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Recursive Nash-in-Nash bargaining solution

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Abstract

The standard Nash-in-Nash bargaining solution is commonly applied in a number of policy applications. However, the Nash-in-Nash framework does not capture renegotiations on off-equilibrium paths or contingent contracts, and as a result, in some situations the predictions of standard Nash-in-Nash are counterintuitive. Thus, we propose a new bargaining solution for interdependent bilateral negotiations, which we call the recursive Nash-in-Nash bargaining solution. The main difference between this bargaining framework and the standard Nash-in-Nash is in the treatment of the disagreement point. In the recursive Nash-in-Nash bargaining solution, the disagreement payoffs are the outcomes of bargaining with knowledge of the disagreement rather than the equilibrium outcomes as in the standard Nash-in-Nash. We show that under some assumptions, the recursive Nash-in-Nash bargaining solution is the same as the Shapley value for the corresponding game in characteristic function form or the more general Myerson value for the corresponding game in partition function form.

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1. Introduction

The Nash bargaining model (Nash (1950)) has been widely used as a framework for analyzing markets involving bargaining in a range of applications.[1,2] Nash's seminal paper considers a two-person game, but many economic problems involve multiple interdependent pairs of bilateral bargaining, such as buyer-seller networks and wage negotiations between a firm and its individual workers. One commonly used extension of the Nash bargaining solution to the problem of bilateral bargaining between multiple pairs is the "Nash equilibrium in Nash bargains," or "Nash-in-Nash" bargaining (Collard-Wexler, Gowrisankaran, and Lee (2019)). It was first proposed by Horn and Wolinsky (1988) to study horizontal mergers given a buyer-seller network and has been applied in various economic environments.[3]

The Nash-in-Nash bargaining solution is defined as the set of bilateral bargaining outcomes consistent with all other bilateral bargaining outcomes, assuming that in the case of an out-of-equilibrium bilateral disagreement all other bargaining outcomes remain at the proposed equilibrium outcomes. That is, each agent believes that all other pairs of bargaining outcomes will remain as predicted by the equilibrium even if there is an out-of-equilibrium breakdown in bargaining between one pair of agents. This may be a good approximation of real-world bargaining outcomes in some situations, but in other situations, agreements of interdependent bargaining pairs may be expected to change if one pair's negotiation breaks down unexpectedly. We propose an alternative extension of the Nash bargaining solution to the multiple bilateral bargaining problems to treat these situations, which we call the "recursive Nash-in-Nash" bargaining solution.[4]

The recursive Nash-in-Nash bargaining solution is similar to the standard Nash-in-Nash in that each bargaining pair splits the surplus bilaterally according to the Nash bargaining solution. The difference is in the disagreement payoffs. In the recursive Nash-in-Nash framework, a bargaining pair's disagreement payoffs are determined assuming that all other bargaining pairs negotiate their agreements expecting the disagreement. We show that under certain assumptions, the recursive Nash-in-Nash bargaining solution is the same as the Shapley value (Shapley (1953)) of a corresponding game in characteristic function form[5] or the Myerson value of a corresponding game in partition function form (Myerson (1977b)) if there are externalities across different groups of bargaining agents.[6] These results simplify the quantification of outcomes using these models for applied work.

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1 In many applications, the Nash bargaining solution is used to identify the division of surplus assuming total gains from trade do not vary over bargaining terms. In such applications, it would probably be more straightforward to describe the outcome as involving splitting the surplus in a constant proportion. In such situations the only operative axioms of the four axioms characterizing the Nash bargaining solution as summarized in the literature (*see*, for example, Osborne and Rubinstein (1990), Chapter 2) are the symmetry and efficiency axioms. However, we will follow the practice in the applied literature and refer to these as involving a Nash bargaining solution.

2 *See, e.g.,* FTC v. ProMedica Health Sys., Inc., 2011-1 Trade Cas. (CCH) ¶ 77,395 (N.D. Ohio Mar. 29, 2011), FCC (2011), Willig (2016), and Farrell (2017).

3 *See, e.g.,* Draganska, Klapper and Villas-Boas (2010), Crawford and Yurukoglu (2012), Grennan (2013), Gowrisankaran, Nevo, and Town (2015), Ho and Lee (2017), Crawford, Lee, Whinston and Yurukoglu (2018), and Sheu and Taragin (2021).

4 A similar bargaining solution called Nash-in-Shapley was independently defined by Froeb, Mares, and Tschantz (2019). That work came to the authors' attention after early drafts of this paper.

5 This result is related to the observations made in Myerson (1977a), Jackson and Wolinsky (1996), Feldman (1996), and Navarro (2007) that the Shapley value or the more general Myerson value is the unique allocation rule that satisfies the "component balance" condition and the "fairness" condition.

6 In the literature, the Myerson value for network games refers to the value derived in Myerson (1977a) (*see e.g.*, Navarro and Perea (2013)), where he generalizes the Shapley value to games with a network structure; the Myerson value for partition

As illustrated in four examples below, the recursive Nash-in-Nash bargaining solution generates materially different outcomes than the standard Nash-in-Nash solution. These differences matter in a number of policy contexts. For example, recently, in the US Copyright Royalty oral hearing for Web V, experts for both Pandora and SoundExchange made reference to an earlier version of this paper[7] and its results when debating whether the appropriate model to apply is Nash-in-Nash, Shapley value, or Myerson value. In Fiedler (2020)'s paper discussing regulation of the health care market, he recognizes the issue of Nash-in-Nash in the setting of a health insurer bargaining with complementary providers and agrees that the recursive Nash-in-Nash bargaining solution has some appealing features in that setting. Boshoff *et al.* (2023) discusses the difference between Nash-in-Nash and Nash-in Shapley (a bargaining solution similar to recursive Nash-in-Nash) in the context of antitrust review of vertical mergers.[8]

The bargaining literature recognizes that the standard Nash-in-Nash disagreement points may not be a good assumption for all real-world situations.[9] Collard-Wexler *et al.* (2019) explained that Nash-in-Nash solutions may not emerge if there are renegotiations upon disagreement, agreements have contingencies, or there are large complementarities on one side of the buyer-seller network. The recursive Nash-in-Nash bargaining solution provides a more plausible alternative in these settings than the standard Nash-in-Nash solution.

Our paper is also related to the literature that establishes the equivalence between non-cooperative extensive form games and the Shapley value or Myerson value.[10] De Fontenay and Gans (2014) showed the equilibrium payoffs of a non-cooperative pairwise bargaining game with externalities are the same as the Myerson value of a related cooperative game. The recursive Nash-in-Nash can be viewed as a reduced form bargaining solution of their extensive form game.

The remainder of the paper is organized as follows. Section 2 provides the definition of the recursive Nash-in-Nash bargaining solution and shows the equivalence between the Shapley value and the Myerson value under certain assumptions. Section 3 uses examples to illustrate the main differences between the recursive Nash-in-Nash and the Nash-in-Nash bargaining solutions. Section 4 presents concluding remarks. The proofs are in the Appendix.

2. Model and Results

We provide the main notation in this section and the rest in the Appendix. Let $N = \{1, \ldots, n\}$ denote the set of agents and let g denote the set of agent pairs who bargain with each other, where g can be viewed as an undirected network such that if agents $i \in N$ and $j \in N$ negotiate with each other, then the unordered pair ij is in g. The pairwise negotiations are over lump-sum transfers.

Following Collard-Wexler *et al.* (2019), we assume that each agent's profits without transfers (called "gross profits" henceforth) do not depend on the transfers negotiated without them, but they can depend on whether agreements are reached in those negotiations. As in Collard-Wexler *et al.* (2019), we take gross profits at all the subsets of bilateral negotiations in g as primitives of

function form games refers to the value derived in Myerson (1977b) (*see e.g.*, Navarro (2007)), where he generalizes the Shapley value to games with externalities across coalitions. We use the term "Myerson value" to refer to the latter one, i.e., the Myerson value for games in partition function form.

7 Yu and Waehrer (2019)

8 As mentioned in footnote 4, the Nash-in-Shapley and the recursive Nash-in-Nash bargaining solutions were independently proposed.

9 *See, e.g.,* Raskovich (2003) and Ho and Lee (2019).

10 *See, e.g.,* Stole and Zweibel (1996), Navarro and Perea (2013), and Bruegemann, Gautier, and Menzio (2019).

the game. Let $\pi: \{g'|g' \subseteq g\} \rightarrow R^{|N|}$ be the gross profit function.[11] It is a vector-valued function whose ith entry $\pi_i(g')$ is agent i's gross profit if the set of agreements reached is $g' \subseteq g$. In practice, agents often negotiate over "actions" that affect their gross profits, such as how much investment to make and the characteristics of the goods being traded. $\pi(g')$ can be viewed as the gross profits when all the agreements in g' include bilaterally efficient actions. We abstract away from how the actions are determined and focus on the surplus division.

A component in our setting is a bargaining group. Let Q_g be the set of N's components in g. Note that Q_g is a partition of N.

Definition. *Given a set of agents N, a set of bilateral negotiations g and a gross profit function $\pi: \{g'|g' \subseteq g\} \rightarrow R^{|N|}$, the recursive Nash-in-Nash bargaining solution is a vector of payoffs $U^g \in R^{|N|}$ such that*

$$\sum_{i \in S} U_i^g = \sum_{i \in S} \pi_i(g), \forall S \in Q_g, \qquad (Component\ balance)$$

$$U_i^g - U_i^{g \backslash ij} = U_j^g - U_j^{g \backslash ij}, \forall ij \in g, \qquad (Fairness)$$

$$U_i^g \geq U_i^{g \backslash ij}, U_j^g \geq U_j^{g \backslash ij}, \forall ij \in g, \qquad (Individual\ rationality)$$

where $U^{g \backslash ij} = \pi(\emptyset)$ if $|g| = 1$ and otherwise $U^{g \backslash ij}$ is the recursive Nash-in-Nash bargaining solution given agents N, bilateral negotiations $g \backslash ij$ and the gross profit function π.

The component balance condition requires that the total payoff of a bargaining group is the sum of its members' gross profits. That is, there are no net transfers across different bargaining groups.[12] The fairness condition, as is commonly assumed in the literature, requires that every pair of negotiating agents splits the gains from trade equally.[13] The individual rationality condition requires that each agent's payoff in an agreement is at least as high as that agent's disagreement payoff.[14] When $|g| = 1$, the recursive Nash-in-Nash solution is the same as the Nash bargaining solution.

The recursive Nash-in-Nash is defined recursively and the disagreement payoffs $U_i^{g \backslash ij}$ and $U_j^{g \backslash ij}$ in the fairness and individual rationality conditions are the recursive Nash-in-Nash bargaining payoffs when ij is removed from the set of negotiations in g assuming all the other contracts are "renegotiated" without this pair's agreement. This is the key difference between recursive Nash-in-Nash and Nash-in-Nash, where $U_i^{g \backslash ij}$ and $U_j^{g \backslash ij}$ in the fairness and individual

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11 We use $|\cdot|$ to denote the size of a set. For example, $|g|$ denotes the number of negotiating pairs in g.

12 This condition is commonly assumed in the literature (sometimes also called "component efficient"). *See, e.g.,* Myerson (1977a), Jackson and Wolinsky (1996), and Navarro (2007).

13 *See*, *e.g.*, Myerson (1977a), Jackson and Wolinsky (1996), Navarro (2007), and De Fontenay and Gans (2014). One could define recursive Nash-in-Nash with unequal bargaining powers as has been done in Nash-in-Nash bargaining models, but if one were to do so, the equivalence with the Shapely and Myerson Values would not hold as those two concepts apply their own form of fairness.

14 While satisfaction of the individual rationality condition here is necessary for the equivalence results (Propositions 1 and 2), it is possible to define the recursive Nash-in-Nash without it. Disagreement at stages in the bargaining can simply be part of the outcome that would be anticipated as part of the disagreement payoffs. One possibility would be to assume that given a set of bargaining pairs $g' \subseteq g$ if the component balance and fairness conditions would result payoffs that violate individual rationality for a set of bargaining pairs $f \subseteq g'$, then $U_i^{g'} = U_i^{g' \backslash f}$ or alternatively $U_i^{g'} = (1/|f|) \sum_{j \in f} U_i^{g' \backslash j}$. As we show in an example in the Appendix, either of these assumptions will result in a solution to the recursive bargaining game even when individual rationality does not hold at every recursive stage. Though such a solution might involve disagreement even for the full set of bargaining pairs g.

rationality conditions would be replaced by i's and j's payoffs assuming all the other contracts remain the same as if i and j have reached an agreement.

The recursive Nash-in-Nash bargaining solution can be solved by induction on the number of negotiating pairs starting from one pair of negotiating agents in g and increasing the number of negotiating agent pairs one by one. Consider the example shown in Figure 1 for an illustration of the induction on the number of negotiation pairs. There are two pairs of negotiations in g: 1 and 3, and 2 and 3. If 1 and 3 disagree, then the set of negotiations becomes g_1, and if 2 and 3 disagree, the set of negotiations becomes g_2. We denote the empty set of negotiations by g_3. One can first solve the recursive Nash-in-Nash bargaining solutions for the negotiation sets g_1 ($U_i^{g_1}, i = 1,2,3$) and g_2 ($U_i^{g_2}, i = 1,2,3$). Then the recursive Nash-in-Nash solution for the negotiations in g ($U_i^{g}, i = 1,2,3$) can be solved using the solutions for g_1 and g_2 as disagreement payoffs in the fairness and individual rationality conditions. For example, the disagreement payoffs for the negotiation between 1 and 3 in g are $U_1^{g\backslash 13} = U_1^{g_1}$ and $U_3^{g\backslash 13} = U_3^{g_1}$.

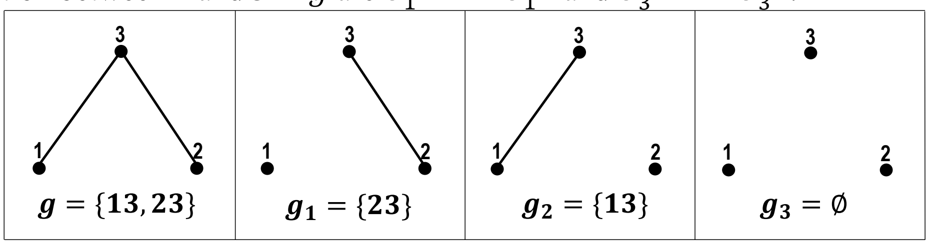

Figure 1: An example of a set of bilateral negotiations g and its subsets g_1, g_2 and g_3.

We now show that under certain assumptions, the recursive Nash-in-Nash bargaining solution is the same as the Shapley value or the more general Myerson value.

Define the restriction of a network g to a coalition S as $g|_S = \{ij \in g: i \in S \text{ and } j \in S\}$.

Assumption 1. (No externality across bargaining groups) $\pi_i(g') = \pi_i(g'|_{S_i^{g'}}), \forall g' \subseteq g, \forall i \in N$, where $S_i^{g'}$ denotes the component of N in g' that i belongs to.

This assumption is used in Proposition 1 but not in Proposition 2. It says that any agent's gross profit depends only on the set of agreements made in their bargaining group. In other words, the agreements made in one bargaining group do not have externalities on agents in other bargaining groups. For example, in Figure 1, this assumption implies that $\pi_1(g_1) = \pi_1(g_3)$ and $\pi_2(g_2) = \pi_2(g_3)$. Externalities on agents in the same bargaining group are allowed. For example, $\pi_1(g)$ and $\pi_1(g_2)$ can be different (due to externality from agents 2 and 3's agreement on agent 1) while satisfying Assumption 1.

Assumption 2. (Monotonicity) $\forall g' \subseteq g, \forall ij \in g', \sum_{k\in N} \pi_k(g') \geq \sum_{k \in N} \pi_k(g'\backslash ij)$.

This assumption says that for any subset of agreements, the agents' total gross profit is weakly higher with these agreements than with one less agreement. For example, in Figure 1, this assumption implies that the three agents' total gross profit given g is weakly higher than that given g_1 or g_2, which in turn is weakly higher than that given g_3. This assumption is used in Proposition 1 to guarantee positive gains from trade.

Proposition 1. *Given a set of agents N, a set of bilateral negotiations g, and a gross profit function $\pi: \{g'|g' \subseteq g\} \to R^{|N|}$ that satisfies Assumptions 1 and 2, the recursive Nash-in-Nash bargaining solution exists and it is the same as the Shapley value of the cooperative game (N, v^g) in characteristic function form, where $v^g(S) = \sum_{i \in S} \pi_i(g|_S)$, $\forall S \subseteq N$.*

The cooperative game (N, v^g) can be interpreted as follows. We want to allocate to each agent in N the total value of the grand coalition, which is the total gross profit of all agents when all the agreements in g are reached, i.e., $v^g(N) = \sum_{i\in N} \pi_i(g)$. The value of each coalition S is the total gross profit of its members when only the subset of agreements within S, $g|_S$, is reached. Proposition 1 says that the Shapley value of this game is the same as the recursive Nash-in-Nash bargaining solution given Assumptions 1 and 2.

In some real-world situations, Assumption 1 is not satisfied, and the recursive Nash-in-Nash is not the same as the Shapley value. Consider the following example adapted from Myerson (1977b). Assume the set of negotiations is g_2 in Figure 1. It has only one subset, $g_3 = \emptyset$. If $\pi_2(g_2) \neq \pi_2(g_3)$, then Assumption 1 is not satisfied. This may happen when agents 1 and 2 are competing retailers, agent 3 is a supplier, and the agreement between retailer 1 and the supplier has negative externality on retailer 2. The recursive Nash-in-Nash bargaining solution given g_2 is different from the Shapley value for the corresponding game $(\{1,2,3\}, v^{g_2})$. This can be seen by comparing agent 2's payoffs. In the recursive Nash-in-Nash bargaining solution, agent 2's payoff is simply $\pi_2(g_2)$ by the component balance condition, whereas agent 2's Shapley value depends on both $\pi_2(g_2)$ and $\pi_2(g_3)$ since it is the average of agent 2's marginal contributions over all possible orders by which the agents arrive at a hypothetical market.

In these situations with externalities across bargaining groups, the recursive Nash-in-Nash bargaining solution is the same as the more general Myerson value for a corresponding game in partition function form if the Myerson value satisfies the individual rationality condition. The following definitions largely follow those in Myerson (1977b).

Given a set of agents N, let PT be the set of partitions of N and let ECL be the set of embedded coalitions; that is, the set of coalitions together with specifications of how the other agents are assigned.[15] Formally:

$$ECL = \{(S, Q) | S \in Q \in PT\}.^{16}$$

A game in partition function form is defined as a vector $w \in R^{|ECL|}$, where $w_{S,Q}$ (the (S,Q)-component of w) is interpreted as the wealth, measured in units of transferable utility, which coalition S would have created if all the agents are aligned into coalitions of partition Q.

Given a network g and a partition Q, let $g|Q$ be the subnetwork of g such that all negotiations in g between agents in different coalitions in Q are removed from network g. That is, $g|Q = \bigcup_{T\in Q} g|_T$. (Note that in our notation, $g|A$ and $g|_A$ are not the same.)

Proposition 2. *Given a set of agents N, a network of potential bilateral negotiations g and a gross profit function $\pi: \{g' | g' \subseteq g\} \to R^{|N|}$, if the recursive Nash-in-Nash solution exists then it is the same as the Myerson value of the corresponding cooperative game in partition function form w^g, where $w^g \in R^{|ECL|}$ and $w^g_{S,Q} = \sum_{i\in S} \pi_i(g|Q)$, $\forall (S,Q) \in ECL$.*

Similar to the game (N, v^g), the cooperative game w^g can be interpreted as follows. We want to allocate to each agent in N the total value of the grand coalition, which is the total gross profit of all agents when all the agreements in g are reached, i.e., $w^g_{N,\{N\}} = \sum_{i\in N} \pi_i(g)$. The value of a coalition S given partition Q is the total gross profit of its members when only the subset of

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15 For example, if $N = \{1,2,3\}$, then for N, $PT = \{\{\{1\},\{2\},\{3\}\},\{\{1,2\},\{3\}\},\{\{1\},\{2,3\}\},\{\{1,3\},\{2\}\},\{\{1,2,3\}\}\}$

16 For the example in the previous footnote, $ECL = \{(\{1\},\{\{1\},\{2\},\{3\}\}), (\{2\},\{\{1\},\{2\},\{3\}\}), (\{3\},\{\{1\},\{2\},\{3\}\}), (\{1,2\},\{\{1,2\},\{3\}\}), (\{3\},\{\{1,2\},\{3\}\}), (\{1\},\{\{1\},\{2,3\}\}), (\{2,3\},\{\{1\},\{2,3\}\}), (\{1,3\},\{\{1,3\},\{2\}\}), (\{2\},\{\{1,3\},\{2\}\}), (\{1,2,3\},\{\{1,2,3\}\})\}$.

agreements "divided" by Q, $g|Q$, is reached. Proposition 2 says that the Myerson value of this game is the same as the recursive Nash-in-Nash bargaining solution if it exists.

Since the Myerson value was proposed as a solution to cooperative games, it does not necessarily satisfy the individual rationality condition typically assumed for bargaining solutions. In a numeric example in the Appendix, we show parameter ranges of the primitives where the Myerson value satisfies or does not satisfy the individual rationality condition in the definition of the recursive Nash-in-Nash solution.

3. Examples

The following examples are based on a similar structure: a set of symmetric manufacturers that all negotiate with all members of a set of symmetric retailers. The gross profit of each manufacturer is $-c(x)$, where x is the number of successful negotiations with downstream firms. The gross profit of each retailer is $R(y, z)$, where y is the number of successful negotiations with manufacturers by this firm and z is the number of successful negotiations by other retailers. Assume $c(0) = 0$ and $R(0, z) = 0$ for any z.

Example 1. Suppose there is one manufacturer and two retailers such that $c(1) = c(2) = K$ (i.e., K is like a fixed cost) and $R(1,1) = r(2)$ and $R(1,0) = r(1)$, where $2r(2) > r(1) > K > 0$.

In the Nash-in-Nash framework, retailer 1's and 2's payments to the manufacturer, t_1 and t_2, satisfy the following equal split of surplus conditions for the negotiation between the manufacturer and retailer i:

$$(t_1 + t_2 - K) - (t_j - K) = (r(2) - t_i) - 0, \qquad i, j = 1, 2,$$

where the left-hand side of the equation is the manufacturer's gains from trade with retailer i and the right-hand side is retailer i's gains from trade. Solving the two equations results in each retailer with payoff equal to half of its revenue, $r(2)/2$, and the manufacturer's payoff equal to $r(2) - K$. Note that the retailers' profits and payments do not depend on the fixed cost K. Another characteristic of the standard Nash-in-Nash solution is that the retailers' payoffs do not depend on the degree to which the two retailers are substitutes, which relates to their incremental contribution to total revenue, $2r(2) - r(1)$. The disagreement payoffs in recursive Nash-in-Nash bargaining framework assume renegotiation. Therefore, the manufacturer's disagreement payoff is $(r(1) - K)/2$, and a retailer's disagreement payoff is 0. With these different disagreement payoffs, an equal split of the surplus implies,

$$(t_1 + t_2 - K) - (r(1) - K)/2 = (r(2) - t_i) - 0, \qquad i = 1, 2.$$

Solving these two equations implies the recursive Nash-in-Nash bargaining payoffs are $(4r(2) - r(1) - K)/6$ for each retailer and $(2r(2) + r(1) - 2K)/3$ for the manufacturer. In contrast to the Nash-in-Nash solution, here the retailers' payoffs depend on the fixed cost K. In addition, the recursive Nash-in-Nash retailers' payoffs are decreasing in $r(1)$ holding $r(2)$ constant and thus, decrease in the substitutability of the retailers.

Example 2. Suppose there are three manufacturers and one retailer who needs inputs from three manufacturers to sell anything. Assume $c(x) = 0$ for all x, $R(3,0) = r > 0$ and $R(y, 0) = 0$ if $y < 3$. The standard Nash-in-Nash will predict that no agreement can be reached despite there being positive surplus from agreement as the retailer needs to pay each manufacturer $r/2$, which totals $3r/2$ in payments, larger than the revenue. Therefore, the Nash-in-Nash solution may not be a good extension of the Nash bargaining solution in multilateral situations with large complementarities on one-side of the market.[17] For example, the Copyright Royalty Board judges in the Web IV and STARS III proceedings found that the three major record companies were "must-haves" for interactive and noninteractive streaming services and for satellite radio service Sirius XM.[18]

Example 3. Now assume two manufacturers and two retailers. For the standard Nash-in-Nash solution the manufacturers' payoffs are $R(2,2) - R(1,2) - c(1)$ and the retailers' payoffs are $R(1,2) - [c(2) - c(1)]$. This solution has the unusual property that the manufacturers' payoffs do not depend on $c(2)$ and the retailers' payoffs do not depend on $R(2,2)$. In a model with an investment stage before the bargaining stage where investments would result in lower costs or higher revenue in the following stage, the standard Nash-in-Nash solution implies some perverse investment incentives.

Assuming that revenue and cost functions are consistent with the individual rationality condition,[19] the recursive Nash-in-Nash solution[20] involves manufacturers' payoffs of

$$\big(R(2,2) - R(1,1)\big)/3 + \big(R(2,2) + R(2,0)\big)/6 - \big(c(2) + c(1)\big)/3,$$

and retailers' payoffs of

$$\big(R(2,2) + R(1,1)\big)/3 - \big(R(2,0) - R(2,2)\big)/6 - \big(2c(2) - c(1)\big)/3.$$

These payoffs include $c(2)$ for the manufacturers and $R(2,2)$ for the retailers, thus avoiding the potential issues described above with the standard Nash-in-Nash solution.[21]

Example 4. Now assume one manufacturer and $n \geq 2$ symmetric retailers. Assume $R(y, z) = r(y + z)$ if $y = 1$. In other words, the gross profit for each retailer with an agreement is $r(x)$ if there are x successful negotiations between the manufacturer and the retailers. Assume $xr(x) - c(x) \geq 0$ and is non-decreasing in x. The Nash-in-Nash payoff for the manufacturer is $(n/2) * \big(r(n) - c(n-1)\big) + (n/2 - 1) * c(n)$. When there are three or more retailers, this solution has the unusual property that the manufacturer's payoff is *increasing* in the cost $c(n)$.

As this example satisfies Assumptions 1 and 2, the recursive Nash-in-Nash solution here is the same as the Shapley value. Using the Shaley value formula, the manufacturer's payoff is $\big[\sum_{i=1}^{n}\big(ir(i) - c(i)\big)\big]/(n+1)$, decreasing in the cost $c(n)$.

4. Concluding Remarks

We introduce a new bargaining solution for interdependent bilateral negotiations that account for renegotiation on off-equilibrium paths or contingent contracts. We show using examples that the recursive Nash-in-Nash bargaining solution may give more reasonable predictions than the Nash-in-Nash bargaining solution in some scenarios and that under certain assumptions that solution corresponds to the Shapley Value or under other conditions corresponds to the more general Myerson value.

The monotonicity condition (Assumption 2) is used to ensure that the individual rationality conditions are satisfied given Assumption 1. Slikker (2007) uses a similar condition. This

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17 As mentioned in the Introduction, this point is also discussed in Collard-Wexler *et al.* (2019).

18 *Web IV* Determination, Federal Register (2016); *SDARS III* Determination, Federal Register (2018).

19 This issue is explored in an example in the Appendix.

20 Solving for the recursive Nash-in-Nash solution in this example by hand is computationally challenging, but we provide the Mathematica code to do the derivation for this example. The code is straightforward to adapt for an arbitrary structure and is available here: http://waehrer.net/RNnNdownloads.htm.

21 It is also notable that the retailers' payoffs include the term $\big(R(2,0) - R(2,2)\big)$, which is the negative externality that is imposed on the other downstream firm when it goes from zero to two successful negotiations with manufacturers.

assumption allows some negative externalities, but they cannot be so large that the total profits decrease due to one bargaining pair's agreement. However, in multilateral bargaining situations where additional agreements imply increased competition between participants, monotonicity may fail to hold if consumers (the beneficiaries of the competition) are not players in the bargaining game.

A limitation of the results is the lack of conditions for guaranteeing the existence of the solution in the case with externalities across bargaining groups. However, as demonstrated in Example 2, there are cases where a solution to the standard Nash-in-Nash does not exist but the recursive Nash-in-Nash exists. We explore existence in an example in the Appendix and find that in that specific example, the recursive Nash-in-Nash bargaining solution can exist even when Assumptions 1 and 2 do not hold. In addition, as described in footnote 14 and implemented in the example in the Appendix, it is possible to generalize the recursive Nash-in-Nash solution to incorporate anticipated equilibrium disagreements such that a solution exists when individual rationality does not hold everywhere.

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Appendix

The following notations mostly follow Jackson (2010) and Navarro (2007). A network in this game is a set of bilateral negotiations. We use $g\backslash ij$ to indicate the subnetwork of a network g such that it is identical to g except that $ij \in g$ is removed.

A path in a network g between agents i and j is a sequence of agents $i_1, \dots, i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, \dots, K-1\}$, with $i_1 = i$ and $i_K = j$, and such that each agent in the sequence $i_1, \dots, i_K$ is distinct. i and j are connected in a network g if there is a path between them in g. i is connected in g if $\exists j \neq i \in N$ such that i and j are connected in g. A network is connected if there is a path between any two agents of the network.

A subset $S \subseteq N$ is called a coalition. A coalition S is a component of N in g if: (1) any two agents in S are connected in g, (2) any two agents such that only one of them is in S are not connected.

1. Proof of Proposition 1

To prove Proposition 1, we will first prove a few lemmas.

Lemma 1. *Let $S = \{1, 2, \dots, s\}$ be a set of agents and g be a connected network of S. Let $U \in R^{|s|}$ be the solution to the following set of linear equations:*

$$U_i - U_j = \Delta_{ij}, \qquad \forall ij \in g, i < j \tag{A1}$$

$$\sum_{i \in S} U_i = \alpha, \tag{A2}$$

where $\alpha, \Delta_{ij} \in R$. If U exists then it is unique.

Proof. For any $k, l \in S$, there is a path from k to l since g is connected. Let the path be $k_1, \dots, k_m$ such that $k_1 = k$ and $k_m = l$. Using (A1) repeatedly along the path, we have

$$\begin{aligned} U_k - U_l &= \left(U_{k_1} - U_{k_2}\right) + \left(U_{k_2} - U_{k_3}\right) + \cdots + \left(U_{k_{m-1}} - U_{k_m}\right) \\ &= \Delta_{k_1 k_2} + \Delta_{k_2 k_3} + \cdots + \Delta_{k_{m-1} k_m} \equiv \delta_{kl}, \end{aligned} \tag{A3}$$

where $\Delta_{ij} \equiv -\Delta_{ji}$ for any $ij \in g$ with $i > j$.

There may be other paths from k to l, but if U exists then $U_k - U_l$ are the same regardless of the path used for the above calculation.

If U exists, then it is the solution to the following set of linear equations:

$$\begin{cases} U_m - U_{m+1} = \delta_{m,m+1}, \qquad m = 1, \dots, s-1, & (A4) \\ \sum_{m \in S} U_m = \alpha, & (A5) \end{cases}$$

This is because (A1) can be derived from (A4) for any ij in g and Eq. (A4) can be derived from Eq. (A1) for any $m = 1, \dots, s-1$.

(A4) and (A5) can be written in matrix form as

$$B\,U = \delta, \qquad (A6)$$

where δ is a vector with s components such that $\delta_m = \delta_{m,m+1}$ for $m = 1, \dots, s-1$ and $\delta_m = \alpha$, for $m = s$; $B = \{b_{i,j}\}_{i,j \in S}$ is an s-by-s matrix such that all components are zero except that $b_{m,m} = 1$, $b_{m,m+1} = -1$ for $m = 1, \dots, s-1$ and $b_{s,m} = 1$ for $m = 1, \dots, s$.

Let $C = \{c_{i,j}\}_{i,j \in S}$ be an s-by-s matrix such that $c_{i,j} = (s-j)/s$ for $j \leq s-1$ and $i \leq j$, $c_{i,j} = -j/s$ for $j \leq s-1$ and $i > j$, and $c_{i,j} = 1/s$ for $j = s$ and $i = 1, \dots, s$. Note that $B * C$ equals the identity matrix. Therefore, B is invertible. Hence, (A6) has a unique solution and thus if U exists then it is unique. ■

Lemma 2. $Sh_i(v^{g'}) - Sh_i(v^{g' \backslash ij}) = Sh_j(v^{g'}) - Sh_j(v^{g' \backslash ij}), \forall g' \subseteq g, \forall ij \in g'$.

Proof. $\forall g' \subseteq g, ij \in g'$, define a cooperative game in characteristic function form $(N, \hat{v}^{ij})$ as follows: $\hat{v}^{ij}(S) = v^{g'}(S) - v^{g' \backslash ij}(S), \forall S \subseteq N$. By the linearity of Shapley value, $Sh_i(\hat{v}^{ij}) = Sh_i(v^{g'}) - Sh_i(v^{g' \backslash ij}), \forall i \in N$.

$\forall S \subseteq N$ such that only one of i and j is in S, $g'|_S = (g' \backslash ij)|_S$ and thus $\hat{v}^{ij}(S) = v^{g'}(S) - v^{g' \backslash ij}(S) = \sum_{k \in S} \pi_k(g'|_S) - \sum_{k \in S} \pi_k((g' \backslash ij)|_S) = 0$. By the symmetry property of the Shapley value, $Sh_i(\hat{v}^{ij}) = Sh_j(\hat{v}^{ij})$. Therefore, $Sh_i(v^{g'}) - Sh_i(v^{g' \backslash ij}) = Sh_j(v^{g'}) - Sh_j(v^{g' \backslash ij})$.■

Lemma 3. *Suppose Assumption 1 (no externality across bargaining groups) holds. Then* $\sum_{i \in C} Sh_i(v^{g'}) = \sum_{i \in C} \pi_i(g')$, $\forall g' \subseteq g, \forall C \in Q_{g'}$.

Proof. Take an arbitrary $g' \subseteq g$, $C \in Q_{g'}$, $i \in C$. First, we will show that $Sh_i(v^{g'}) = Sh_i(v^{g'|C})$. Consider i's marginal contribution in game $(N, v^{g'})$. When adding i to a coalition, the gross profits of the agents in other bargaining groups do not change and the gross profits of the agents in i's bargaining group change in the same way as if the game is $(N, v^{g'|C})$ by Assumption 1. Similarly, in game $(N, v^{g'|C})$, when adding i to a coalition, the gross profits of the agents in other bargaining groups do not change by Assumption 1. Therefore, i's marginal contribution to any coalition is the same in $(N, v^{g'})$ and $(N, v^{g'|C})$. Hence, i's Shapley values in these two games are the same.

Second, we will show that $\sum_{i \in C} Sh_i(v^{g'|C}) = \sum_{i \in C} \pi_i(g'|_C)$. In game $(N, v^{g'|C})$, the marginal contribution of an agent j who is not in C to any coalition is simply their gross profit $\pi_j(g'|_C)$ by the definition of $v^{g'|C}$. Therefore, the Shapley value of these agents are simply their gross profits given the set of agreements $g'|_C$. Since the sum of all agents' Shapley value is their total gross profits given agreements $g'|_C$, we have $\sum_{i \in C} Sh_i(v^{g'|C}) = \sum_{i \in C} \pi_i(g'|_C)$.

Lastly, $\sum_{i \in C} \pi_i(g'|_C) = \sum_{i \in C} \pi_i(g')$ by Assumption 1, and we have shown that $\sum_{i \in C} Sh_i(v^{g}) = \sum_{i \in C} Sh_i(v^{g|C})$. Therefore, $\sum_{i \in C} Sh_i(v^{g'}) = \sum_{i \in C} \pi_i(g')$.■

Lemma 4. Suppose Assumptions 1 (no externality across bargaining groups) and 2 (monotonicity) hold. Then $Sh_i(v^{g'}) \geq Sh_i(v^{g'\setminus ij})$ and $Sh_j(v^{g'}) \geq Sh_j(v^{g'\setminus ij})$, $\forall g' \subseteq g, \forall ij \in g'$.

Proof. Take an arbitrary $g' \subseteq g$ and $ij \in g'$. Let C be the bargaining group that ij is in given g'. Firstly, note that a coalition's total gross profit is the same in games $(N, v^{g'})$ and $(N, v^{g'\setminus ij})$ if the coalition does not include both i and j. This is because the only difference between the two games is whether agreement ij is possible and a coalition without both ij does not allow ij to reach an agreement.

Secondly, the total gross profit of a coalition S with both i and j in $(N, v^{g'})$ is weakly larger than in $(N, v^{g'\setminus ij})$. This is because the total gross profit of all agents in N is weakly larger given agreements $g'|_S$ than given agreements $g'\setminus ij|_S$ by Assumption 2 and the total gross profit for agents not in coalition S is the same given agreements $g'|_S$ and $(g'\setminus ij)|_S$ by Assumption 1.

Given these two observations, i's marginal contribution to any coalition is weakly larger in $(N, v^{g'})$ than in $(N, v^{g'\setminus ij})$. Therefore, its Shapley value is also weakly larger in $(N, v^{g'})$ than in $(N, v^{g'\setminus ij})$. The same arguments can be used to show $Sh_j(v^{g'}) \geq Sh_j(v^{g'\setminus ij})$. ■

Proof of Proposition 1. Proof by induction.

First, we will show that $\forall g_1 \subseteq g$ with $|g_1| = 1$, the recursive Nash-in-Nash bargaining solution given N, g_1 and π is the same as the Shapley value for the cooperative game (N, v^{g_1}).

Let the negotiating pair in g_1 be i and j. Solving the two equations defined by the component balance and fairness conditions, we get $U_i^{g_1} = \pi_i(\emptyset) + \left(\pi_i(g_1) + \pi_j(g_1) - \pi_i(\emptyset) - \pi_j(\emptyset)\right)/2$, $U_j^{g_1} = \pi_j(\emptyset) + \left(\pi_i(g_1) + \pi_j(g_1) - \pi_i(\emptyset) - \pi_j(\emptyset)\right)/2$, and $U_k^{g_1} = \pi_k(g_1), \forall k \neq i, j$. Moreover,

$$\begin{aligned}
&\left(U_i^{g_1} + U_j^{g_1}\right) - \left(\pi_i(\emptyset) + \pi_j(\emptyset)\right) \\
&= \pi_i(g_1) + \pi_j(g_1) - \pi_i(\emptyset) - \pi_j(\emptyset) \\
&= \left(\pi_i(g_1) + \pi_j(g_1) + \sum_{k \neq i,j} \pi_k(g_1)\right) - \left(\pi_i(\emptyset) + \pi_j(\emptyset) + \sum_{k \neq i,j} \pi_k(g_1\setminus ij)\right),
\end{aligned}$$

where the last equality uses Assumption 1. Therefore, $\left(U_i^{g_1} + U_j^{g_1}\right) - \left(\pi_i(\emptyset) + \pi_j(\emptyset)\right) \geq 0$ by Assumption 2. Hence, U^{g_1} is the recursive Nash-in-Nash bargaining solution given N, g_1 and π. Solving the Shapley value for the cooperative game (N, v^{g_1}) yields that the Shapley value is the same as U^{g_1}.

Now suppose we have shown that $\forall g_m \subseteq g$ with $|g_m| = m$, the recursive Nash-in-Nash bargaining solution given N, g_m and π is the same as the Shapley value for the cooperative game (N, v^{g_m}). By Lemmas 2-4, the Shapley value for the cooperative game $(N, v^{g_{m+1}})$, $\forall g_{m+1} \subseteq g$ with $|g_{m+1}| = m + 1$ satisfies component balance, fairness and individual rationality. By Lemma 1, the recursive Nash-in-Nash bargaining solution is unique if it exists. Therefore, the Shapley value for game (N, v^g) is the same as the recursive Nash-in-Nash bargaining solution given N, g, and π.■

2. Proof of Proposition 2

To prove Proposition 2, we will first prove Lemmas 5 and 6.

Lemma 5. $\Phi_i(w^{g'}) - \Phi_i\left(w^{g'\setminus ij}\right) = \Phi_j(w^{g'}) - \Phi_j\left(w^{g'\setminus ij}\right)$, $\forall g' \subseteq g, \forall ij \in g'$.

Proof. Similar to the proof of Lemma 2, $\forall g' \subseteq g, \forall ij \in g'$, define a cooperative game in partition function form w^{ij} as follows: $\widehat{w}^{ij}_{S,Q} = w^{g'}_{S,Q} - w^{g'\setminus ij}_{S,Q}, \forall (S,Q) \in ECL$. By the linearity of the Myerson value, $\Phi_k(\widehat{w}^{ij}) = \Phi_k(w^{g'}) - \Phi_k(w^{g'\setminus ij}), \forall k \in N$.

$\forall (S,Q) \in ECL$ such that i and j are in different elements of Q, $g'|Q = (g'\setminus ij)|Q$, and thus $\widehat{w}^{ij}_{S,Q} = w^{g'}_{S,Q} - w^{g'\setminus ij}_{S,Q} = \sum_{k\in S}\pi_i(g'|Q) - \sum_{k\in S}\pi_i((g'\setminus ij)|Q) = 0$. By the symmetry property of the Myerson value (implied by Value Axiom 1 of Myerson (1977b)), $\Phi_i(\widehat{w}^{ij}) = \Phi_j(\widehat{w}^{ij})$. Therefore, $\Phi_i(w^{g'}) - \Phi_i(w^{g'\setminus ij}) = \Phi_j(w^{g'}) - \Phi_j(w^{g'\setminus ij})$. ■

Lemma 6. *For any $g' \subseteq g$ and for any $C \in Q_{g'}$, $\sum_{i\in C}\Phi_i(w^{g'}) = \sum_{i\in C}\pi_i(g')$.*

Proof. Recall the definition of decomposability in Myerson (1977b): Given $Q \in PT$ and $w \in R^{|ECL|}$, we say that w is Q-decomposable if and only if:

$$\forall(\tilde{S},\tilde{Q}) \in ECL, w_{\tilde{S},\tilde{Q}} = \sum_{S\in Q} w_{\tilde{S}\cap S,\tilde{Q}\cap Q},$$

where for any $Q \in PT$ and $\tilde{Q} \in PT$, $\tilde{Q} \cap Q \in PT$ is defined as $\tilde{Q} \cap Q = \{\tilde{S} \cap S | \tilde{S} \in \tilde{Q}, S \in Q, \tilde{S} \cap S \neq \emptyset\}$. Also recall Corollary 1 of Myerson (1977b): If $w \in R^{ECL}$ is Q-decomposable, then, for any $S \in Q$, $\sum_{n\in S}\Phi_n(w) = w_{S,Q}$.

Note that $Q_{g'}$ is a partition of N and $g'|Q = g'|Q \cap Q_{g'}, \forall Q \in PT$. $w^{g'}$ is $Q_{g'}$-decomposable because for any $(S,Q) \in ECL$,

$$w^{g'}_{S,Q} = \sum_{i\in S}\pi_i(g'|Q) = \sum_{i\in S}\pi_i(g'|Q \cap Q_{g'}) = \sum_{C\in Q_{g'}}\sum_{i\in S\cap C}\pi_i(g'|Q \cap Q_{g'}) = \sum_{C\in Q_{g'}} w^{g'}_{S\cap C,Q\cap Q_{g'}}.$$

By Corollary 1 of Myerson (1977b), for any $C \in Q_{g'}$, $\sum_{i\in C}\Phi_i(w^{g'}) = w^{g'}_{C,Q_{g'}}$. Since $w^{g'}_{C,Q_{g'}} = \sum_{i=c}\pi_i(g'|Q_{g'}) = \sum_{i=c}\pi_i(g')$, we have $\sum_{i\in C}\Phi_i(w^{g'}) = \sum_{i\in C}\pi_i(g')$. ■

Proof of Proposition 2. This proof is similar to the proof of Proposition 1. By Lemma 1, the recursive Nash-in-Nash bargaining solution is unique if it exists, and it is given recursively by the component balance and fairness conditions. By Lemmas 5 and 6, the Myerson values of the corresponding cooperative games satisfy the component balance and fairness conditions. Therefore, the recursive Nash-in-Nash bargaining solution is the same as the Myerson value if it exists. ■

3. Example 3 when Assumptions 1 and 2 are violated

In order to explore the issue of when the individual rationality condition holds and the implications of it failing to hold, consider a variation of Example 3. Alter the assumptions of Example 3 to add the following structure to the revenue and cost functions. Assume $c(x) = 0$ for all x and $R(y,z) = \max\{3y - \alpha z, 0\}$, where $3 > \alpha > 0$.

This structure is inconsistent with Assumption 1 (No externalities across bargaining groups). Note that $\{(1,3),(2,4)\}$ includes two bargaining groups and the payoff for $(1,3)$ depends on whether there is agreement by $(2,4)$. For $\alpha > 1$, this structure is also inconsistent with Assumption 2 (Monotonicity) as $R(2,0) > R(2,1) + R(1,2)$.

However, for $9/5 \geq \alpha > 1$, the component balance and fairness conditions solve for payoffs at each recursive stage that satisfy individual rationality and thus the recursive Nash-in-Nash solution exists and is the same as the Myerson Value even though Assumptions 1 and 2 are not satisfied. For these parameter values, the payoffs to the manufacturers is $3 - 2/3\alpha$ and the retailers is $3 - 4/3\alpha$.[22]

For $\alpha > 9/5$, not all individual rationality conditions hold. Labeling the manufacturers 1 and 2 and the retailers 3 and 4, the set of bargaining pairs is $g = \{(1,3), (1,4), (2,3), (2,4)\}$. For $9/5 < \alpha \leq 9/4$, the individual rationality condition fails for $g' \subseteq g$ such that $|g'| = 3$.[23] For example, for $g' = \{(1,3), (1,4), (2,3)\}$, individually rationality fails for bargaining pair (1,4) because the negative externality associated with inclusion of retailer 4 and having to compensate 4 to participate can be avoided through disagreement between 1 and 4. Thus, for these parameter values the recursive Nash-in-Nash as defined in the main text does not exist.

However, as suggested in footnote 14, it is possible to incorporate the anticipation of disagreement into the recursive Nash-in-Nash solution. One such assumption is to assume that if individual rationality fails for bargaining pairs $f \subseteq g'$, then the payoffs are defined such that $U_i^{g'} = U_i^{g' \setminus f}$. Implementing this approach for $9/5 < \alpha \leq 9/4$, results in payoffs of $(15 - 4\alpha)/4$ for manufacturers and $(9 - 4\alpha)/4$ for retailers. Notice that at $\alpha = 9/4$, the retailers' payoff is zero and thus, with an additional assumption regarding anticipated disagreements a solution for the recursive Nash-in-Nash can be recovered.

Now consider the standard Nash-in-Nash solution for this example. Example 3 shows the solution as $R(2,2) - R(1,2) - c(1)$ for manufacturers and $R(1,2) - [c(2) - c(1)]$ for retailers. Applying the structure on gross profits assumed here, these payoffs are 3 for manufacturers and $\max\{3 - 2\alpha, 0\}$ for retailers. Thus, for the standard Nash-in-Nash, the retailer's payoff is zero when $3 > \alpha \geq 3/2$.

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22 These solutions were derived and individual rationality conditions checked using the Mathematica code available here: http://waehrer.net/RNnNdownloads.htm.

23 For $\alpha > 9/4$, the individual rationality condition fails at additional recursive stages.