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Inception-expansion-bursting bubbles in the BRICS-dollar exchange rates

Geraldo E. Silva jr.

*Department of Economics, Federal University of São
Carlos*

Sidney M. Caetano

*Department of Economics, Federal University of Juiz de
Fora*

Abstract

The new economic and geopolitical landscape of the BRICS bloc has stirred the interest of politicians and policymakers in exploring potential changes in exchange rate policy, including the prospect of a new currency. Assuming the exchange rate as a fundamentals-based stock price, we employed the SADF and GSADF tests to verify nominal exchange rate explosivity, and demonstrate how explosiveness can be detected in the ratio of the exchange rate relative to the random walk fundamentals and Purchasing Power Parity (PPP) fundamentals. The results revealed positive and enduring positive bubbles for Brazil, minor bubbles for South Africa, and prolonged negative bubbles for China across various exchange rate policy scenarios. We conclude that the diverse array of scenarios presented numerous options for bubble currencies, and the possibility of a new BRICS exchange rate common policy or a new currency could accommodate these variations.

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Contact: Geraldo E. Silva jr. - gedmundos@ufscar.br, Sidney M. Caetano - sidney.caetano@ufjf.br.

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1 Introduction

The ascent of economic power and geopolitical influence among the BRICS nations commenced in 2006 and was formalized in 2010 with the admission of South Africa as a member. Coined initially by Goldman Sachs, the term "BRICS" represents the initials of Brazil, Russia, India, China, and South Africa. The objectives of this organization include institutional development, akin to the International Monetary Fund (IMF) and World Bank, facilitated through the establishment of a New Development Bank. Furthermore, plans for expansion in 2024 involve the inclusion of new members such as Saudi Arabia, Egypt, the United Arab Emirates, Ethiopia, and Iran. The primary focus is on creating a new financial framework, encompassing a Contingent Reserve Agreement, a BRICS payment system, and the potential adoption of a common currency.

Saji (2019) explored the prospects of forming a monetary union among BRICS countries. The results, using the Markov-Switching Model, showed a divergence in real exchange rate behavior among BRICS members when comparing pre-BRICS and post-BRICS periods. Such results demand possible coordination and interaction between members to enable the creation of a currency union. The proposal should enhance the advantages and capabilities of the members. Zharikov (2023) suggested the use of a digital currency, analogous to the experiences of China and Russia from 2020. This hypothesis posits that a digital currency would share financial risk through a risk-sharing mechanism. This currency would be used as an asset in portfolio investment, superior to a cryptocurrency. Finally, Ullah et al. (2024) documented the economic and financial differences among BRICS members, highlighting significant gaps and asymmetries. These gaps and asymmetries in exchange rates, fiscal policies, and monetary policies would pose a significant financial challenge to the adoption of a common currency. Thus, this aim could be an idyllic dream on one hand and a concrete reality on the other, if possible coordination is adopted in the manner of the European currency.

Achieving these goals necessitates the synchronization or coordination of domestic exchange rate policies, mirroring the process that led to the creation of the Euro currency in Europe. Drawing *insights* from recent episodes can shed light on the feasibility of synchronizing or coordinating exchange rate policies within this potential new arrangement. Mainly, it is a group of countries with a history of very different exchange rate regimes with possible bubbles. Gupta et al. (2023) use the multi-scale Log-Periodic Power Law Singularity (LPPLS) confidence indicator approach to detect both positive and negative bubbles¹ at short-, medium- and long-run for the stock markets of the BRICS countries. Maldonado et al. (2018) and Maldonado et al. (2019) employed a regime-switching equation, confirming the existence of rational bubbles in the dynamics of the exchange rate across BRICS countries.² Numerous economic episodes have been characterized by what is commonly referred to as the "bubbles" phenomenon. Bubbles are regularly associated with remarkable increases in asset prices, reaching a point of collapse when market values surpass the asset's fundamentals by a considerable margin (Brunnermeier, 2016).³ Issues related to exchange rate policy have expanded both theoretical and empirical literature, gaining prominence on the political agenda. Notable events include the collapse of the *Bretton Woods* System in the early 1970s,

¹Positive (negative) bubbles occur when the average price during the explosive period is greater (less) than the initial price at the start of the explosive period, see Etienne et al. (2014).

²However, the dynamics of the exchange rate series for each country are subject to controversy. The researchers considered cointegration as evidence of the transmission of shocks among BRICS members.

³The rational expectations revolution, coupled with contemporary discussions in the literature, has significantly advanced the understanding of these phenomena.

the gas bubbles resulting from the Oil Shock studied by Hamilton (2009) and Caspi et al. (2015), and the subprime mortgage market crisis in the U.S. that had global repercussions, as explored by Phillips and Yu (2011), Phillips et al. (2015), and Phillips and Shi (2018). The literature has identified the Great Recession as a crucial turning point.⁴ Obstfeld and Rogoff (1996) views the exchange rate as a stock price based on fundamentals, linking the present to future times. We focused on two main formulations to study rational exchange rate bubbles, the *random walk* formulation and the Purchasing Power Parity (PPP) decomposed into tradable goods and nontradable ones. Then, we attempt to contribute to the empirical literature that examines and analyzes exchange rate bubbles. Our empirical arrangement aims to study the inception, expansion or duration, and bursting of bubbles in BRICS-dollar exchange rate, utilizing robust tests to present data-stamping for the identification of single and multiple bubbles. We use these tools for testing and date-stamping periods of mildly explosive dynamics (exuberance). Our findings indicate the presence of both positive and negative bubbles for Brazil, China, and South Africa. Notably, we observed a Brazilian positive bubble lasting more than thirty-one years, a negative bubble persisting for nine years in China, and smaller bubbles in the case of South Africa.

The remainder of the paper is organized as follows. Section 2 contains the model, empirical strategy, and dataset. The empirical results and analysis are presented in Section 3. The conclusion is provided in Section 4.

2 Methodology

2.1 Present value model of exchange rate and empirical strategy

Following Obstfeld and Rogoff (1996), where “the nominal exchange rate must be viewed as an asset price”⁵, we propose to use the same procedure as Bettendorf and Chen (2013) that assume the present value model of exchange rate⁶ (ER_t):

$$ER_t = (1 - \gamma) \sum_{j=0}^k \gamma^j E_t[f_{t+j}] + \gamma^{k+1} E_t[ER_{t+k+1}], \quad (1)$$

in which γ represents discount factor and f_t is the value of foundations in the period t . Taking transversality conditions $\lim_{k \rightarrow \infty} \gamma^k E_t[ER_{t+k+1}] = 0$, in the long run the exchange rate will only depend on future expected fundamentals. If the transversality condition fails, the nominal exchange rate can present an explosive rational bubble. Let us suppose that bubbles follow the autoregressive process $AR(1)$:

$$b_t = \frac{1}{\gamma} b_{t-1} + \varepsilon_t, \quad (2)$$

⁴In the pre-Great Recession period, seminal contributions from Mehra and Prescott (1985) and Diba and Grossman (1987) on rational bubbles sparked extensive research efforts, focusing on establishing robust tests and theoretical foundations. Various essential variables such as exchange rates, stock prices, hyperinflation, *commodity* prices, and rental prices became subjects of empirical studies. In the post-Great Recession period, the use of robust tests such as ADF (Augmented Dickey-Fuller test), RTADF (Right Tail ADF test), SADF (Supreme to the ADF test), and GSADF (General SADF) elevated the study of bubble phenomena, encompassing recent aspects related to prices, especially cryptocurrencies, as evidenced by Al-Anaswah and Wilfling (2011), Phillips et al. (2011), and Phillips et al. (2015).

⁵This view implies that it is determined by current and expected values of fundamentals.

⁶The model is in line with Engel and West (2005) and León-Ledesma and Mihailov (2013).

where $\frac{1}{\gamma} > 1$ represents an explosive process; and, $\varepsilon_t \sim NID(0, \sigma^2)$ random errors. As equations (1) and (2), we can decompose the exchange rate into two components: the fundamentals (F_t^f) added to rational bubbles (b_t),

$$ER_t = F_t^f + b_t \Rightarrow (ER_t - F_t^f) = b_t, \quad (3)$$

in with F_t^f denotes the discounted sum of all future economic fundamentals, it is linearly dependent on the economic fundamental f_t that is a $I(1)$ process.⁷ We need to propose a model for the fundamentals (F_t^f), so we suggest two traditional models: 1) *random walk*, which assumes that the variation in the exchange rate is identically equal to zero - that is, the exchange rate is expected to remain, on average, unchanged; and 2) Purchasing Power Parity - PPP, where nominal exchange rate in logarithm can be expressed as the difference between domestic and foreign prices. Formally,

$$F_{1t}^f = ER_{t-1}, \quad (4)$$

$$F_{2t}^f = p_t - p_t^*. \quad (5)$$

Based on Engel (1999), Bettendorf and Chen (2013) show that the price differential (F_{2t}^f) can be decomposed into two components, the traded goods component (F_{2t}^{fT}), and the nontraded goods component (F_{2t}^{fN}):

$$\underbrace{(p_t - p_t^*)}_{F_{2t}^f} = \underbrace{(p_t^T - p_t^{T*})}_{F_{2t}^{fT}} + \underbrace{\alpha(p_t^N - p_t^T) - \beta(p_t^{N*} - p_t^{T*})}_{F_{2t}^{fN}}, \quad (6)$$

where α (β) is the share of the domestic (foreign country) nontraded goods component. We use Producer Price Index (PPI) to represent tradable goods price level⁸:

$$F_{2t}^{fT} = \ln(PPI_t) - \ln(PPI_t^*). \quad (7)$$

The relative nontraded goods component is constructed from the aggregate consumer price indexes (CPI) relative to aggregate PPI⁹:

$$F_{2t}^{fN} = \ln(CPI_t) - \ln(PPI_t) - [\ln(CPI_t^*) - \ln(PPI_t^*)]. \quad (8)$$

The focal point is to verify nominal exchange rate explosivity ER_t , and demonstrate how explosiveness can be detected in the ratio of the exchange rate relative to the three types of economic fundamentals.

⁷See Bettendorf and Chen (2013) and Engel and West (2005).

⁸Although, using PPI fitted to local production, excludes marketing and other consumption services to those goods.

⁹Note that no assumption is made about α and β . Algebraically, is easily checked $f_t^N = \alpha(p_t^N - p_t^T) - \beta(p_t^{N*} - p_t^{T*}) = (p_t - p_t^T) - (p_t^* - p_t^{T*})$.

2.2 Statistical tests: ADF, SADF, and GSADF

At the heart of all the methodologies considered lies the following ADF regression equation¹⁰:

$$\Delta y_t = \mu_{r_1, r_2} + \gamma_{r_1, r_2} y_{t-1} + \sum_{j=1}^J \phi_{r_1, r_2}^j \Delta y_{t-j} + \varepsilon_t, \quad (9)$$

where y_t denotes a generic time series, Δy_{t-j} with $j = 1, \dots, k$ are lagged first differences of the series included to accommodate serial correlation, $\varepsilon_t \sim N(0, \sigma_{r_1, r_2}^2)$ are the Gaussian residuals, and μ_{r_1, r_2} , γ_{r_1, r_2} and ϕ_{r_1, r_2}^j with $j = 1, \dots, k$ are regression coefficients. The subscripts r_1 and r_2 denote fractions of the total sample size T that specify the starting and ending points of a subsample period. The null hypothesis of interest is that of a unit root, $H_0 : \gamma_{r_1, r_2} = 0$, against the alternative of explosive behavior in y_t , $H_1 : \gamma_{r_1, r_2} > 0$. The ADF test statistic corresponding to this null is given by

$$ADF_{r_1}^{r_2} = \frac{\hat{\gamma}_{r_1, r_2}}{SE(\hat{\gamma}_{r_1, r_2})}. \quad (10)$$

The statistic in equation (10) is obtained by estimating the regression given in equation (9) on the full sample of observations, i.e., by setting $r_1 = 0$ and $r_2 = 1$. Testing for exuberance entails comparing the ADF_0^1 statistic with the right-tailed critical value (CV) from its limit distribution.¹¹ Phillips et al. (2011) developed the supremum ADF (SADF) test that is consistent with a single boom-bust episode. It involves recursively estimating equation (9) using a forward expanding sample. In this setting, the beginning of the subsample is held constant at $r_1 = 0$, while the end of the subsample, r_2 , increases from r_0 (the minimum window size) to one (the entire sample period). Recursive estimation of equation (9) yields a sequence of $ADF_{r_0}^{r_2}$ statistics. The supremum of this sequence is defined by

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}. \quad (11)$$

Similarly to the standard ADF test, rejection of the null of a unit root requires that the SADF statistic exceeds the right-tailed critical value from its limit distribution.¹² Phillips et al. (2015) proposed an extension of the SADF test (GSADF) which has the same alternative hypothesis as the SADF but which covers a larger number of subsamples. Specifically, given a minimum window size r_0 , the methodology involves estimating the regression given in Equation (9) for all possible subsamples by allowing both the ending point, r_2 , and the starting point, r_1 , to change. This extra flexibility on the estimation window results in substantial power gains and makes the GSADF test better suited to detect the presence of multiple regime changes. Formally,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \quad (12)$$

Again, rejection of the unit root hypothesis in favor of the alternative hypothesis of explosive behav-

¹⁰In this paper we used the R package *exuber* for testing and date-stamping periods of mildly explosive dynamics (exuberance) in time series, as explained in Vasilopoulos et al. (2022). The package computes test statistics for the supremum augmented Dickey-Fuller test (SADF), the generalized SADF (GSADF), and the panel GSADF. In addition to the traditional ADF, of course.

¹¹When $ADF_0^1 > CV$, H_0 is rejected in favor of H_1 .

¹²However, the alternative hypothesis of the SADF test is that of explosive dynamics in some part(s) of the sample.

ior in some part(s) of the sample requires that the GSADF statistic exceeds the right-tailed critical value from its limit distribution.

2.3 Dataset

Our study focuses on the bilateral exchange rates between the United States and BRICS countries. We work with monthly data because a higher frequency of price data is not available. Note in Table I that the samples vary in periods and, therefore, in the number of observations. The dataset used refers to the exchange rate, consumer price index, and producer price index of each country.¹³ All times series are transformed into logarithm. In the following section, we demonstrate how explosiveness can be detected in the log nominal exchange rates, i.e, $S_t^i = ER_t^i$, where $i =$ Brazil (BR/US), South Africa (SA/US), China (CH/US), India (IN/US), and Russia (RU/US) as well as in the ratio of the exchange rate relative to the three types of economic fundamentals:

$$S_t^{iRW} = ER_t^i - F_{1t}^{if}, \quad (13)$$

$$S_t^{ifT} = ER_t^i - F_{2t}^{ifT}, \quad (14)$$

$$S_t^{ifN} = ER_t^i - F_{2t}^{ifN}. \quad (15)$$

The issue with rational bubbles is primarily presented in the context of nominal exchange rates. A discussion on this involves studying its fundamentals. First of all, Obstfeld and Rogoff (1996) considered the exchange rate as an asset price that reflects current and expected values of its fundamentals. Subsequently, Bettendorf and Chen (2013), Jiang et al. (2015), and Hu and Oxley (2017) also used nominal exchange rates in their studies. Bettendorf and Chen (2013) proposed studying exchange rate fundamentals by analyzing the relative prices of tradable and nontradable goods. According to Jiang et al. (2015), the occurrence of a rational bubble in nominal exchange rates indicates that no long-run relationship exists between the nominal exchange rate and prices. Similar to Bettendorf and Chen (2013), they suggest that a decomposition between tradable and nontradable goods serves as a guide to studying fundamentals. Finally, Hu and Oxley (2017) extended this idea to G10 and emerging market economies.

3 Empirical Results and Analysis

We applied the statistics tests to all time series of nominal exchange rate transformed into logarithm (S_t^i), see results in the Table I. We use the rules-of-thumb proposed by Phillips et al. (2015) to compute the minimum window size and the minimum duration of an episode of exuberance.

We set the lag order to zero for all time series.¹⁴ The standard ADF test fails to reject the null at the 1% significance level, which is in line with the low power of the ADF test to detect bubbles that burst in-sample.¹⁵ As Evans (1991), although widely employed, standard unit root tests have extremely low power in detecting episodes of explosive dynamics when these are interrupted by market crashes. Vasilopoulos et al. (2022) also report that the standard ADF test is not consistent

¹³The data were obtained from the BACEN (<https://www.bcb.gov.br/>), the IPEADATA (<http://www.ipeadata.gov.br/Default.aspx>), and the Fed St. Louis (<https://fred.stlouisfed.org/>).

¹⁴According to Vasilopoulos et al. (2022), with respect to the selection of lag, simulation evidence indicates the proposed right-tailed unit root methodologies work well when the number of lags is fixed at a small value, i.e., 0 or 1. On the contrary, lag selection based on information criteria can result in severe size distortion.

¹⁵Alternatively, we can see results which reports whether the null of a unit root is rejected and, if so, at which level of significance. For reasons of space, we omitted the table that can be requested by email.

Table I: Tests for explosive behavior in the exchange rate

Brazil	Sample: 1960:01-2023:08 (MWS = 57 and MD = 7)						
	$S_t^{BR/US}$	$S_t^{BR/US-RW}$	$S_t^{BR/US-fT}$	$S_t^{BR/US-fN}$	CV 10%	CV 5%	CV 1%
ADF	-2.54	-12.9	-2.41	-2.37	-0.450	-0.0814	0.564
SADF	22.0	-7.85	-1.69	20.9	1.24	1.51	2.03
GSADF	25.6	-0.746	2.48	23.3	2.06	2.29	2.76
China	Sample: 1999:01-2022:12 (MWS = 33 and MD = 6)						
	$S_t^{CH/US}$	$S_t^{CH/US-RW}$	$S_t^{CH/US-fT}$	$S_t^{CH/US-fN}$	CV 10%	CV 5%	CV 1%
ADF	-1.33	-9.39	0.326	-0.0933	-0.420	-0.0662	0.579
SADF	21.1	12.6	8.37	9.38	1.13	1.41	1.95
GSADF	21.1	12.6	8.40	9.42	1.87	2.13	2.69
India	Sample: 2013:08-2023:07 (MWS = 20 and MD = 5)						
	$S_t^{IN/US}$	$S_t^{IN/US-RW}$	$S_t^{IN/US-fT}$	$S_t^{IN/US-fN}$	CV 10%	CV 5%	CV 1%
ADF	-0.0296	-10.7	-0.0414	-1.53	-0.439	-0.0835	0.621
SADF	0.694	-5.68	1.36	-0.822	1.02	1.34	1.90
GSADF	2.02	-1.42	2.19	1.38	1.71	1.99	2.53
Russia	Sample: 2015:01-2022:01 (MWS = 17 and MD = 4)						
	$S_t^{RU/US}$	$S_t^{RU/US-RW}$	$S_t^{RU/US-fT}$	$S_t^{RU/US-fN}$	CV 10%	CV 5%	CV 1%
ADF	-1.61	-1301.	-2.71	-1.30	-0.410	-0.0636	0.611
SADF	-1.23	-704.	-1.51	-0.901	0.958	1.30	1.87
GSADF	0.638	-0.298	0.826	1.50	1.63	1.94	2.54
South Africa	Sample: 1994:01-2022:12 (MWS = 37 and MD = 6)						
	$S_t^{SA/US}$	$S_t^{SA/US-RW}$	$S_t^{SA/US-fT}$	$S_t^{SA/US-fN}$	CV 10%	CV 5%	CV 1%
ADF	-1.21	-13.7	-2.47	-0.947	-0.441	-0.0871	0.644
SADF	2.45	-1.40	2.26	2.59	1.16	1.43	1.97
GSADF	3.81	-0.790	2.35	4.31	1.91	2.16	2.74

Note: MWS = minimum window size; MD = Minimum duration of an explosive period; CV = Critical

Values are obtained from Monte-Carlo simulations with 10000 replications.

with regime changes, so it exhibits extremely low power in the presence of boom-bust episodes. Initially, we generate finite-sample critical values based on Monte Carlo method with 10000 replications, see Table I. A comparison of the SADF and GSADF test statistics to their critical values suggests that the null hypothesis of a unit root against the alternative of explosive behavior is rejected at all conventional levels of significance for Brazil, South Africa and China. However, the SADF test statistics for India and Russia does not reject the null hypothesis at all conventional levels of significance. According to the GSADF test statistics, we can reject the null hypothesis at the 5% significance level for India, but we can not reject H0 of a unit root against the alternative of explosive behavior for Russia. To be more robust, we use the univariate wild bootstrap method. This approach involves applying a wild bootstrap re-sampling scheme to construct the bootstrap analogue of the Phillips et al. (2015) test which is asymptotically robust to non-stationary volatility, see Table IV. A comparison of the SADF and GSADF test statistics to their critical values suggests that the null hypothesis of a unit root against the alternative of explosive behavior is only rejected at conventional levels of 1% for Brazil and China. Having established the statistical sig-

nificance of the SADF and GSADF test statistics at the 5% level, we can turn to the identification of the episode(s) of exuberance.

To visualize these episodes, see Figures 1, 2, and 3 which returns a plot of the estimated GSADF statistics together with the sequence of 95% critical values. Shaded areas in the plot indicate periods during which the GSADF statistic exceeds its critical value and, thus, we see that there is evidence of exuberance using the rule-of-thumb proposed by Phillips et al. (2015) to compute the minimum duration of an episode of exuberance. Table II and III show the ‘exact’ dates of the origination and collapse of the bubble and the bubble duration based on Monte Carlo and bootstrap methods, respectively. The critical values by wild bootstrap, which is robust to non-stationary volatility, show a greater number of bubbles for Brazil, China and South Africa. A visual inspection of the graphs using Monte Carlo method concerning to BRICS countries reveals that once the bubble starts, the GSADF test statistics gradually increase and eventually exceed their critical values, rejecting the null hypothesis of no explosiveness. When the bubble bursts, both statistics fall below the critical bound almost instantaneously and remain there until the end of the sample. For Brazil, Figure 1, the results indicate that the bubble started on 1964-10-01 under the GSADF procedure and continued until 1965-05-01. During the *Bretton Woods* era, results pointed out a small bubble with seven periods. The justification is that the organism enabled an adjustment due to the balance of payments problems. Also, the longest bubble occurred from 1975 to 2007, covering a period longer than thirty-one years for exchange rate levels and non-tradable formulation.

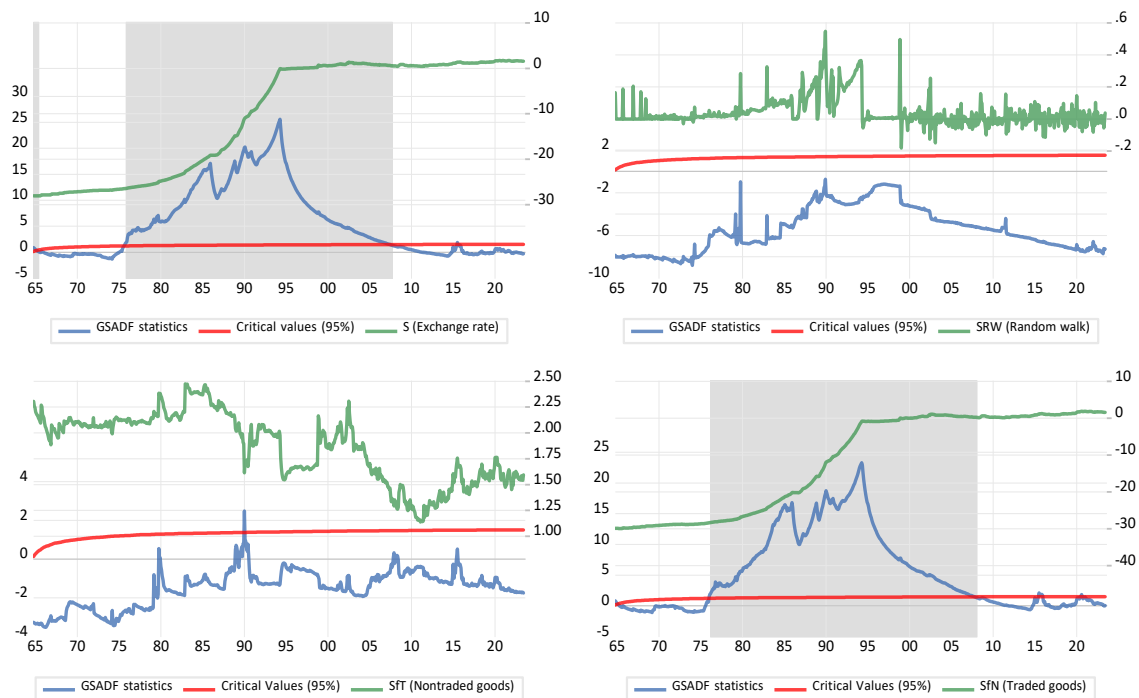


Figure 1: Brazil

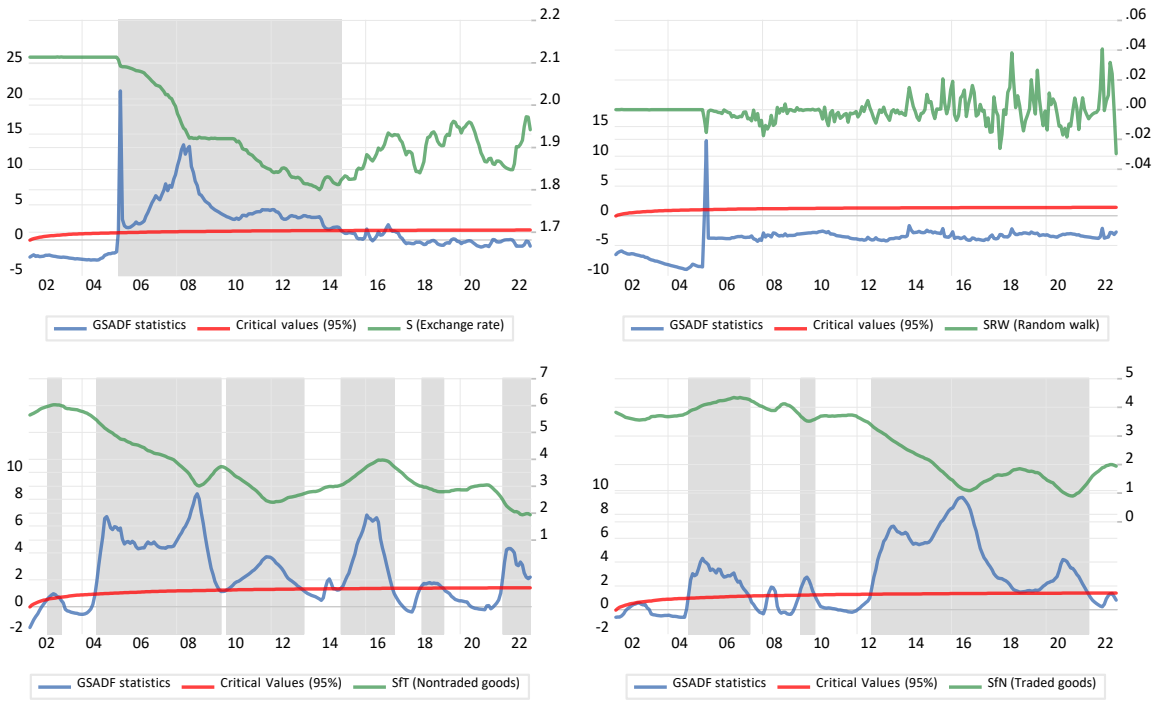


Figure 2: China

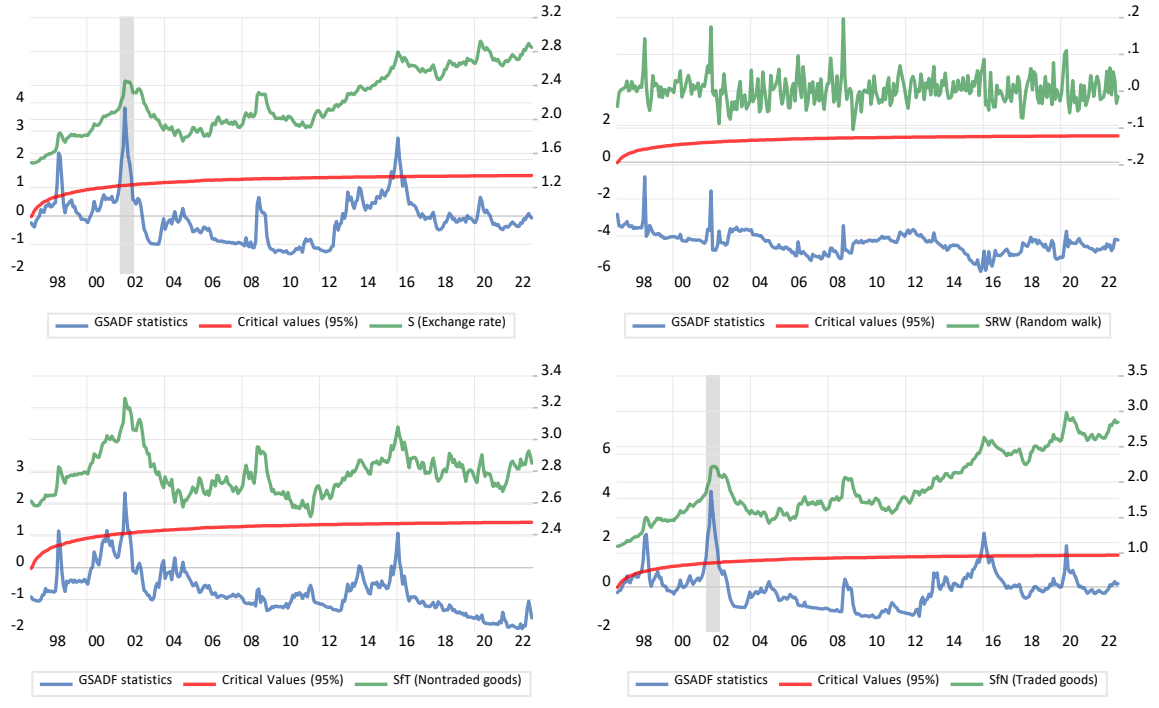


Figure 3: South Africa

Table II: Dates of the origination and collapse of the bubble and the bubble duration

Brazil (MD = 7)					
	Start	Peak	End	Duration	Signal
$S_t^{BR/US}$	1964-10-01	1964-12-01	1965-05-01	7	positive
	1975-11-01	1994-06-01	2007-09-01	382	positive
	Start	Peak	End	Duration	Signal
$S_t^{BR/US-fN}$	1976-03-01	1994-06-01	2008-01-01	382	positive
China (MD = 6)					
	Start	Peak	End	Duration	Signal
$S_t^{CH/US}$	2005-07-01	2005-08-01	2014-12-01	113	negative
	Start	Peak	End	Duration	Signal
$S_t^{CH/US-fT}$	2002-07-01	2002-10-01	2003-02-01	7	positive
	2004-08-01	2008-11-01	2009-11-01	63	negative
	2010-02-01	2011-10-01	2013-05-01	39	negative
	2014-12-01	2016-01-01	2017-03-01	27	positive
	2018-05-01	2018-09-01	2019-04-01	11	negative
	2021-10-01	2022-01-01	2022-12-01	15	negative
	Start	Peak	End	Duration	Signal
$S_t^{CH/US-fN}$	2004-11-01	2005-06-01	2007-06-01	31	positive
	2009-08-01	2009-11-01	2010-03-01	7	negative
	2012-08-01	2016-06-01	2021-10-01	110	negative
South Africa (MD = 6)					
	Start	Peak	End	Duration	Signal
$S_t^{SA/US}$	2001-09-01	2001-12-01	2002-05-01	8	positive
	Start	Peak	End	Duration	Signal
$S_t^{SA/US-fN}$	2001-09-01	2001-12-01	2002-05-01	8	positive

Note: Datestamping with the GSADF tests. MD = Minimum duration of an explosive period.

During this period, the Brazilian exchange rate policy emphasized a dirty floating process using minidevaluations and maxidevaluations and a crawling peg system. They used the exchange rate as a nominal anchor for almost two years, sequentially employing high-interest rate levels to avoid speculative attacks. China, Figure 2, presented peculiar results combining positive and negative bubbles. We found positive bubbles lasting seven periods in 2002/2003, from 2014/2017, and 2004/2007, presenting twenty-seven and thirty-one months, respectively. During this phase, the Chinese exchange rate policy noted the use of a nominal anchor in a fixed rule perspective with an interest rate policy following American levels. Also, post-Great Recession event, they used the strategy of maintaining the lower level for the Yuan currency as a definitive strategy and changes in reserves levels as a tactic of exchange rate policy. The consequence was the occurrence of all negative bubbles, some with almost ten years of duration. These events caused a twin surplus for the Chinese economy during the period. South Africa's data results, Figure 3, presented two small

bubbles for exchange rate and non-tradable formulation, each lasting eight periods. This phase explains the result for a dirty floating regime in South African exchange rate policy.

4 Concluding Remarks

The economic power and geopolitical influence of the BRICS bloc worldwide pose intriguing questions in the present time. The focus on the New Development Bank and potential exchange rate policy arrangements is a key objective for the bloc, with a particular interest in the prospect of a common new currency or the utilization of existing currencies. This paper aims to provide a detailed account of the rational inception-expansion-bursting bubble of members to serve as a guide toward realizing such a perspective. The examination of individual occurrences and collapses of these bubbles is crucial for formulating a new arrangement. The results, as outlined by Maldonado et al. (2018), present varying perspectives. These authors identified periodically collapsing cointegration among BRICS, utilizing a different strategy and model-generated bubble results. A long run perspective of nominal exchange rate policy justifies a duration of 382 periods. In contrast to Maldonado et al. (2019), our findings suggest numerous bubble alternatives, with the exception of India and Russia, which did not exhibit bubbles. Brazil, China, and South Africa displayed bubbles with distinct characteristics and sizes. South Africa, for instance, experienced only two small bubbles at the beginning of the century. Brazil and China showcased the longest positive bubbles in the study, surpassing thirty years, related to exchange rate levels and nontradable factors. The results highlight significant differences in PPP between Brazil and its trade partners, with Brazil's balance of payments influencing exchange rate policy during events such as Oil Shocks, Interest Rate Shocks in the 1970s, and the Debt Crisis in the 1980s. The Brazilian commodity export-drive mechanism as the main instrument to promote economic growth explains those results. China's scenario was more intricate, featuring both positive and negative bubbles. The negative bubbles, lasting over three years, emerged after the Great Recession Crisis and contributed to the twin-surplus problem. Positive bubbles characterized China's exchange rate policy before the Great Recession, with a single positive bubble initiated in 2014 and concluding in 2017 due to considerations regarding a new exchange rate policy arrangement. The negative bubble made part of China's strategy to guarantee cheaper imported raw materials and, after this bubble, they changed the plan to promote exports. These scenarios underscore the complexity of guiding BRICS members in domestic exchange rate policy arrangements and shed light on the investigation into the use of a new currency or the utilization of individual currencies within this evolving economic and geopolitical bloc.

References

- AL-ANASWAH, N. AND B. WILFLING (2011): "Identification of speculative bubbles using state-space models with Markov-switching," *Journal of Banking & Finance*, 35, 1073–1086.
- BETTENDORF, T. AND W. CHEN (2013): "Are there bubbles in the Sterling-dollar exchange rate? New evidence from sequential ADF tests," *Economics Letters*, 120, 350–353.
- BRUNNERMEIER, M. K. (2016): "Bubbles," in *Banking Crises*, Springer, 28–36.
- CASPI, I., N. KATZKE, AND R. GUPTA (2015): "Date stamping historical periods of oil price explosivity: 1876–2014," *Energy Economics*, 48, 1–13.

- DIBA, B. T. AND H. I. GROSSMAN (1987): "On the inception of rational bubbles," *The Quarterly Journal of Economics*, 102, 697–700.
- ENGEL, C. (1999): "Accounting for US real exchange rate changes," *Journal of Political Economy*, 107, 507–538.
- ENGEL, C. AND K. D. WEST (2005): "Exchange rates and fundamentals," *Journal of Political Economy*, 113, 485–517.
- ETIENNE, X. L., S. H. IRWIN, AND P. GARCIA (2014): "Bubbles in food commodity markets: Four decades of evidence," *Journal of International Money and Finance*, 42, 129–155.
- EVANS, G. W. (1991): "Pitfalls in testing for explosive bubbles in asset prices," *The American Economic Review*, 81, 922–930.
- GUPTA, R., J. NEL, AND J. NIELSEN (2023): "US monetary policy and BRICS stock market bubbles," *Finance Research Letters*, 51, 103435.
- HAMILTON, J. D. (2009): "Causes and Consequences of the Oil Shock of 2007–08," *Brookings Papers on Economic Activity*, Spring, 215–261.
- HU, Y. AND L. OXLEY (2017): "Are there bubbles in exchange rates? Some new evidence from G10 and emerging market economies," *Economic Modelling*.
- JIANG, C., Y. WANG, T. CHANG, AND C.-W. SU (2015): "Are there bubbles in Chinese RMB–dollar exchange rate? Evidence from generalized sup ADF tests," *Applied Economics*, 47, 6120–6135.
- LEÓN-LEDESMA, M. AND A. MIHAILOV (2013): "Advanced International Macroeconomics and Finance," .
- MALDONADO, W. L., J. RIBEIRO, AND O. A. F. TOURINHO (2019): "Testing Four Types of Bubbles in BRICS Exchange Rates," *Emerging Markets Finance and Trade*, 57, 1–21.
- MALDONADO, W. L., O. A. TOURINHO, AND J. A. D. ABREU (2018): "Cointegrated Periodically Collapsing Bubbles in the Exchange Rate of BRICS," *Emerging Markets Finance and Trade*, 54, 54–70.
- MEHRA, R. AND E. C. PRESCOTT (1985): "The equity premium: A puzzle," *Journal of Monetary Economics*, 15, 145–161.
- OBSTFELD, M. AND K. ROGOFF (1996): *Foundations of International Macroeconomics*, vol. 30, MIT press Cambridge, MA.
- PHILLIPS, P. C., S. SHI, AND J. YU (2015): "Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500," *International Economic Review*, 56, 1043–1078.
- PHILLIPS, P. C. AND S.-P. SHI (2018): "Financial bubble implosion and reverse regression," *Econometric Theory*, 34, 705–753.

- PHILLIPS, P. C., Y. WU, AND J. YU (2011): “Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values?” *International Economic Review*, 52, 201–226.
- PHILLIPS, P. C. AND J. YU (2011): “Dating the timeline of financial bubbles during the subprime crisis,” *Quantitative Economics*, 2, 455–491.
- SAJI, T. G. (2019): “Can BRICS Form a Currency Union? An Analysis under Markov Regime-Switching Framework,” *Global Business Review*, 20, 151–165.
- ULLAH, I., S. M. HAIDER, AND M. GOHAR (2024): “BRICS Expansion: Prospects and Challenges,” *Research Journal of Human and Social Aspects*, 2, 52–64.
- VASILOPOULOS, K., E. PAVLIDIS, AND E. MARTÍNEZ-GARCÍA (2022): “exuber: Recursive Right-Tailed Unit Root Testing with R,” *Journal of Statistical Software*, 103, 1–26.
- ZHARIKOV, M. V. (2023): “Digital Money Options for the BRICS,” *International Journal of Financial Studies*, 11, 2–17.

A Appendix

Table III: Dates of the origination and collapse of the bubble and the bubble duration

Brazil (MD = 7; Lag = 0; Wild Bootstrap)					
	Start	Peak	End	Duration (in months)	Signal
$S_t^{BR/US}$	1964-10-01	1964-12-01	1965-10-01	12	positive
	1971-05-01	1971-09-01	1971-12-01	7	positive
	1975-08-01	1994-06-01	2008-11-01	399	positive
	2015-07-01	2015-09-01	2016-03-01	8	positive
$S_t^{BR/US-fN}$	1964-10-01	1964-12-01	1965-05-01	7	positive
	1969-09-01	1970-12-01	1972-02-01	29	positive
	1975-11-01	1994-06-01	2008-11-01	396	positive
	2009-10-01	2009-10-01	2010-07-01	9	positive
	2015-07-01	2015-09-01	2016-03-01	8	positive
	2020-02-01	2020-10-01	2022-03-01	25	positive
China (MD = 06; Lag = 0; Wild Bootstrap)					
$S_t^{CH/US-fT}$	2002-07-01	2002-10-01	2003-04-01	9	positive
	2004-07-01	2008-11-01	2009-11-01	64	negative
	2009-12-01	2016-01-01	2017-04-01	88	positive
	2018-03-01	2018-09-01	2020-05-01	26	negative
	2021-11-01	2022-01-01	2022-12-01	14	negative
$S_t^{CH/US-fN}$	2002-07-01	2002-10-01	2003-03-01	8	negative
	2004-10-01	2005-06-01	2007-08-01	34	positive
	2009-08-01	2009-11-01	2010-04-01	8	negative
	2012-05-01	2016-06-01	2021-10-01	113	negative
South Africa (MD = 06; Lag = 0; Wild Bootstrap)					
$S_t^{SA/US}$	1997-11-01	1998-07-01	1998-10-01	11	positive
	2000-10-01	2001-12-01	2002-05-01	19	positive
	2013-11-01	2014-01-01	2014-05-01	6	positive
	2014-06-01	2016-01-01	2016-04-01	22	positive
$S_t^{SA/US-fN}$	1997-07-01	1998-08-01	1998-10-01	15	positive
	2001-03-01	2001-12-01	2002-05-01	14	positive
	2015-01-01	2016-01-01	2016-09-01	20	positive

Note: Datestamping with the GSADF tests. MD = Minimum duration of an explosive period.

Table IV: Diagnostics

Brazil (Lag = 0; Wild Bootstrap)		
SADF	S:	Rejects H0 at the 1% significance level
	SRW:	Cannot reject H0
	SfT:	Cannot reject H0
	SfN:	Rejects H0 at the 1% significance level
GSADF	S:	Rejects H0 at the 1% significance level
	SRW:	Cannot reject H0
	SfT:	Cannot reject H0
	SfN:	Rejects H0 at the 1% significance level
China (Lag = 0; Wild Bootstrap)		
SADF	S:	Rejects H0 at the 5% significance level
	SRW:	Cannot reject H0
	SfT:	Rejects H0 at the 1% significance level
	SfN:	Rejects H0 at the 1% significance level
GSADF	S:	Cannot reject H0
	SRW:	Cannot reject H0
	SfT:	Rejects H0 at the 1% significance level
	SfN:	Rejects H0 at the 1% significance level
India (Lag = 0; Wild Bootstrap)		
SADF	S:	Cannot reject H0
	SRW:	Cannot reject H0
	SfT:	Rejects H0 at the 5% significance level
	SfN:	Cannot reject H0
GSADF	S:	Cannot reject H0
	SRW:	Cannot reject H0
	SfT:	Rejects H0 at the 10% significance level
	SfN:	Cannot reject H0
Russia (Lag = 0; Wild Bootstrap)		
SADF	S:	Cannot reject H0
	SRW:	Cannot reject H0
	SfT:	Cannot reject H0
	SfN:	Cannot reject H0
GSADF	S:	Cannot reject H0
	SRW:	Cannot reject H0
	SfT:	Cannot reject H0
	SfN:	Cannot reject H0
South Africa (Lag = 0; Wild Bootstrap)		
SADF	S:	Rejects H0 at the 5% significance level
	SRW:	Cannot reject H0
	SfT:	Rejects H0 at the 10% significance level
	SfN:	Rejects H0 at the 5% significance level
GSADF	S:	Rejects H0 at the 5% significance level
	SRW:	Cannot reject H0
	SfT:	Cannot reject H0
	SfN:	Rejects H0 at the 5% significance level

Note: CV - Critical Values obtained from 5000 bootstrap repetitions.