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# Responses of unemployment to productivity changes for a general matching technology

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## Abstract

Workers separate from jobs, search for jobs, accept jobs, and fund consumption with their wages. Firms recruit workers to fill vacancies. Search frictions prevent firms from instantly hiring available workers. Unemployment persists. These features are described by the Diamond--Mortensen--Pissarides modeling framework. In this class of models, how unemployment responds to productivity changes depends on resources that can be allocated to job creation. Yet, this characterization has been made when matching is parameterized by a Cobb--Douglas technology. For a canonical DMP model, I (1) demonstrate that a unique steady-state equilibrium will exist as long as the initial vacancy yields a positive surplus; (2) characterize responses of unemployment to productivity changes for a general matching technology; and (3) show how a matching technology that is not Cobb--Douglas implies unemployment responds more to productivity changes, which is independent of resources available for job creation, a feature that will be of interest to business-cycle researchers.

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# 1. Introduction

Each month, workers actively search for jobs and firms actively recruit workers. Despite workers being available to start jobs and firms having posted job openings for vacant positions that could start within 30 days, in any given month, millions of unemployed workers cannot find jobs and millions of vacancies go unfilled. It takes time for a worker to sift through job boards and fill out applications. And it is costly for a firm to post a vacancy. Search frictions give rise to unemployment: a firm cannot instantly hire a worker. A large class of models known as DMP models—short for Diamond–Mortensen–Pissarides models—use these features to study unemployment dynamics.

Within this class of models, Ljungqvist and Sargent (2017) show that the response of unemployment to changes in productivity depends almost entirely on resources that can be allocated to vacancy creation. To understand this idea, consider what happens when a job is created.

Imagine that Eva is unemployed. While unemployed, Eva searches for work, cooks, cleans, and collects unemployment-insurance benefits, which in total amounts to $z$. Upon finding a job, Eva uses the technology at the firm to produce $y$. The match generates $y - z$. This amount—at least some of it—can be allocated to vacancy creation. Ljungqvist and Sargent (2017) call $y - z$ the fundamental surplus. Viewed as a fraction of output, $(y - z)/y$, potential resources for vacancy creation are increasing in $y$. The derivative with respect to $y$ is $z/y^2$. The change will be large only when $z$ is large, which occurs when $z$ is close to $y$. This is crucial:

The fundamental surplus must be small to produce high unemployment volatility during business cycles, because a change in productivity will generate a large change in resources devoted to vacancy creation. Other factors that affect unemployment can be ignored because they are bounded by "a consensus about reasonable parameter values" (Ljungqvist and Sargent, 2017, 2636).[1]

There are two key issues:

**Decomposition for general matching technology.** For the DMP class of models, Ljungqvist and Sargent (2017) establish that responses of unemployment to productivity changes depend on two factors. The two-factor decomposition, though, is based on workers and firms forming productive matches through a particular form of constant-returns-to-scale technology: Cobb–Douglas. Do conclusions about the two-factor, multiplicative decomposition hold for a general matching technology?

**Whether matching technology matters.** Under the decomposition, only one of the two factors is economically meaningful: the upper bound on resources, as a fraction of output, that the invisible hand can allocate to vacancy creation—the fundamental surplus fraction. The factor that does not matter includes an economy's matching technology. Is matching technology inconsequential for labor-market volatility?

The analysis here adopts the DMP framework and a matching technology that exhibits constant returns to scale and satisfies standard regularity assumptions. Within this framework, a ratio of vacancies to unemployment—or labor-market tightness—drives unemployment dynamics. I establish that Ljungqvist and Sargent's (2017) two-factor, multiplicative decomposition of the

[1] Two essential contributions to the development of DMP models were made by Pissarides (1985) and Mortensen and Pissarides (1994). Diamond (1982a,b) made fundamental earlier contributions. See Pissarides (2000), the Economic Sciences Prize Committee (2010), and Petrosky-Nadeau and Wasmer (2017) for further background.

elasticity of tightness with respect to productivity holds for a general matching technology. The factor that includes the fundamental surplus matters, as predicted, but, unlike the Cobb–Douglas case, the second factor is not bounded by professional consensus. Instead, the second factor is bounded by the elasticity of matching with respect to unemployment. There is no reason for this bound to be constant over the business cycle and the second factor—and therefore matching technology—could influence unemployment dynamics.

For conventional parameters, however, I find that the fundamental surplus is significantly more meaningful. Which suggests an economy's matching technology does not influence unemployment volatility. Nevertheless, there is scope for matching technology to matter some. Going beyond the local analysis of the elasticity at steady-state, in a comparative steady-state analysis as a shortcut for analyzing model dynamics, I show that a non-Cobb–Douglas matching technology delivers larger responses of unemployment to productivity changes. This matching technology, with nonconstant elasticity of matching with respect to unemployment, offers a partial solution to the Shimer or unemployment-volatility puzzle, the failure of the canonical DMP model to match the observed volatility of unemployment (Shimer, 2005; Pissarides, 2009).

In addition, I provide an alternative interpretation of the existence and uniqueness of an economy's non-stochastic, steady-state equilibrium. An equilibrium will exist as long as it is profitable to post an initial vacancy. As far as I know, a unique equilibrium is typically posited to exist, which often holds for good economic reasons (see, e.g., Pissarides [2000, 19–20]). While my idea may be well understood by DMP researchers, a benefit is providing an explicit requirement for parameter values and range in which equilibrium tightness is guaranteed to fall.

Understanding how a matching technology fits into interpretations of the fundamental surplus is important. The fundamental surplus, as Ljungqvist and Sargent (2021, 49) emphasize, offers a single channel for explaining how diverse features like "sticky wages, elevated utility of leisure, bargaining protocols that suppress the influence of outside values, a frictional credit market that gives rise to a financial accelerator, fixed matching costs, and government policies like unemployment benefits and layoff costs" can generate high unemployment volatility during business cycles. All these features may interact with matching.

The DMP class of models explicates unemployment, which is a source of misery for many people. Understanding how matching affects unemployment dynamics in this class of models will help policymakers improve public policies that affect the unemployed.

## 2. Model Environment

To characterize unemployment dynamics, I begin with a canonical DMP model. The basic features include linear utility, random search, workers with identical capacities for work, wages determined as the outcome of Nash bargaining, job creation that drives the value of posting a vacancy to zero, and exogenous separations. The environment differs from the one studied by Ljungqvist and Sargent (2017) in a single way: Instead of specifying the matching technology as Cobb–Douglas, I use a general matching technology. The matching technology exhibits constant returns to scale in vacancies and the number of unemployed workers; probabilities of finding and filling a job fall within 0 and 1; and certain regularity conditions for limiting behavior hold.[2]

[2] After the "preliminaries" of describing the economic environment, Ljungqvist and Sargent (2017, 2635) "proceed under the assumption that the matching function has the Cobb–Douglas form, $M(u, v) = Au^{\alpha}v^{1-\alpha}$, where $A > 0$, and $\alpha \in (0, 1)$ is the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\theta/q(\theta)$."

The main result establishes that the fundamental insights of Ljungqvist and Sargent (2017) hold.

### 2.1 Description of the Model Environment

The environment is populated by a unit measure of identical, infinitely-lived workers. Workers are risk neutral with a discount factor of $\beta = (1 + r)^{-1}$ and are either employed or unemployed. They aim to maximize discounted income. Employed workers earn labor income. Unemployed workers earn no labor income, look for work, and experience the value of nonwork, denoted by $z > 0$.

The environment is also populated my a large measure of firms. Firms are either active or inactive. An active firm is either in a productive match with a worker or actively recruiting. An inactive firm becomes active by posting a vacancy, which incurs a cost each period.

Once matched with a worker, a firm operates a production technology that converts an indivisible unit of labor into $y$ units of output. The production technology exhibits constant returns to scale in labor. Each active firm matched with a worker employs a single worker. While matched, a firm earns $y - w$, where $w$ is the per-period wage paid to the worker. Wages are determined by the outcome of Nash bargaining.

All matches are exogenously destroyed with per-period probability $s$. Free entry by the large measure of firms implies that a firm's expected discounted value of posting a vacancy equals zero.

A matching function $M$ determines the number of successful matches in a period. Its arguments are the aggregate measures of unemployed workers, $u$, and vacancies, $v$. The function $M(u, v)$ is increasing in both its arguments. More workers searching for jobs for a given level of vacancies leads to more matches and more vacancies for a given level of unemployment leads to more matches. In addition, $M$ exhibits constant returns to scale in $u$ and $v$.

Labor-market tightness, $\theta$, is defined as the ratio of vacancies to unemployed workers, $\theta := v/u$. Under random matching, the probability that a firm fills a vacancy is given by $q(\theta) := M(u, v)/v = M\left(\theta^{-1}, 1\right)$ and the probability that an unemployed worker matches with a firm is given by $\theta q(\theta) = M(u, v)/u = M(1, \theta)$. Each unemployed worker faces the same likelihood of finding a job because firms lack a recruiting technology that selects a particular candidate and workers do not direct their search effort. The matching technology embodies frictions that generate involuntary unemployment.

### 2.2 Key Bellman Equations

Key Bellman equations in the economy include a firm's value of a filled job and a posted vacancy; and a worker's value of employment and unemployment.

A firm's value of a filled job, $\mathcal{J}$, and a posted vacancy, $\mathcal{V}$, satisfy

$$\mathcal{J} = y - w + \beta\left[s\mathcal{V} + (1 - s)\mathcal{J}\right] \tag{1}$$

$$\mathcal{V} = -c + \beta\left\{q(\theta)\mathcal{J} + \left[1 - q(\theta)\right]\mathcal{V}\right\}. \tag{2}$$

The asset value of a filled job equals flow profit, $y - w$, plus the expected discounted value of continuing the match. The match ends with probability $s$, providing the firm an opportunity to post a vacancy; and the match endures with probability $1 - s$, providing the value of a filled job. The asset value of a vacancy equals the flow posting cost, $c$, plus the expected discounted value of matching with a productive worker. A productive match occurs with probability $q(\theta)$ and the vacancy remains unfilled the following period with probability $1 - q(\theta)$.

A worker's value of employment, $\mathcal{E}$, and unemployment, $\mathcal{U}$, satisfy

$$\mathcal{E} = w + \beta \left[ sU + (1 - s) \mathcal{E} \right] \tag{3}$$

$$\mathcal{U} = z + \beta \left\{ \theta q(\theta) \mathcal{E} + \left[ 1 - \theta q(\theta) \right] \mathcal{U} \right\}. \tag{4}$$

The asset value of employment equals the flow wage plus the expected discounted value of being unemployed with probability $1 - \theta q(\theta)$ or employed with probability $\theta q(\theta)$ the following period.

Convention in the economy dictates that a worker and a firm split the surplus generated from a match through Nash bargaining. Surplus from a match, $\mathcal{S}$, is the benefit to a firm from operating as opposed to maintaining a vacancy plus the benefit to a worker from earning a wage as opposed to experiencing nonwork: $\mathcal{S} = (\mathcal{J} - \mathcal{V}) + (\mathcal{E} - \mathcal{U})$. Nash bargaining depends on the parameter $\phi \in [0, 1)$, which measures a worker's relative bargaining power. The outcome of Nash bargaining specifies that what the worker stands to gain equals their share of surplus and the firm receives the remainder: $\mathcal{E} - \mathcal{U} = \phi \mathcal{S}$ and $\mathcal{J} - \mathcal{V} = (1 - \phi) \mathcal{S}$.

The next section defines an equilibrium and establishes conditions for existence and uniqueness.

### 2.3 Equilibrium

The size of the labor force is normalized to 1. A steady-state equilibrium requires that the number of workers who separate from jobs, $s(1 - u)$, equals the number of unemployed workers who find employment, $\theta q(\theta) u$, so that the unemployment rate remains constant. The steady-state condition implies $u = s / [s + \theta q(\theta)]$, which yields a Beveridge-curve relationship that is negative in $u$–$v$ space. "When there are more vacancies, unemployment is lower because the unemployed find jobs more easily" (Pissarides, 2000, 20). Consistent with this theory, data on vacancies and unemployment exhibit a negative relationship. The data are shown in figure 4 of appendix A.1.[3]

A steady-state equilibrium is a list of values $\langle u, \theta, w \rangle$ that satisfy the Bellman equations (1) – (4) along with the free-entry condition that requires $\mathcal{V} = 0$, the stipulation that wages are determined by the outcome of Nash bargaining, and the steady-state unemployment rate. These equations can be manipulated to yield a single expression in $\theta$ alone:

$$y - z = \frac{r + s + \phi \theta q(\theta)}{(1 - \phi) q(\theta)} c. \tag{5}$$

Details for arriving at the expression in (5) are provided in appendix D.

While Ljungqvist and Sargent (2017) do not explicitly establish the existence of a unique $\theta$ that solves (5), it is straightforward to do so. I state and sketch a proof here because the steps yield a requirement for parameters that has an intuitive interpretation. Appendix D.3 provides more detail.

**Proposition 1.** Suppose $y > z$, which says that workers produce more of the homogeneous consumption good at work than at home, and suppose that $(1 - \phi)(y - z) / (r + s) > c$. Then a unique $\theta \in (0, (1 - \phi)(y - c) / (\phi c))$ solves (5). The condition that $(1 - \phi)(y - z) / (r + s) > c$ requires that the value of an initial job opening be positive.

*Proof.* To establish existence, I define the function

$$\mathcal{T}(\tilde{\theta}) = \frac{y - z}{c} - \frac{r + s + \phi \tilde{\theta} q(\tilde{\theta})}{(1 - \phi) q(\tilde{\theta})}.$$

[3]Barlevy et al. (2023) provide a discussion of the negative relationship and Elsby et al. (2015) provide an overview within the context of the DMP framework.

Using the fact that $\lim_{\tilde{\theta}\to 0} q\left(\tilde{\theta}\right) = 1$ and the requirement that the value of posting an initial vacancy is positive, $\lim_{\tilde{\theta}\to 0} \mathcal{T}\left(\tilde{\theta}\right) = (y-z)/c - (r+s)/(1-\phi) > 0$. The inequality, as will be shown, can be interpreted as an initial posted vacancy having positive value. Next, I define $\tilde{\theta}^{\bullet} = (1-\phi)(y-z)/(\phi c) > 0$, where the inequality comes from the fact that $y > z$ and $\phi \in [0,1)$ by assumption. Then $\mathcal{T}\left(\tilde{\theta}^{\bullet}\right) < 0$. Because $\mathcal{T}$ is a combination of continuous functions, it is also continuous. An application of the intermediate-value theorem establishes that there exists $\theta \in (0, (1-\phi)(y-z)/(\phi c))$ such that $\mathcal{T}(\theta) = 0$. Uniqueness follows from the fact that $\mathcal{T}$ is everywhere decreasing.

The condition that $(1-\phi)(y-z)/(r+s) > c$ requires that the value of posting an initial vacancy is profitable. The following thought experiment illustrates why.

Starting from a given level of unemployment, which is guaranteed with exogenous separations, the value of posting an initial vacancy is computed as $\lim_{\theta\to 0} \mathcal{V}$. In the thought experiment, the probability that the initial vacancy is filled is 1, as $\lim_{\theta\to 0} q(\theta) = 1$. The following period the firm earns the value of a productive match, which equals the flow payoff $y - w$ plus the value of a productive match discounted by $\beta(1-s)$. The value of $\mathcal{J}$ is thus $(y-w)/[1-\beta(1-s)]$.

The wage rate paid by the firm in this scenario is $\lim_{\theta\to 0} w = \phi y + (1-\phi) z$, making $\mathcal{J} = (1-\phi)(y-z)/[1-\beta(1-s)]$. Using this expression in the value of an initial vacancy yields

$$\lim_{\theta\to 0} \mathcal{V} = \lim_{\theta\to 0} \langle -c + \beta\{q(\theta)\mathcal{J} + [1-q(\theta)]\mathcal{V}\}\rangle = -c + \beta\frac{(1-\phi)(y-z)}{1-\beta(1-s)} > 0.$$

The inequality stipulates that in order to start the process of posting vacancies, the first vacancy needs to be profitable. Developing this inequality yields $(1-\phi)(y-z)/(r+s) > c$, establishing the condition listed in proposition 1.

To arrive at the equilibrium, other profit-seeking firms post vacancies. Filling a vacancy is no longer guaranteed, which raises the expected cost of maintaining a vacancy until it is filled, lowering $\mathcal{V}$. Recruitment efforts eventually drive $\mathcal{V}$ to 0. □

Checking that parameters generate an equilibrium can be useful, especially in complicated models. Not only does a steady state permit a comparative-static analysis, but not checking can lead to key channels being mistakenly downplayed (Christiano et al., 2021; Ljungqvist and Sargent, 2021, 47n18). In addition, knowing the interval that contains equilibrium tightness is useful for finding its numerical value.

### 2.4 The Elasticity of Labor-Market Tightness with Respect to Productivity

Unemployment dynamics are driven by labor-market tightness within the DMP class of models. Big responses of unemployment to the driving force of productivity require a high elasticity of tightness with respect to productivity, $\eta_{\theta,y}$. My main result establishes that the two-factor, multiplicative decomposition of $\eta_{\theta,y}$ holds for a general matching technology, not only for the Cobb–Douglas case. The result is stated in proposition 2. Appendix D.5 provides details.

**Proposition 2.** In the canonical DMP search model, which features a general matching technology, random search, linear utility, workers with identical capacities for work, exogenous separations, and no disturbances in aggregate productivity, the elasticity of market tightness with respect to productivity can be decomposed as

$$\eta_{\theta,y} = \left[1 + \frac{(r+s)\left(1-\eta_{M,u}\right)}{(r+s)\,\eta_{M,u} + \phi\theta q\left(\theta\right)}\right] \frac{y}{y-z} =: \Upsilon \frac{y}{y-z} < \frac{1}{\eta_{M,u}} \frac{y}{y-z}, \tag{6}$$

where the second factor is the inverse of fundamental surplus fraction and the first factor is bounded below by 1 and above by $1/\eta_{M,u}$: $1 < \Upsilon < 1/\eta_{M,u}$.

The bound $1/\eta_{M,u}$ exceeds 1 because $\eta_{M,u} \in (0,1)$, a well-known result that is proved in appendix B in proposition 3 for completeness.

For the Cobb–Douglas case, $\eta_{M,u}$ is constant. Estimates for its value and a "consensus" about reasonable values for the other terms in (6) imply that the factor $\Upsilon$ contributes little to the elasticity of market tightness (Ljungqvist and Sargent, 2017, 2636). Only the second factor, the inverse of the fundamental surplus fraction, $y/(y-z)$, can possibly generate unemployment dynamics observed in the data. Because a diverse set of DMP models allow a similar two-factor decomposition, the influence of the fundamental surplus is a single, common channel for explaining unemployment volatility. Any additional feature added to a DMP model must run through this channel.

The decomposition, though, suggests that an economy's matching technology, subsumed in $\Upsilon$, does not matter for unemployment dynamics. In general, however, $\eta_{M,u}$ is not constant and depends on $\theta$, which varies meaningfully over the business cycle.

This variability is shown in figure 1, which depicts $1/\eta_{M,u}$ for two prominent matching technologies. While details will be provided in the computational experiment described below, the main takeaway is that the bound warrants looking at whether matching technology can matter for unemployment volatility. For the Cobb–Douglas parameterization, $1/\eta_{M,u}$ equals the constant $1/\alpha$. In contrast, the nonlinear series is depicted for values of $\theta$ observed in the US economy from December 2000 onwards. The variability and magnitude of the series provide scope for investigating whether a matching technology matters for unemployment volatility.

## 3. Generating Larger Unemployment Responses to Productivity Perturbations

A comparative-equilibrium exercise is carried out by looking at how unemployment varies with productivity, a main goal of DMP models, for two matching functions. This shortcut for analyzing model dynamics is feasible because unemployment is a fast-moving stock variable and productivity shocks exhibit high persistence. The novelty is the comparison between matching functions.

I compare two parameterization of $M\left(u,v\right)$: $Au^{\alpha}v^{1-\alpha}$ and $\mathcal{A}uv\left(u^{\gamma}+v^{\gamma}\right)^{-1/\gamma}$. The first is the familiar and empirically successful Cobb–Douglas parameterization (Bleakley and Fuhrer, 1997; Petrongolo and Pissarides, 2001). The second is a nonlinear parameterization suggested by den Haan et al. (2000). To motivate the nonlinear form, imagine that each unemployed person contacts other agents randomly. The probability that the other agent is a firm is $v/(u+v)$. There are $u \times v/(u+v)$ matches. The general form used here captures thick and thin market externalities.

The computational experiment begins by replicating Ljungqvist and Sargent (2017, 2644, fig. 2). I use their parameters for comparison: The model period is one day, avoiding job-finding and -filling probabilities above 1. The discount factor is $\beta = 0.95^{1/365}$, which corresponds to an annual interest rate of 5 percent. The daily separation rate is $s = 0.001$, which corresponds to a job lasting on average 2.8 years. The bargaining parameter is $\phi = 0.5$, which is the midpoint of its range. The

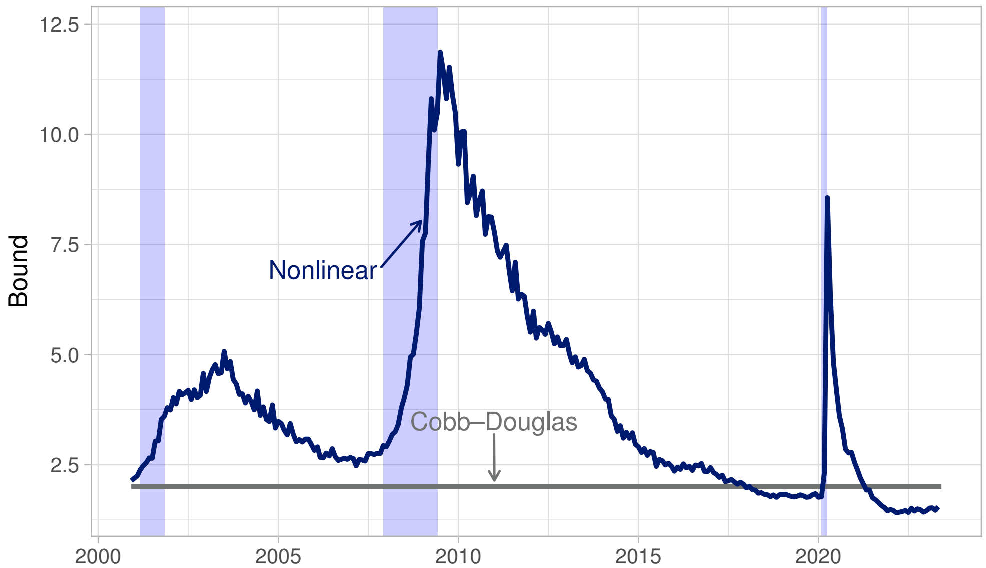


Figure 1: Upper bounds for $\Upsilon$, inverses of the elasticity of matching with respect to unemployment, $1/\eta_{M,u}$, for two matching technologies.

*Notes:* The inverses, $1/\eta_{M,u}$, are upper bounds for $\Upsilon$, the first factor in the decomposition in (6). For the Cobb–Douglas technology, $1/\eta_{M,u} = 1/\alpha$, evaluated at $\alpha = 0.5$. For the nonlinear technology, $1/\eta_{M,u} = (1+\theta^{\gamma})/\theta^{\gamma}$, evaluated at $\gamma = 1.27$ and values for $\theta$ observed in US data after December 2000. These parameter values are used in the computational experiment described in figure 2. Shaded areas indicate US recessions.
*Sources:* Author's calculations using data from the US Bureau of Labor Statistics. Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNEMPLOY. Job Openings: Total Nonfarm [JTSJOL], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/JTSJOL.

flow cost of posting a vacancy is $c = 0.1$. The value of nonwork is $z = 0.6$ and values of workers' productivity are investigated above $z$ and substantially less than unity.[4]

The remaining parameters specify the matching technology. For the Cobb–Douglas matching technology $\alpha = 0.5$, so that $\eta_{M,u}$ equals the bargaining parameter. This choice satisfies Hosios's (1990) efficiency condition. So far, these parameters agree with those adopted by Ljungqvist and Sargent (2017). For the nonlinear matching technology, I set $\gamma = 1.27$ to agree with den Haan et al. (2000, 491, table 1). The only remaining parameters to choose are the matching-efficiency parameters, $A$ and $\mathcal{A}$.

The matching-efficiency parameters along with productivity levels $y \in Y = \{0.61, 0.63, 0.65\}$ index six economies. For each $y \in Y$, $A$ and $\mathcal{A}$ are calibrated to make the unemployment rate 5 percent. Some values of matching efficiency can cause finding and filling probabilities to rise above 1, as established in appendix C, but this is avoided by adopting a daily time period for the calibration (Ljungqvist and Sargent, 2017, 2639n6). When I perturb productivity around $y$ for each economy, all other parameters remain fixed.

How the steady-state unemployment rate responds to productivity perturbations is shown in

[4] Appendix D.4 shows how a different choice of $c$ will produce the same equilibrium level of unemployment and job-finding (but not job-filling) through a different level of matching efficiency. Kiarsi (2020) emphasizes the importance of the cost of posting a vacancy in this class of models.

figure 2. Unemployment increases when productivity falls and decreases when productivity rises, regardless of matching technology. Magnitudes of unemployment responses, however, depend on at least two features.

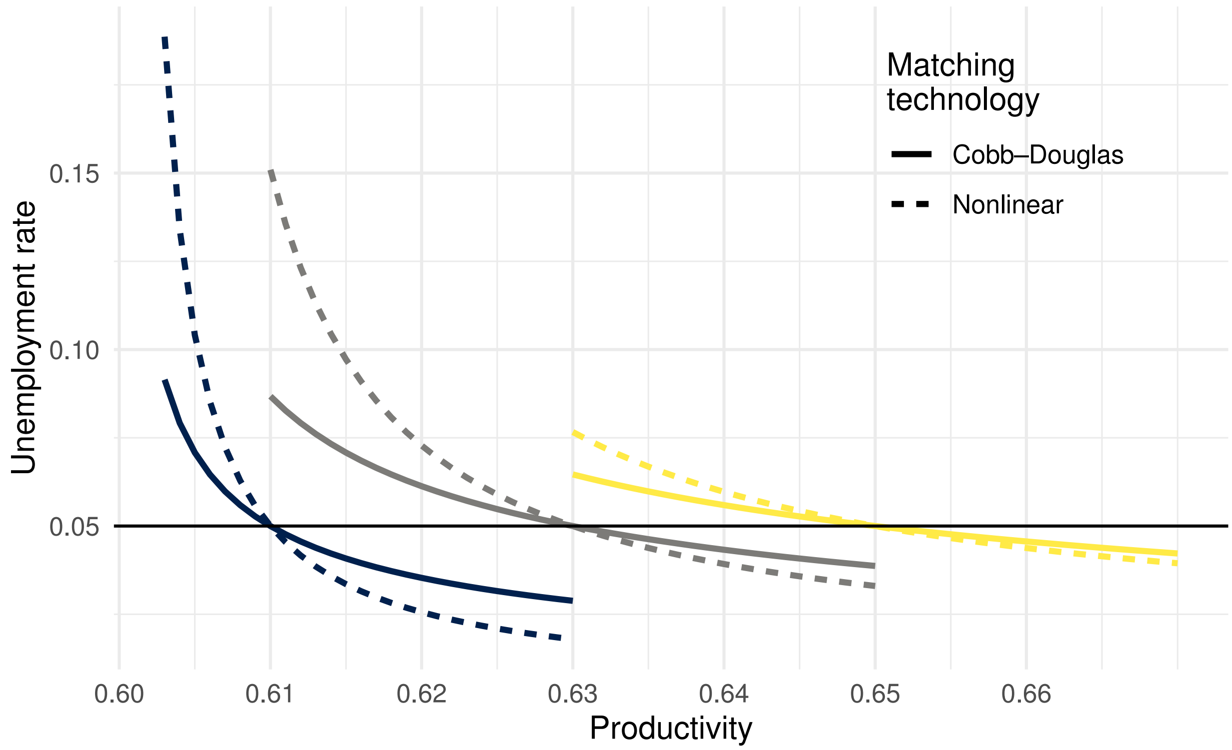


Figure 2: Responses of unemployment to productivity perturbation.

*Notes:* The six economies are indexed by three productivity levels and two matching technologies. For each economy, matching efficiency for each matching technology is adjusted to generate 5 percent unemployment at productivity levels 0.61, 0.63, and 0.65. The matching technologies are Cobb–Douglas, $M(u,v) = Au^{\alpha}v^{1-\alpha}$, and nonlinear, $\mathcal{M}(u,v) = \mathcal{A}uv\,(u^{\gamma}+v^{\gamma})^{-1/\gamma}$. In each economy, steady-state unemployment rates are shown for perturbations in productivity around each economy's baseline productivity level.

First, looking from left to right, the closer $y$ is to $z$, the smaller is the fundamental surplus. The smaller the fundamental surplus, as predicted by equation (6), the more $\theta$ and thus unemployment respond to productivity changes. For the two dark, navy curves at left, where line pattern indexes matching technology, the fundamental surplus fraction is smallest and unemployment responses are largest. For the two light, yellow curves at right, where line pattern again indexes matching technology, the fundamental surplus fraction is largest and unemployment responses are smallest.

Second, the relationship between unemployment and productivity depends on an economy's matching technology. This point can be seen by comparing solid lines to broken lines. For each $y \in Y$, the nonlinear matching technology, causes unemployment to respond more to changes in productivity. This result will be useful to those who want address the Shimer or unemployment-volatility puzzle (Shimer, 2005; Pissarides, 2009).

## 4. Conclusion

For a canonical DMP model, I showed that the elasticity of labor-market tightness with respect to productivity can be decomposed into two multiplicative factors for a general matching technology. One of the factors depends on the fundamental surplus and this factor has the largest influence on unemployment dynamics in the computational experiment. The other factor is bounded above by the inverse of the elasticity of matching with respect to unemployment, which, in general, varies meaningfully over the business cycle. The finding leads to the conclusion that matching technology can matter for unemployment dynamics. As features like sticky prices, sticky wages, idiosyncratic shocks, and composition effects of employment are added to the DMP class of models, investigating interactions with the matching technology may be worth considering, not just the fundamental-surplus channel.

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