. 256–259) and Lambertini and Orsini (2004, p. 127).

2.3 A stand-alone effect: Introduction of a strictly positive stand-alone value and the equilibrium
Using the perspective of Rohlfs (2001), as quoted earlier, we introduce a strictly positive stand-alone value, which is independent of the expected network sizes, into the utility function as follows: $u(\theta) = N(S^e)\theta + a$. In particular, we derive the following inverse demand function:

$$P(X, S^e) = N(S^e)(1-X) + a, \tag{5}$$

where $N(1) > a > c \geq 0$.[8] Equation (5) implies that all consumers share the same valuation for the stand-alone benefits (see Remark 3). Thus, the FOC is given by:

$$\frac{\partial \Pi}{\partial X} = N(S^e)(1-2X) + a - c = 0, \tag{6}$$

[8] If $c \geq a > 0$, we obtain the same results as in Section 2.1 and Remark 1.

where $X \in [0,1]$ and $S^e \in [0,1]$. The SOC is given by: $\frac{\partial^2 \Pi}{\partial X^2} = -2N(S^e) \leq 0$. It also holds that $\left.\frac{\partial \Pi}{\partial X}\right|_{X=0} = N(S^e) + a - c > 0$ and $\left.\frac{\partial \Pi}{\partial X}\right|_{X=1} = -N(S^e) + a - c < 0$ [9] for $S^e \in [0,1]$.

Using equation (6), we have the fulfilled expectation equilibrium which is satisfied with the following conditions.

$$X = \frac{1}{2}\left\{1 + \frac{a-c}{N(S^e)}\right\} \text{ and } S^e = X, \text{ where } X \in [0,1] \text{ and } S^e \in [0,1]. \quad (7)$$

For example, we assume that $N(S^e) = \beta S^e$, $\beta = 0.1$, and $a - c = 0.01$. We can use this to draw Figure 3, which illustrates that there exists a stable equilibrium, i.e., $X^{\#} \simeq 0.585$. That is, as argued by Rohlfs (2001), a start-up problem can be solved by introducing a strictly positive stand-alone value.

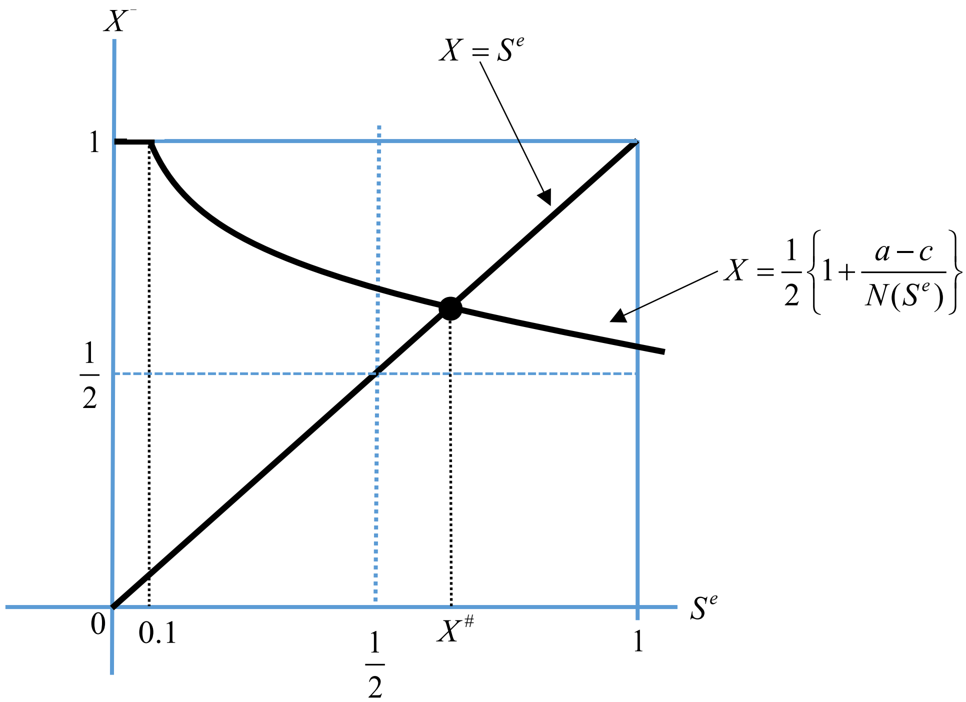

Figure 3. A stand-alone effect in the case of a pure network good

Remark 3. Additively separable network effects and a mixed network good[10]

In previous subsections, we consider the case of a pure network good, which is expressed as a function of multiplicatively added network effects, associated with either zero or a strictly positive stand-alone value. Here, we assume the following utility function with additively separable network effects, which expresses that the intrinsic utility is independent of the network effects, as mentioned in Lambertini and Orsini

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[9] This is a necessary condition for a partial output equilibrium (i.e., less than unity) to exist.

[10] Remark 3 is basically similar to that of Belleflamme and Peitz (2015, pp. 561–563). We deal with the case that network effects are not too strong, i.e., $\alpha > N'(S^e)$. In this case, there exists a stable and unique equilibrium.

(2004).

$$u(\theta)=\alpha\theta+N(S^e), \quad \alpha>c\geq 0.^{11} \tag{8}$$

where $\alpha>N'(S^e)$ and $1>N(1)$. Equation (8) implies heterogeneous stand-alone benefits (see equation (5)). In this case, we derive the following inverse demand function:

$$P(X,S^e)=\alpha(1-X)+N(S^e). \tag{9}$$

In particular, following the definitions of Amir and Lazzati (2011), we examine the case of a mixed network good, i.e., $P(X,S^e=0)=\alpha(1-X)>0$. To simplify, assume $\alpha=1>c\geq 0$. The profit function is given by: $\Pi(X,S^e)=\{P(X,S^e)-c\}X=\{1-X+N(S^e)-c\}X$. This profit function is basically similar to that of Lambertini and Orsini (2004), except for the assumption of consumer expectations.[12] In this case, we have the following FOC.

$$\frac{\partial\Pi}{\partial X}=1-2X+N(S^e)-c=0, \tag{10}$$

where $X\in[0,1]$ and $S^e\in[0,1]$. We have the fulfilled expectation equilibrium which is satisfied with the following conditions.

$$X=\frac{1}{2}\{1-c+N(S^e)\} \text{ and } S^e=X, \text{ where } X\in[0,1] \text{ and } S^e\in[0,1]. \tag{11}$$

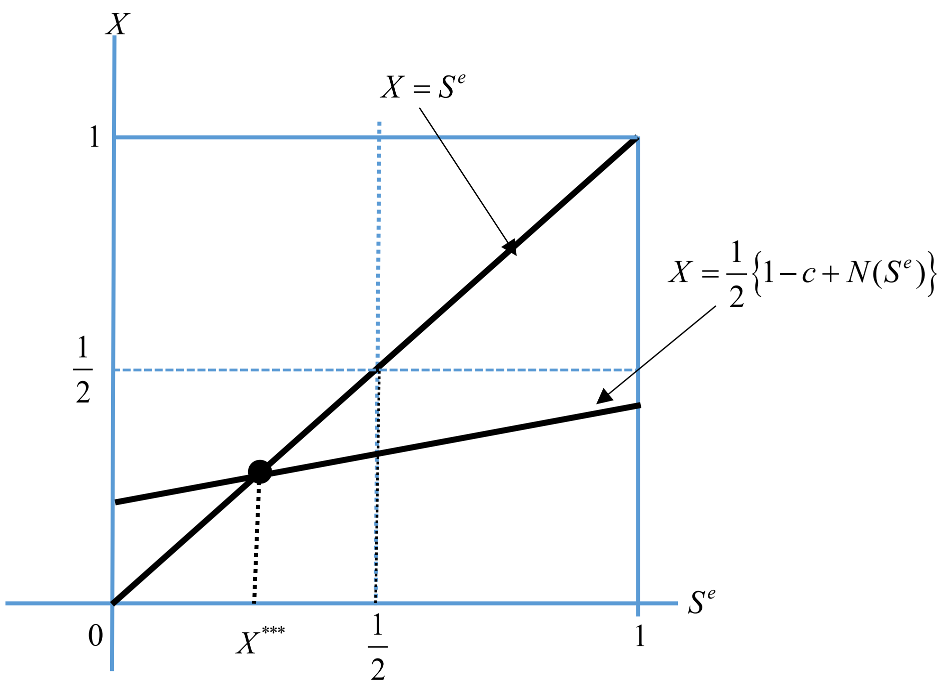

Figure 4. The case of a mixed network good

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[11] If $c\geq\alpha>0$, we obtain the same results as in Section 2.1 and Remark 1.

[12] Under responsive expectations, i.e., $S^e=X$, the profit function is rewritten as: $\Pi(X,X)=\{1-X+N(X)-c\}X$. We then obtain the same result as Lambertini and Orsini (2004).

For example, we assume that $N(S^e) = \beta S^e$, $\beta = 0.5$, and $c = 0.5$. We can draw Figure 4, which illustrates that there is a single stable equilibrium, i.e., $X^{***} = \frac{1}{3}$.

3. Concluding Remarks

In this note, we reconsidered the coordination problem in the case of a monopoly with a pure network good. As addressed in Lambertini and Orsini (2004), the coordination problem relating to the issue of critical mass has not been associated with the presence of network effects and depends on consumer expectations, i.e., passive expectations. Furthermore, even with passive expectations, in the case of a mixed network good in which the intrinsic utility is independent of the expected network sizes, the problem does not necessarily arise. Assuming a mixed network good and responsive expectations, Lambertini and Orsini (2004) show that the coordination problem relating to critical mass does not arise. However, assuming a pure network good and passive expectations, we demonstrate the perspective of Rohlfs (1974, 2001), i.e., the coordination problem associated with critical mass and the role of a stand-alone effect can be used to solve the problem. However, we should note that providing stand-alone benefits may be very costly for the monopoly. In this paper, we assume that the monopoly does not incur the introduction cost of a stand-alone value. However, many firms planning to enter network industries do face the problem of incurring significant costs to secure a certain volume of customers. We should then consider the case of the costly introduction of a stand-alone value.

As in Rohlfs (1974. 2001) and Lambertini and Orsini (2004), in this note we consider a one-sided market associated with network effects, which positively affects the utilities of consumers (i.e., within-group direct network effects). In other words, we do not consider network effects on the supply side (sellers and suppliers). However, in the digital economy as it exists, we observe the enormous trade of products and services ubiquitously operating through various platforms on Internet systems. For this reason, we should focus on the indirect network effects in two-sided (or even multi-sided) markets with platforms (intermediaries) and reconsider the start-up problem in these markets. That is, the number of suppliers (sellers) affects the utilities of consumers (buyers) whereas the number of consumers affects the revenues of suppliers. In this case, is it possible for a coordination failure and a start-up, i.e., a "chicken and egg" problem to arise?[13] However, we appreciate the role of platforms as an organizer (or a market maker) to solve the problem. For instance, although we assume that a stand-alone value is exogenously given in our model, we appreciate that platforms serve stand-alone utilities for sellers and/or buyers to sustain a market, in other words, to solve start-up issues. In particular, platforms manage network effects in the market.[14] Although important in the digital economy, we do not address the issue in this note and defer our attention to future study.

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[13] See, for example, Caillaud and Jullien (2003). Recently, Belleflamme and Peitz (2021) consider the concepts and strategy of platforms.

[14] Relating to Section 2.3, we can interpret the monopoly as a supplier providing a network good and a platform serving a stand-alone utility.

Appendix. The stability condition at the fulfilled expectation equilibrium

The viable market implies that the fulfilled expectation monopoly equilibrium is stable. We now prove the stability condition.

Using equation (3), i.e., $N(S^e)\left(1-2X\right)-c=0$, we derive the following equation.

$$\frac{dX}{dS^e}=\frac{N'[S^e]}{N[S^e]}\frac{1-2X}{2}=\frac{N'[S^e]S^e}{N[S^e]}\frac{1-2X}{2S^e}>0,$$

where $1-2X>0$.[15]

It is stable for the fulfilled expectation equilibrium to hold such that $\left.\frac{dX}{dS^e}\right|_{S^e=X}<1$. That is, we obtain the following condition: $\frac{N'[X]X}{N[X]}<\frac{2X}{1-2X}$, where $\frac{N'[X]X}{N[X]}$ denotes the network effect elasticity in relation to the network sizes. For example, as in Figure 1, where the elasticity is unity. Thus, the stability condition is $1<\frac{2X}{1-2X}$ at $X=X^{**}$. In this case, we derive $0.25<X^{**}\doteq 0.361$ at the equilibrium, which is satisfied with the stability condition.

[15] In Section 2.3, given equation (6), it holds that $1-2X<0$, because $a-c>0$. Thus, in general, the stability condition is that the absolute value is less than unity, i.e., $\left|\left.\frac{dX}{dS^e}\right|_{S^e=X}\right|<1$.

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